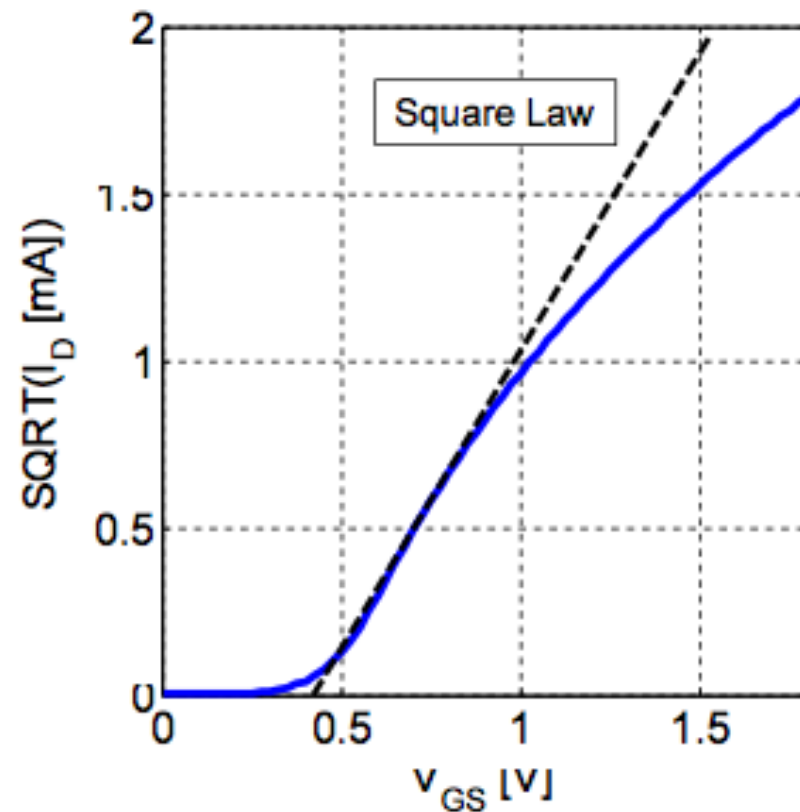
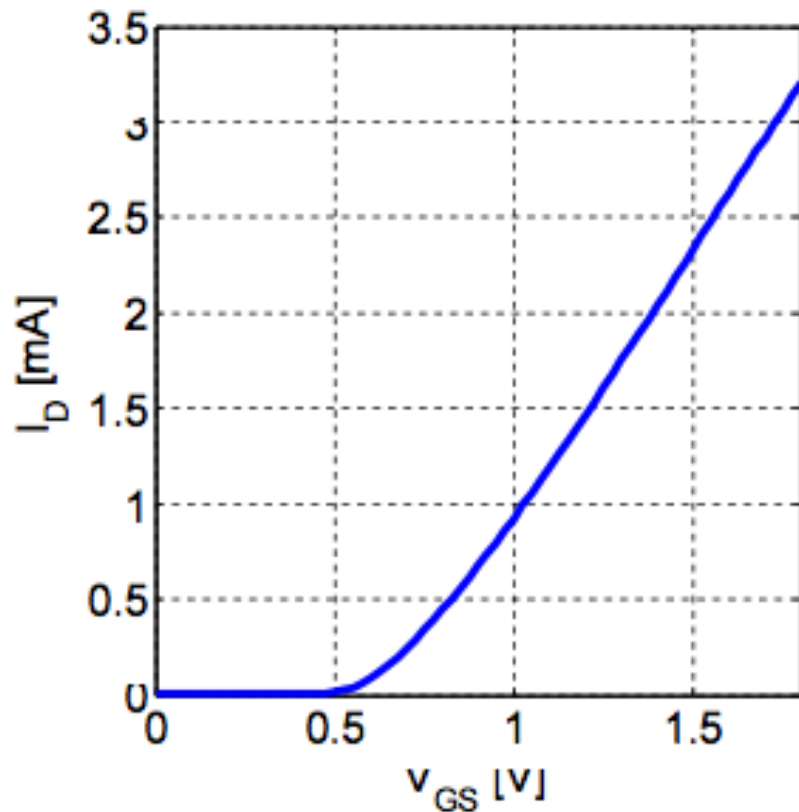


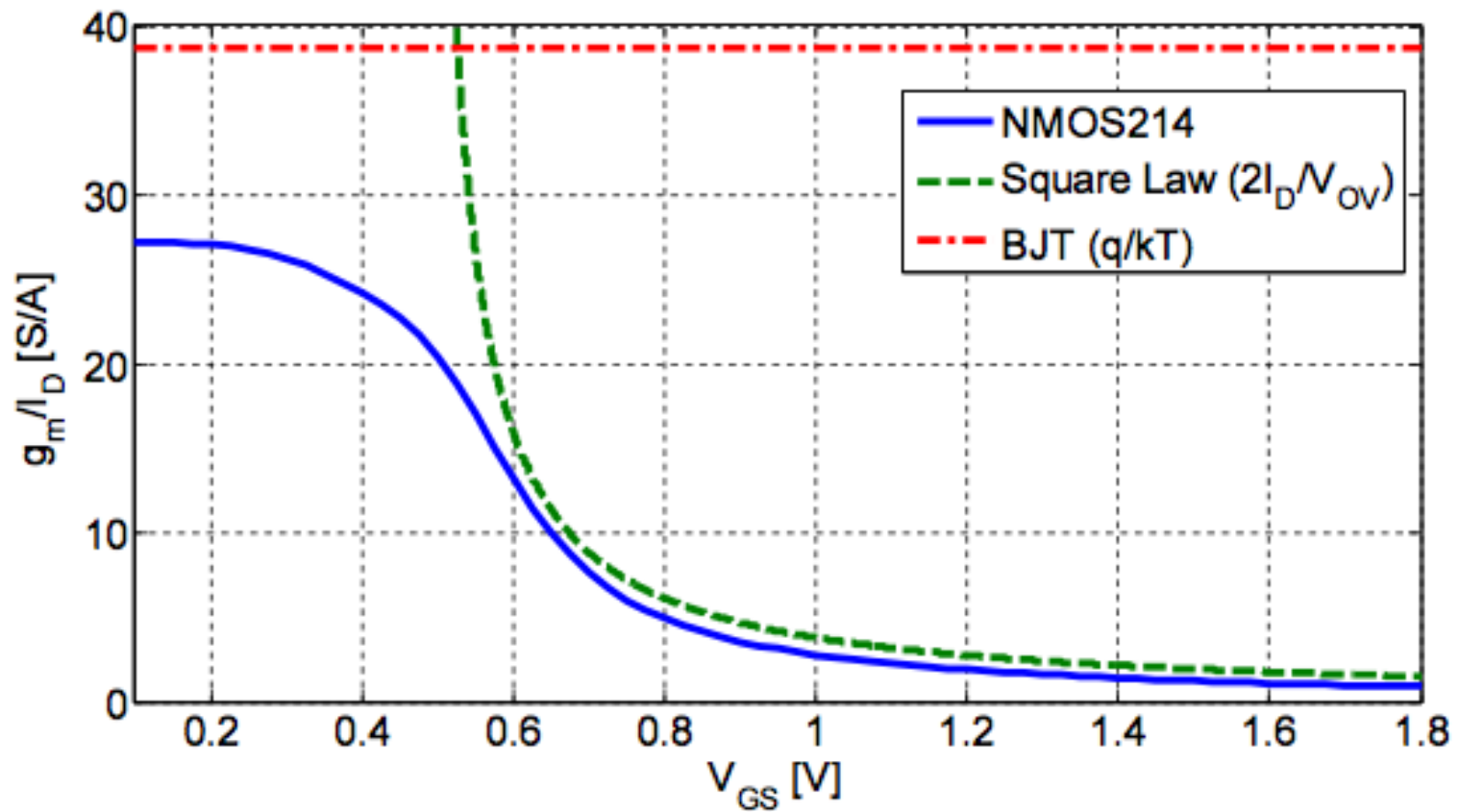
Short Channel MOSFET  
vs.  
Long Channel MOSFET

Issues with the Square Law model

## Simulation (NMOS, 5/0.18 $\mu\text{m}$ , $V_{DS}=1.8\text{V}$ )

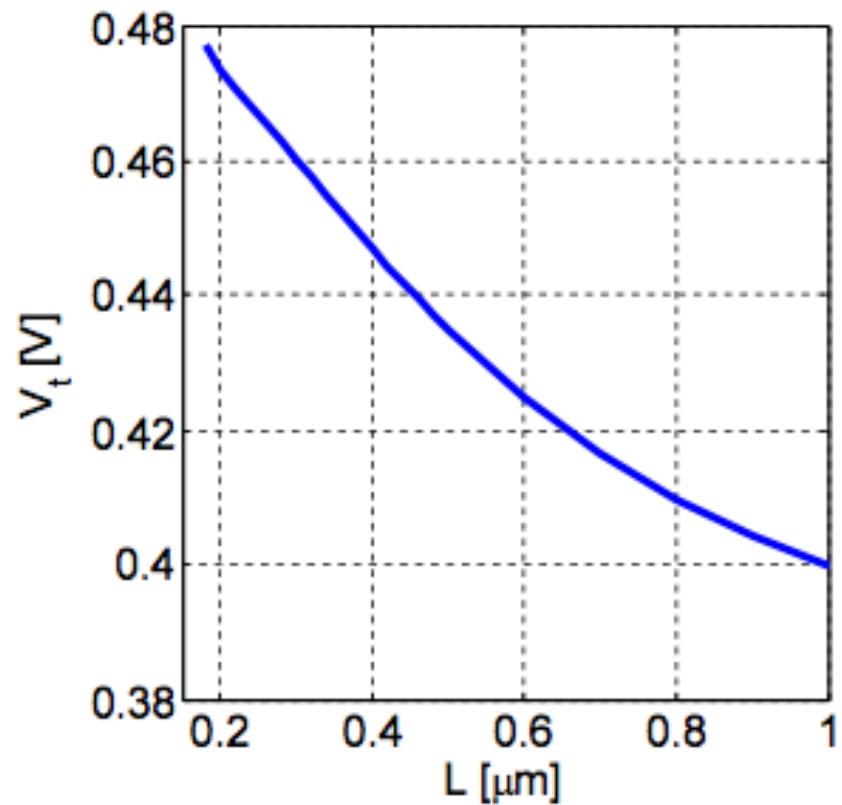
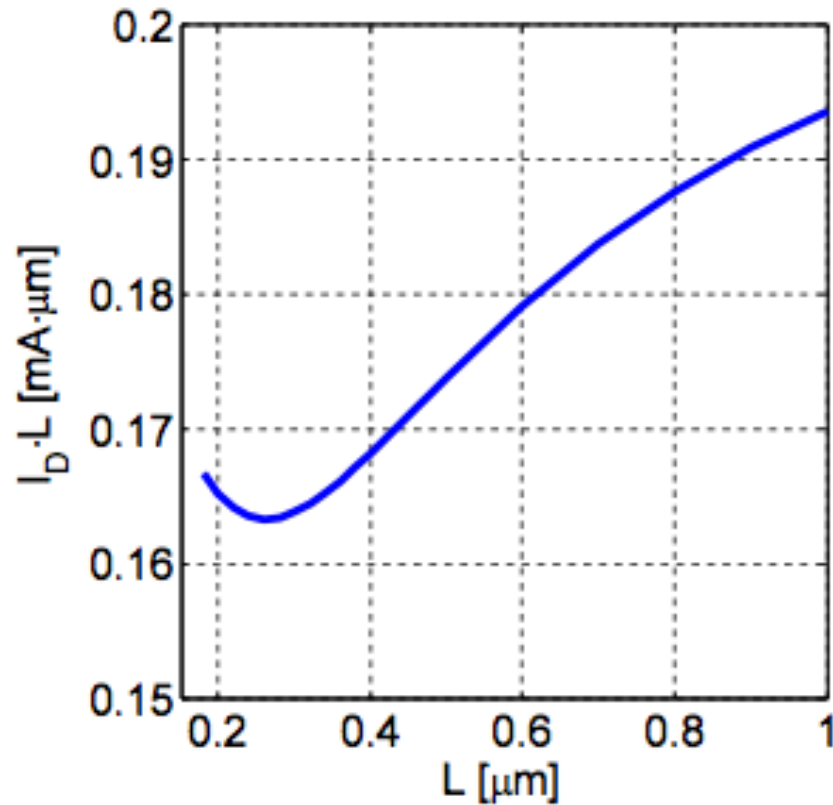


- Two observations
  - The transistor does not abruptly turn off at some  $V_t$
  - The current is not perfectly quadratic in  $(V_{GS}-V_t)$



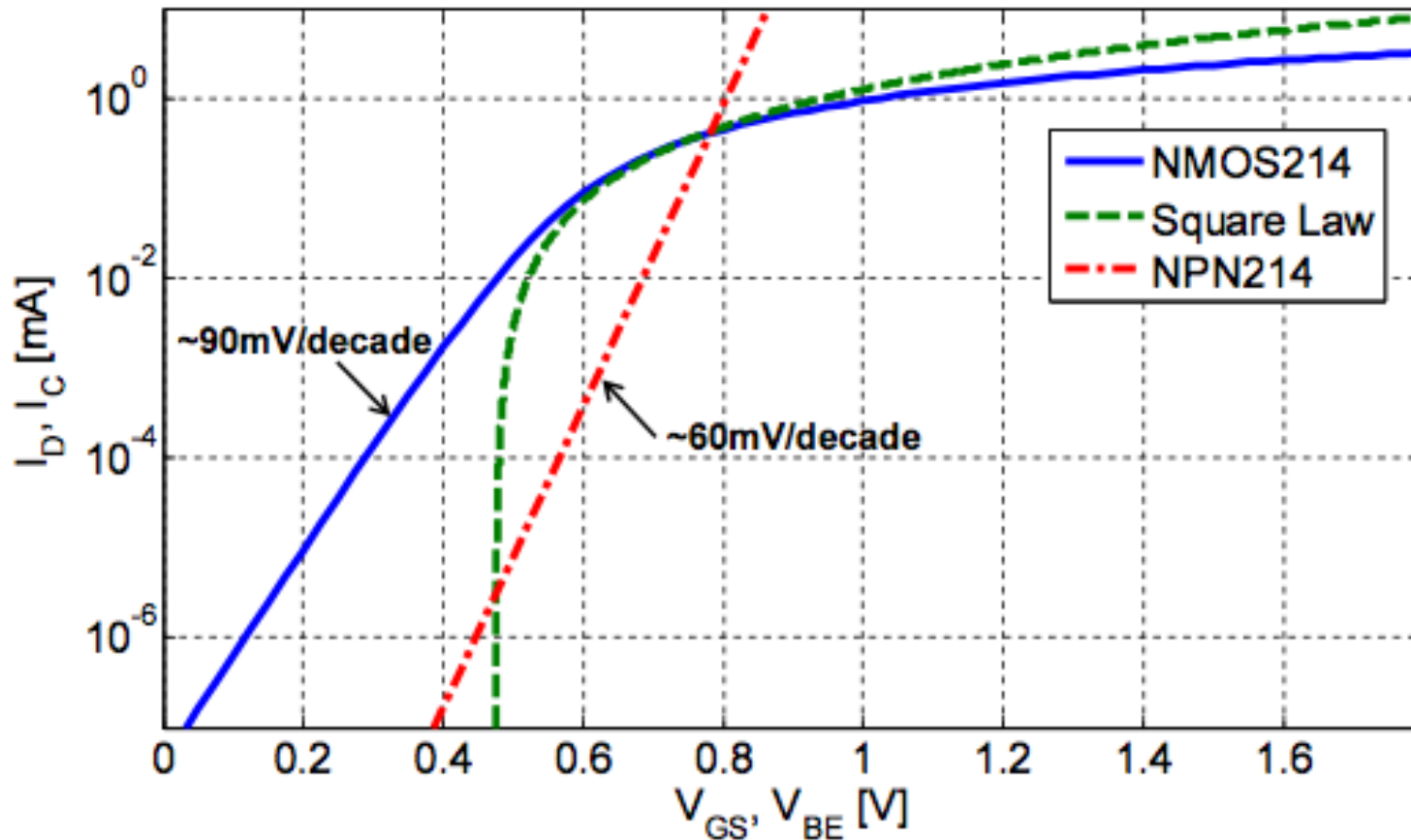
- The square law fails miserably at predicting  $g_m/I_D$  for low  $V_{GS}$

## Additional Issues



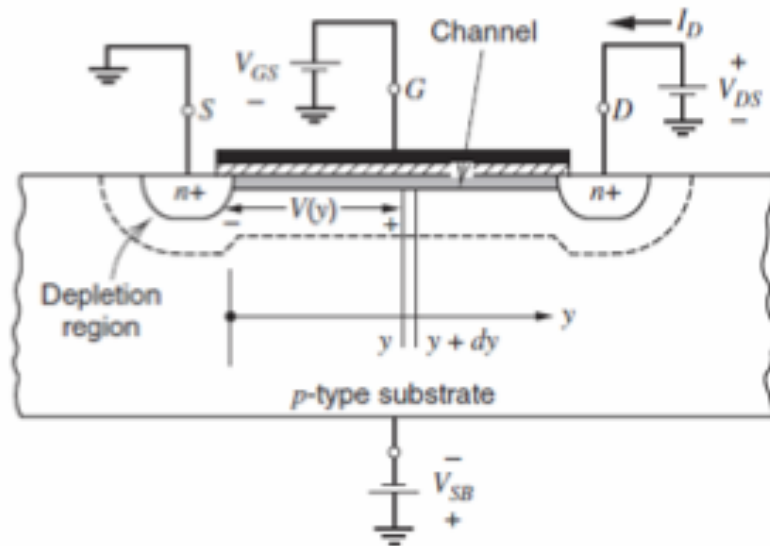
- The current does not scale perfectly with  $1/L$  ( $I_D \cdot L \neq \text{const.}$ )
- The threshold voltage of the device depends on the channel length

## Currents on a Log Scale



- What is  $V_t$ , anyway? The device does not turn off at all, but really approaches an exponential IV law for low  $V_{GS}$
- What determines the current at low  $V_{GS}$ ?

## Definition of $V_t$

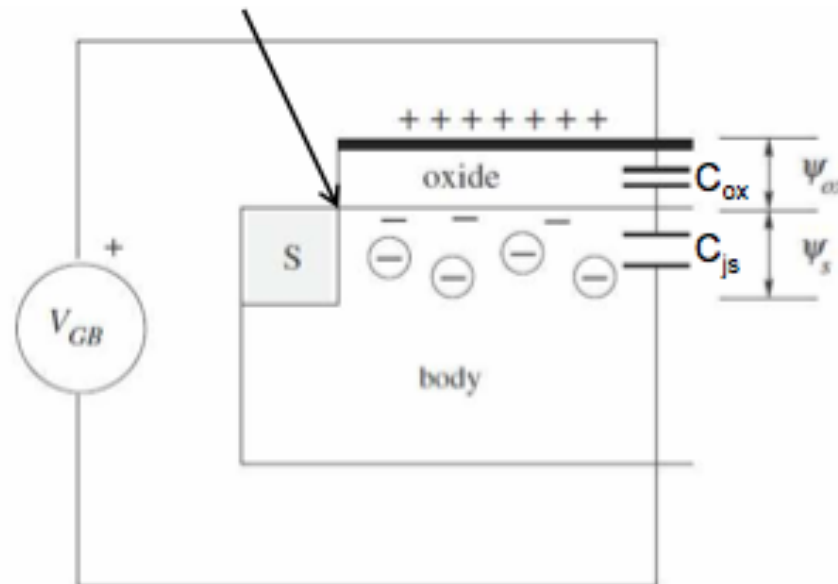


- $V_t$  is (roughly speaking) defined as the  $V_{GS}$  at which the number of electrons at the surface equals the number of doping atoms
- Seems arbitrary, but makes sense in terms of surface charge control
  - This is the point where the surface becomes inverted (no more holes to fill) and the relationship between mobile charge and gate voltage becomes linear,  $Q_n \propto C_{ox}(V_{GS}-V_t)$
  - Exactly what is assumed in the square law model

## Weak Inversion

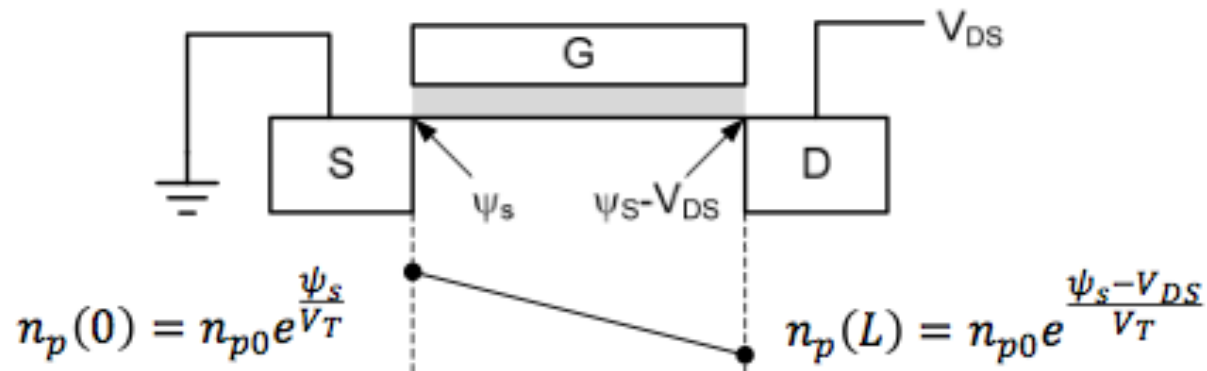
- Before inversion occurs, the electrostatic field from the gate forward-biases the source-side pn junction at the surface
- Physics governed by the “gated diode” model

Potential at this point is higher than body potential → forward bias



D.L. Pulfrey, *Understanding Modern Transistors and Diodes*,  
Cambridge University Press, 2010.

## Resulting Diffusion Current



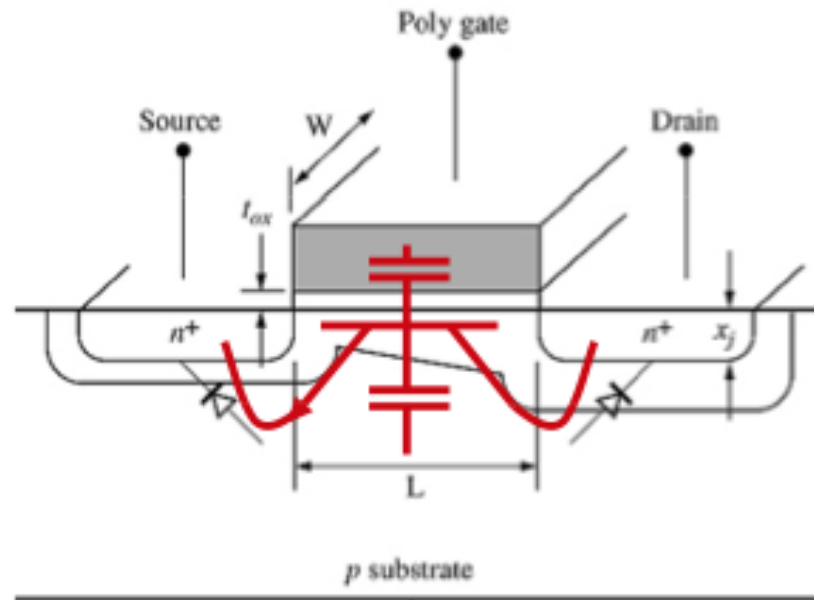
$$I_D = qAD_n \frac{n_p(L) - n_p(0)}{L}$$

$$I_D = qAD_n n_{p0} e^{\frac{\psi_s}{V_T}} (1 - e^{-\frac{V_{DS}}{V_T}})$$

- The current grows exponentially with  $\psi_s$
- The current becomes independent of  $V_{DS}$  for  $V_{DS} > 3V_T$  ( $\sim 78\text{mV}$ )



## BJT Similarity



- We have
  - An NPN sandwich, mobile minority carriers in the P region
- This is a BJT!
  - Except that the base potential is here controlled through a capacitive divider, and not directly by an electrode

## Capacitive Divider

$$\frac{d\psi_s}{dV_{GS}} = \frac{C_{ox}}{C_{js} + C_{ox}} = \frac{1}{n}$$

- $n$  is called “subthreshold factor” or “nonideality factor”
- $n \cong 1.45$  in the EE214B technology
- After including this relationship between  $\psi_s$  and  $V_{GS}$  and after a few additional manipulations, the final expression for the drain current becomes

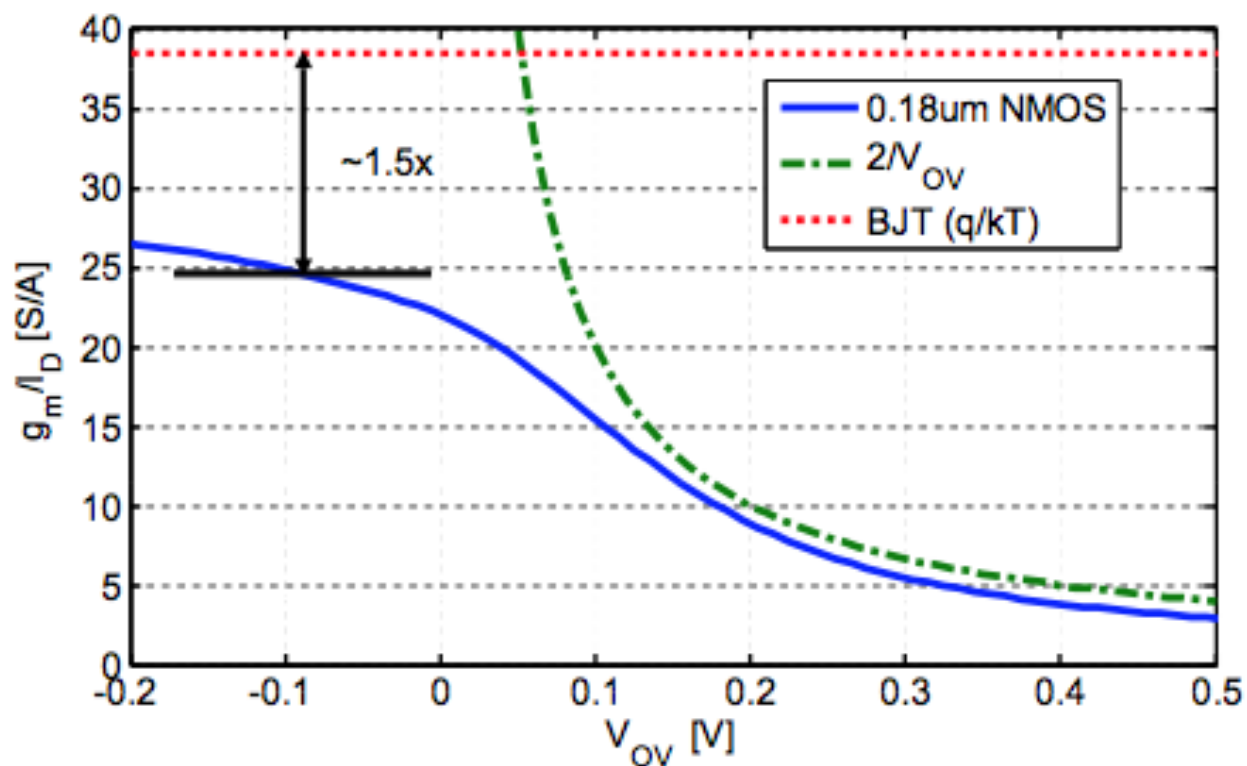
$$I_D = I_{D0} e^{\frac{V_{GS}-V_t}{V_t}} \left( 1 - e^{-\frac{V_{DS}}{V_t}} \right)$$

where  $I_{D0}$  depends on technology ( $I_{D0} \cong 0.43\mu\text{A}$  for EE214B technology)

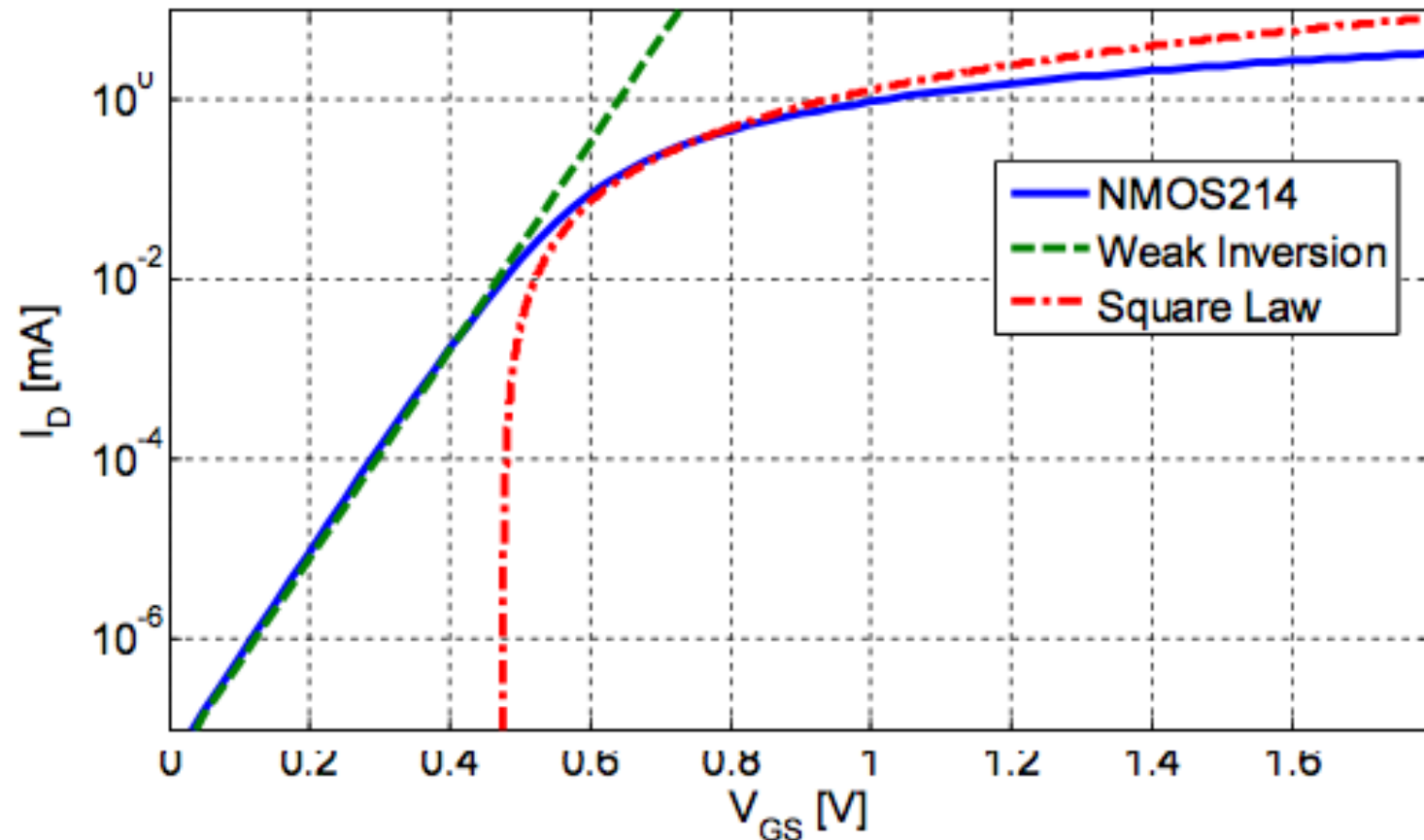
## Subthreshold Transconductance

$$g_m = \frac{dI_D}{dV_{GS}} = \frac{1}{n} \frac{I_D}{V_T} \qquad \frac{g_m}{I_D} = \frac{1}{nV_T}$$

- Similar to BJT, but unfortunately  $n$  ( $\cong 1.5$ ) times lower

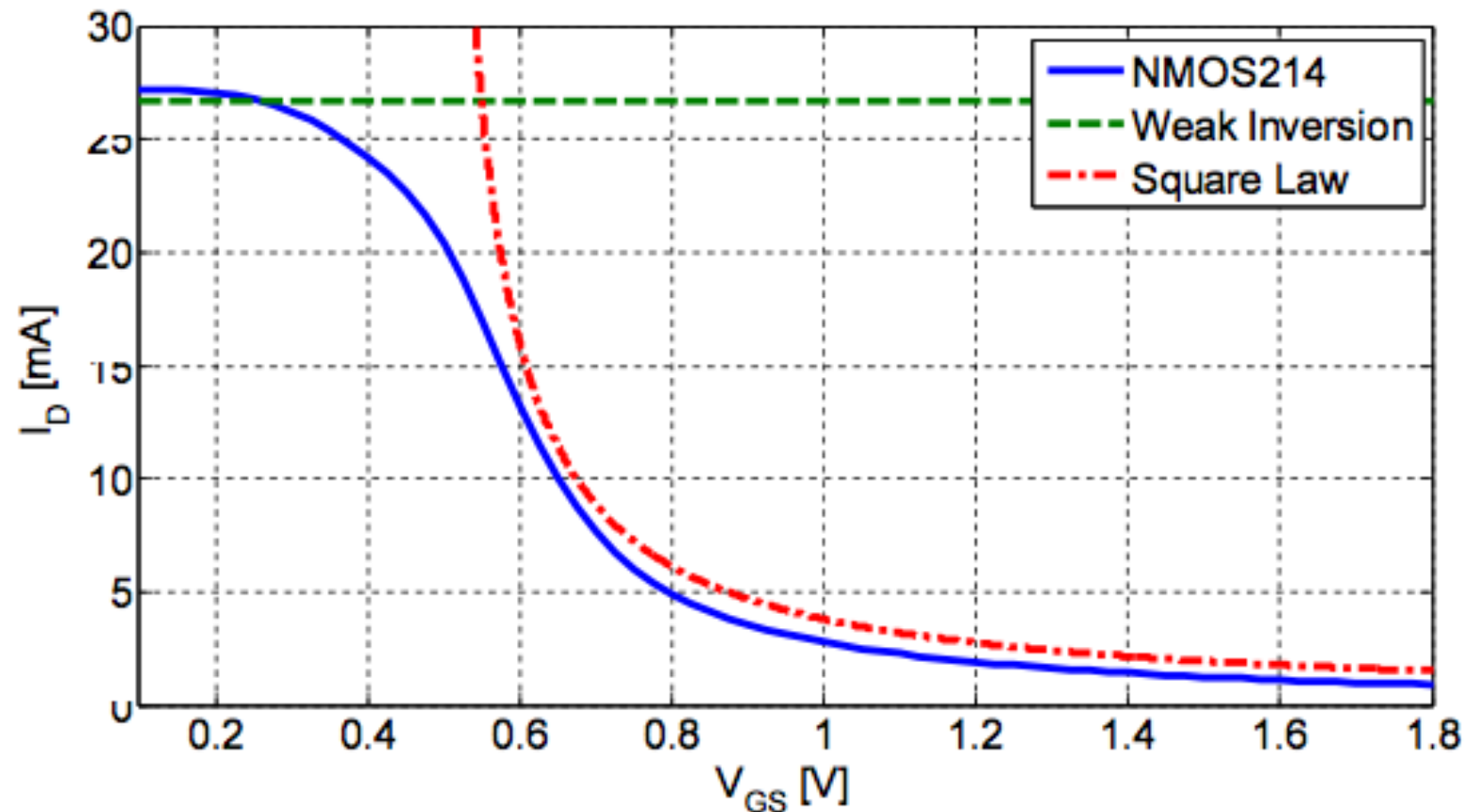


## Combining the Weak Inversion Expression and Square Law



- Two remaining problems
  - The weak inversion expression and square law are disconnected
  - We still do not know what causes the discrepancies at high  $V_{GS}$

$$g_m/I_D$$



- We now have a better idea now about the maximum possible  $g_m/I_D$ , but this does not help in the transistor region between the two IV laws

## Moderate Inversion

- In the transition region between weak and strong inversion, the drain current consists of both drift and diffusion currents
- One can show that the ratio of drift/diffusion current in moderate inversion and beyond is approximately  $(V_{GS}-V_t)/(kT/q)$
- This means that the square law equation (which assumes 100% drift current) does not work unless the gate overdrive is several  $kT/q$ 
  - Recall that in EE214A, you used the square law model only for  $V_{GS}-V_t > 150\text{mV} \cong 6 kT/q$

## Moderate Inversion

- In the transition region between subthreshold and strong inversion, we have two different current mechanisms

$$\text{Drift (MOS)} \quad v = \mu E$$

$$\text{Diffusion (BJT)} \quad v = D \frac{dn}{dx} = \frac{kT}{q} \mu \frac{dn}{dx}$$

- Both current components are always present
  - Neither one clearly dominates in moderate inversion
- Can show that ratio of drift/diffusion current  $\sim (V_{GS} - V_t)/(kT/q)$ 
  - MOS equation becomes dominant at several  $kT/q$

## Short Channel Effects

- The sub-square behavior at large  $V_{GS}$  is primarily due to a number of issues that fall under the category of “short channel effects”
- Onset of velocity saturation due to high lateral field
- Mobility degradation due to high vertical field
- Strong  $V_{DS}$  dependence of drain current and output resistance
- Threshold voltage depends on channel length and width
- Many more issues exist; we will once again only discuss the most relevant subset



## Figures of Merit for Design

- Transconductance efficiency
  - Want large  $g_m$ , for as little current as possible

$$\frac{g_m}{I_D}$$

$$= \frac{2}{V_{OV}}$$

- Transit frequency
  - Want large  $g_m$ , without large  $C_{gg}$

$$\frac{g_m}{C_{gg}}$$

$$\approx \frac{3 \mu V_{OV}}{2 L^2}$$

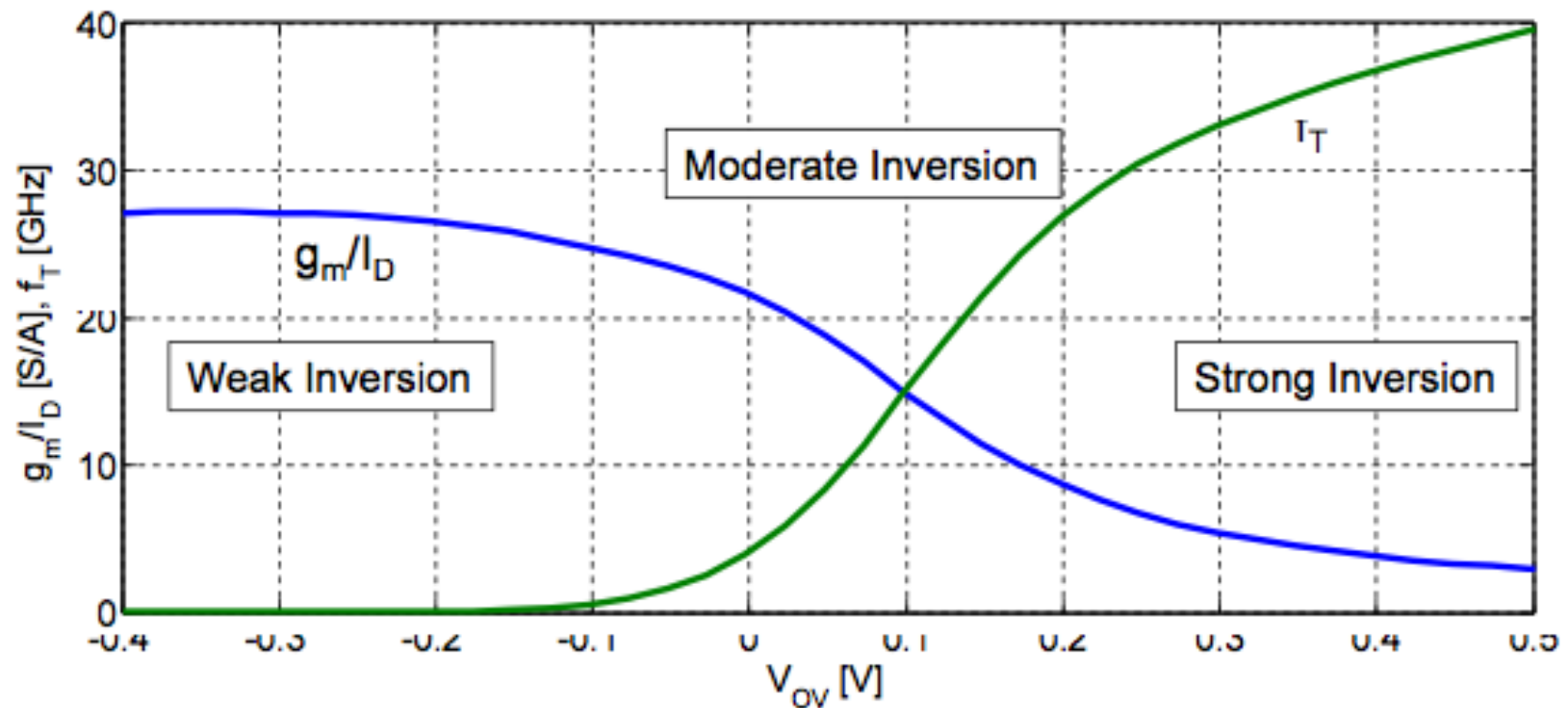
- Intrinsic gain
  - Want large  $g_m$ , but no  $g_o$

$$\frac{g_m}{g_o}$$

$$\approx \frac{2}{\Lambda V_{OV}}$$

### Square Law

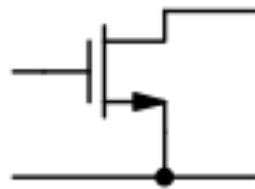
## Design Tradeoff: $g_m/I_D$ and $f_T$



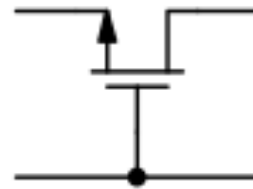
- Weak inversion: Large  $g_m/I_D$  ( $>20$  S/A), but small  $f_T$
- Strong inversion: Small  $g_m/I_D$  ( $<10$  S/A), but large  $f_T$

## Elementary Amplifier Configurations

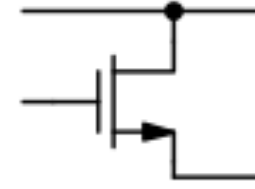
MOS



Common  
Source

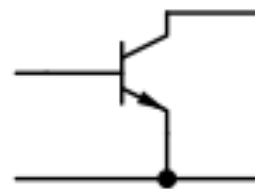


Common  
Gate

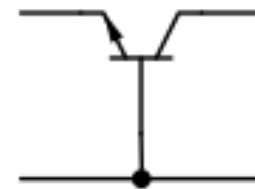


Common  
Drain

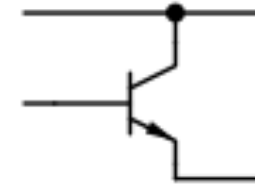
BJT



Common  
Emitter



Common  
Base



Common  
Collector

*Transconductance  
Stage*

*Current  
Buffer*

*Voltage  
Buffer*