Name: Solution

EE303 - Final Exam

Closed Book:
Two 8.5"x11" sheet of handwritten notes permitted
Calculator permitted

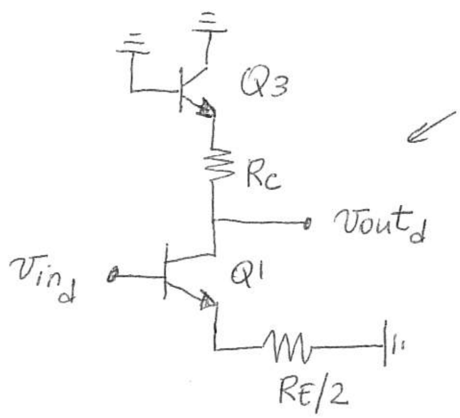
Important Notes:

- Read each problem completely and thoroughly
- Summarize all your answers in the boxes provided on these exam sheets
- Make sure to mark the units on your answers!
- Do all your work on the exams sheets provided. If you use any additional sheets, please turn them in, so we can consider all work for partial credit
- Do not forget to put your name in the space above

Problem #	Points	Score
1	10	
2	20	
3	20	
4	20	
5	20	
TOTAL	**90**	

solution - problem 1

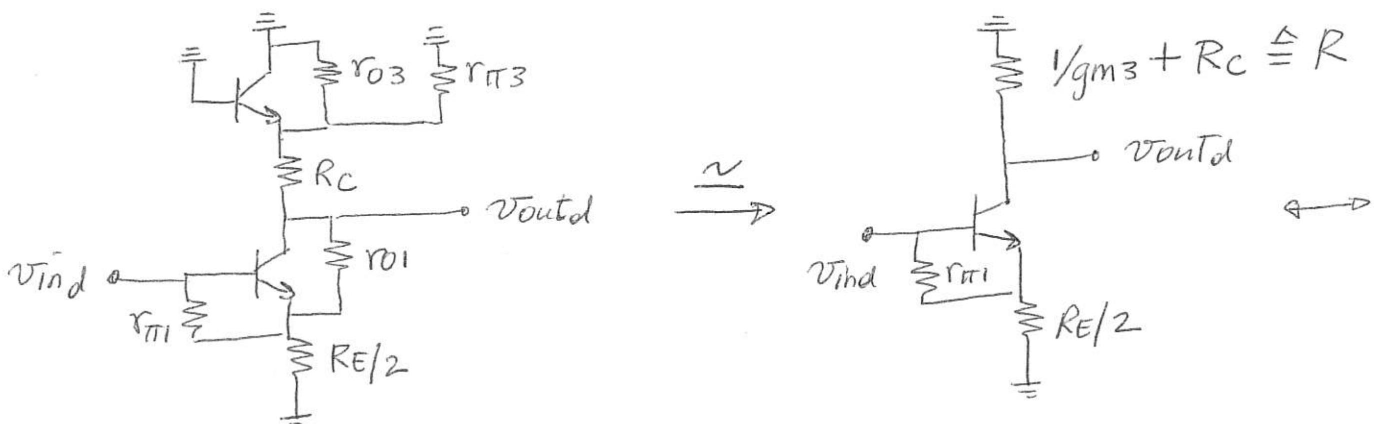
(b) differential half circuit

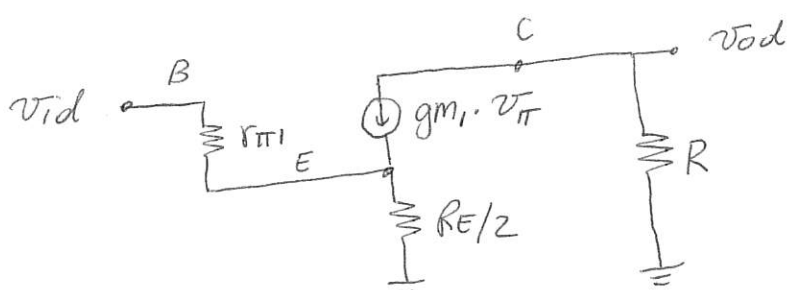


differential half circuit (AC)

(a) assume $r_o = \infty$ and $\beta = g_m \cdot r_\pi >> 1 \leftrightarrow r_\pi >> \frac{1}{g_m}$

$$\frac{v_{outd}}{v_{ind}} = \frac{-R}{\frac{1}{g_{m1}} + \frac{R_E}{2} + \frac{R_E}{2 \cdot \beta_1}} \underset{\beta >> 1}{\approx} \frac{-R}{\frac{1}{g_{m1}} + \frac{R_E}{2}}$$





$$v_{outd} = -g_{m1} \cdot v_\pi \cdot R$$

$$v_\pi = v_{id} - \frac{R_E}{2}\left(g_{m1} \cdot v_\pi + \frac{v_\pi}{r_{\pi 1}}\right)$$

$$v_\pi \left(1 + g_{m1}\frac{R_E}{2} + \frac{R_E}{2 r_{\pi 1}}\right) = v_{id}$$

$$v_{outd} = -g_{m1} \cdot R \cdot \frac{v_{id}}{1 + g_{m1}\frac{R_E}{2} + \frac{R_E}{2 \cdot r_{\pi 1}}}$$

Name:

Problem 1 [10 pts]

Consider the differential pair illustrated in Fig. 10.70. Assuming perfect symmetry and $V_A = \infty$, and $\beta >> 1$.

(a) Determine the voltage gain.

(b) Draw the differential AC half circuit

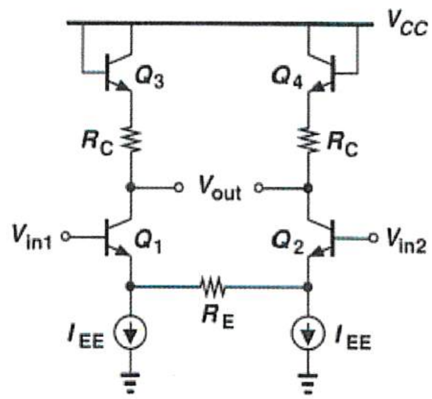
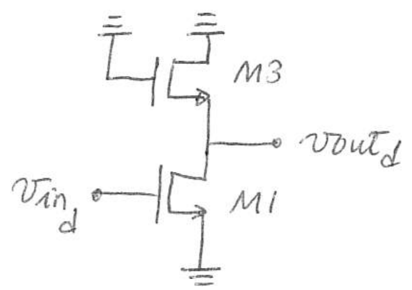


Figure 10.70

Vout/(Vin1−Vin2)	

solution - problem 2

(a)

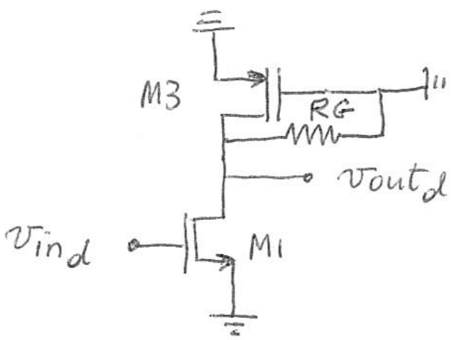


assume finite r_o

differential half AC circuit

$$\frac{v_{od}}{v_{id}} = -g_{m1} \cdot \left(r_{o1} \,||\, r_{o3} \,||\, \frac{1}{g_{m3}}\right)$$

(b) assume finite r_o



$R_G \triangleq R_1 = R_2$

$$\frac{v_{out_d}}{v_{in_d}} = -g_{m1}\left(r_{o1} \,||\, R_G \,||\, r_{o3}\right)$$

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Problem 2 [20 pts]

Calculate the differential voltage gain of the circuits depicted in Fig. 10.76. Assume perfect symmetry and $\lambda > 0$.

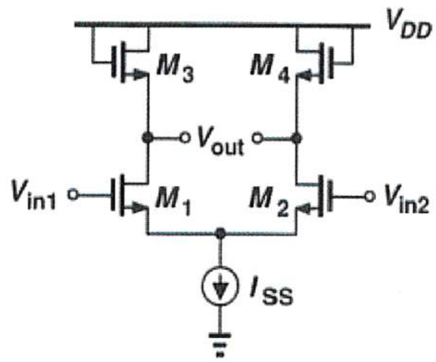


Fig. 10.76 (a)

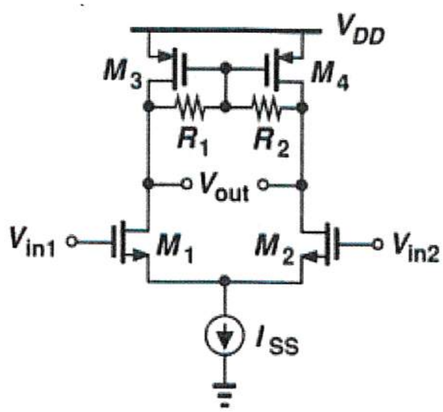
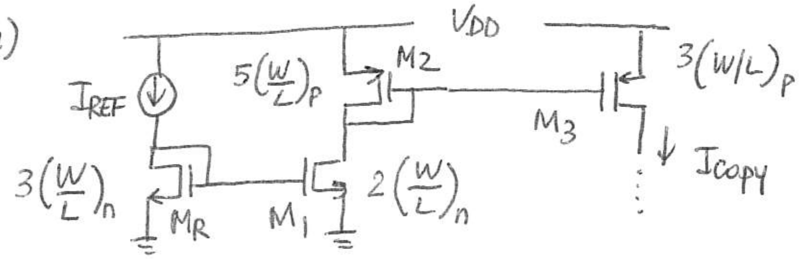


Fig. 10.76 (b)

(a) $v_{out}/(v_{in1}-v_{in2})$ =	
(b) $v_{out}/(v_{in1}-v_{in2})$ =	

solution problem 3

(a)

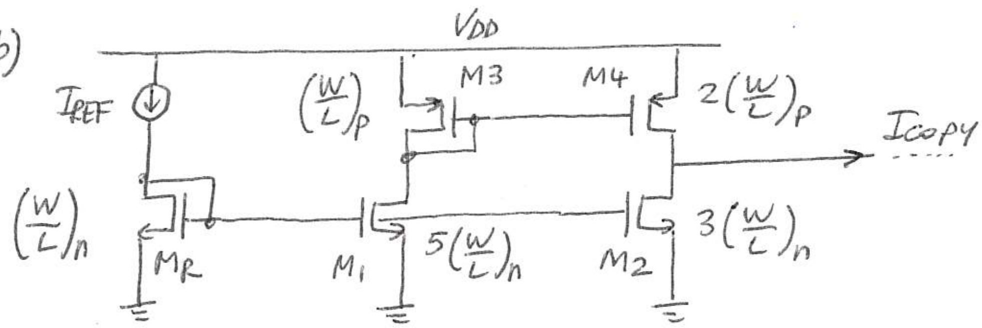


$V_{GS_R} = V_{GS_1} \rightarrow I_{D1} = \frac{2}{3} I_{REF}$ $\qquad I_{D1} = I_{D2}$

$V_{GS2} = V_{GS3} \rightarrow \frac{I_{copy}}{I_{D1}} = \frac{3(W/L)_p}{5(W/L)_p} = \frac{3}{5}$

$I_{copy} = \frac{3}{5} I_{D1} = \frac{3}{5} \cdot \frac{2}{3} I_{REF} = \frac{2}{5} I_{REF}$

(b)



$I_{copy} = I_{D4} - I_{D2}$

$I_{D1} = I_{D3} = I_{REF} \cdot 5 \leftarrow$ since: $V_{GS_R} = V_{GS1}$

$V_{GS3} = V_{GS4} \rightarrow \frac{I_{D4}}{I_{D3}} = \frac{2(W/L)_p}{(W/L)_p} \rightarrow I_{D4} = 2 I_{D3} =$
$= 10 I_{REF}$

$V_{GS_R} = V_{GS2} \rightarrow \frac{I_{D2}}{I_{REF}} = \frac{3(W/L)_n}{(W/L)_n} \rightarrow I_{D2} = 3 I_{REF}$

In conclusion:

$I_{copy} = 10 I_{REF} - 3 I_{REF} = 7 I_{REF}$

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Problem 3 [15 pts]

Calculate I_{copy} in each of the circuits shown in Fig. 9.71. Assume all of the transistors operate in saturation.

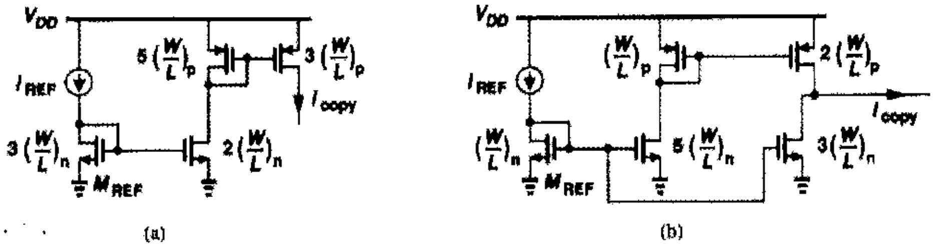


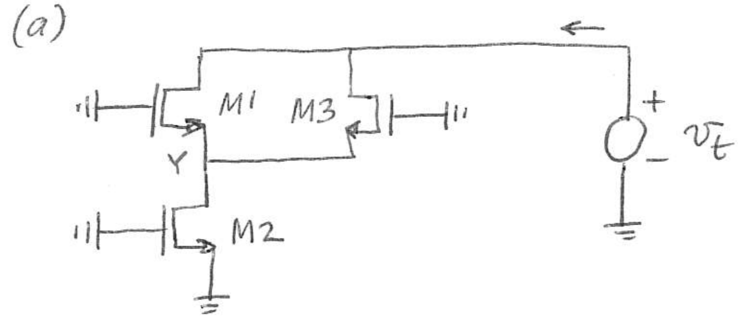
Figure 9.71

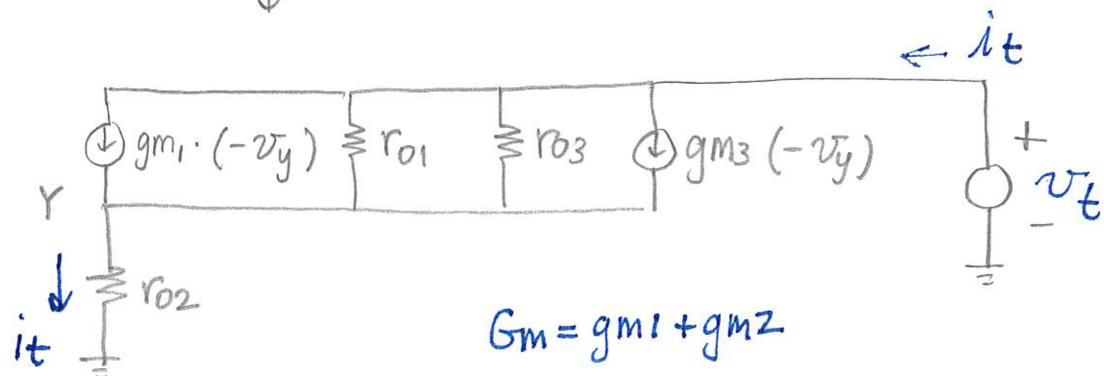
a) I_{copy} =	
b) I_{copy} =	

solution problem 4

assume $g_m r_o >> 1$

(a)





$G_m = g_{m1} + g_{m2}$

$$v_y = r_{o2} \cdot i_t$$

$$i_t = \frac{(v_t - v_y)}{r_{o1}} + \frac{(v_t - v_y)}{r_{o3}} - v_y\,(g_{m1} + g_{m3})$$

$$i_t = \frac{v_t}{r_{o1}} + \frac{v_t}{r_{o3}} - \frac{r_{o2}}{r_{o1}} i_t - \frac{r_{o2}}{r_{o3}} i_t - r_{o2}\, i_t \cdot G_m$$

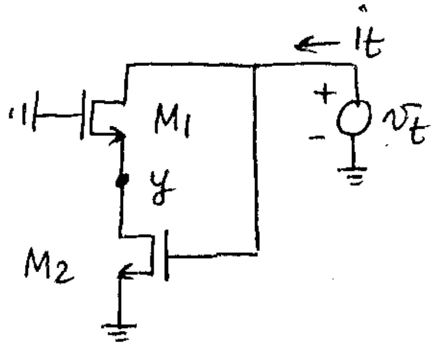
$$i_t \left(1 + \frac{r_{o2}}{r_{o1}} + \frac{r_{o2}}{r_{o3}} + r_{o2} G_m\right) = v_t \left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right)$$

$$\frac{v_t}{i_t} = (r_{o1} \| r_{o3}) \cdot \left[1 + r_{o2}/(r_{o1} \| r_{o3}) + r_{o2} G_m\right] =$$

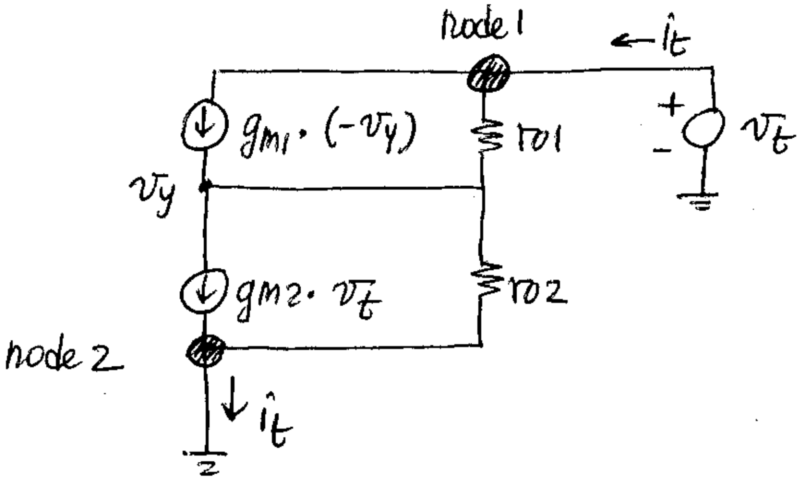
$$= (r_{o1} \| r_{o3}) + r_{o2} + r_{o2}\,(r_{o1} \| r_{o3})\, G_m =$$

$$\approx r_{o2}\,(r_{o1} \| r_{o3})\, G_m$$

(b)



$\updownarrow$



KCL at node 1 :

$$i_t = -g_{m1} v_y + \frac{v_t - v_y}{r_{o1}} \rightarrow$$

$$i_t = -g_{m1} v_y + \frac{v_t}{r_{o1}} - \frac{v_y}{r_{o1}} \rightarrow i_t = \frac{v_t}{r_{o1}} - v_y\left(g_{m1} + \frac{1}{r_{o1}}\right)$$

KCL at node 2 :

$$i_t = g_{m2} v_t + \frac{v_y}{r_{o2}} \rightarrow v_y = r_{o2}\left(i_t - g_{m2} v_t\right) \rightarrow$$

$$v_y = r_{o2}\cdot i_t - g_{m2} r_{o2}\cdot v_t$$

let's take the result from KCL at node 1 and right away assume $r_{o1} g_{m1} >> 1$:

$$i_t = \frac{v_t}{r_{o1}} - v_y \left(\frac{g_{m1} r_{o1} + 1}{r_{o1}} \right) \approx \frac{v_t}{r_{o1}} - v_y \, \frac{g_{m1} \cancel{r_{o1}}}{\cancel{r_{o1}}}$$

replace the value of v_y found applying KCL at node 2 :

$$i_t \approx \frac{v_t}{r_{o1}} - \frac{g_{m1} \cancel{r_{o1}}}{\cancel{r_{o1}}} \left[r_{o2} i_t - g_{m2} r_{o2} v_t \right] \rightarrow$$

$$i_t \approx \frac{v_t}{r_{o1}} - g_{m1} \cdot r_{o2} \cdot i_t + g_{m1} g_{m2} r_{o2} \cdot v_t \rightarrow$$

$$i_t \left(1 + g_{m1} r_{o2} \right) = v_t \left(\frac{1}{r_{o1}} + g_{m1} g_{m2} r_{o2} \right)$$

$$\frac{v_t}{i_t} \approx \frac{1 + g_{m1} r_{o2}}{\frac{1}{r_{o1}} + g_{m1} g_{m2} r_{o2}}$$

$$\frac{v_t}{i_t} \approx \frac{r_{o1} \left(1 + g_{m1} r_{o2} \right)}{1 + g_{m1} g_{m2} r_{o1} r_{o2}} \cong \quad g_m r_o >> 1$$

$$\cong \frac{r_{o1} \left(1 + g_{m1} r_{o2} \right)}{g_{m1} g_{m2} r_{o1} r_{o2}} = \frac{1 + g_{m1} r_{o2}}{g_{m1} g_{m2} r_{o2}} \approx$$

$$\approx \frac{g_{m1} \cdot r_{o2}}{g_{m1} g_{m2} r_{o2}} \approx \frac{1}{g_{m2}}$$

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Problem 4 [20 pts]

Compute the output resistance of the circuits depicted in Fig. 9.50. Assume all of the transistors operate in saturation and $g_m r_O \gg 1$.

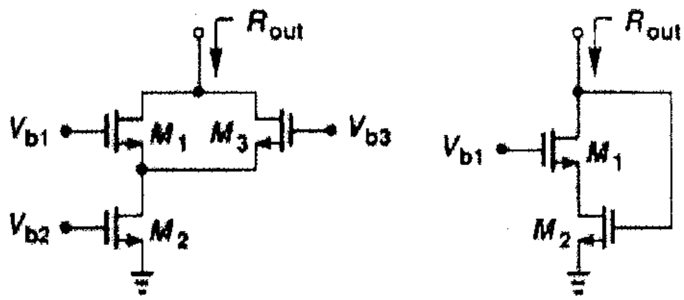
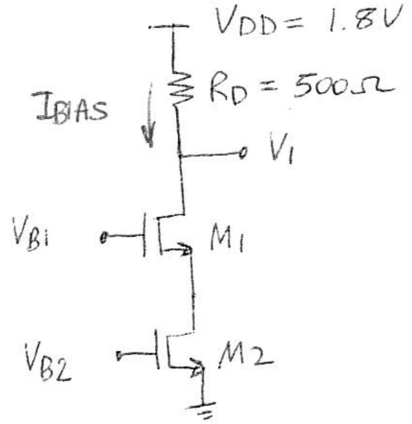


Fig 9.50 (a) Fig 9.50 (b)

a) R_{out} =	
b) R_{out} =	

solution problem 5



$$(W/L)_1 = \frac{20}{0.18}$$

$$(W/L)_2 = \frac{40}{0.18}$$

$$\mu C_{ox} = 100\ \mu A/V^2$$

$$V_{TH} = 0.4\ V$$

$$I_{BIAS} = 1mA$$

(a) highest allowable value of V_{B1}

$$I_{BIAS} = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)_2 V_{ov_2}^2$$

$$V_{ov_2}^2 = \frac{2\,I_{BIAS}}{\mu C_{ox}(W/L)_2} \rightarrow V_{ov_2} = \sqrt{\frac{2m \cdot 0.18}{100\mu \cdot 40}} =$$

$$\simeq 300\,mV$$

$$V_{B2} = V_{GS2} = V_{ov2} + V_{TH} = 700\,mV$$

$$V_1 = V_{DD} - R_D \cdot I_{BIAS} = 1.8 - 500 \cdot 1m = 1.3\,V$$

$$V_{ov1} = \sqrt{\frac{2\,I_{BIAS}}{\mu C_{ox}(W/L)_1}} = \sqrt{\frac{2m \cdot 0.18}{100\mu \cdot 20}} \simeq 424.26\,mV$$

$$V_{GS1} = V_{ov1} + V_{TH} = 824.26\,mV$$

$$\underbrace{V_1 - V_{S1}}_{V_{DS1}} \geq \underbrace{V_{GS1} - V_{TH}}_{V_{OV1}} = V_{B1} - \cancel{V_{S1}} - V_{TH}$$

$$V_{B1} \leq V_1 + V_{TH} = 1.3V + 0.4V = 1.7\,V$$

Name:

Problem 5 [10 pts]

Consider the circuit shown in Fig. 9.49, where $V_{DD} = 1.8$ V, $(W/L)_1 = 20/0.18$, and $(W/L)_2 = 40/0.18$. Assume $\mu_n C_{ox} = 100\ \mu A/V^2$ and $V_{TH} = 0.4$ V.

(a) If we require a bias current of 1 mA and $R_D = 500\ \Omega$, what is the highest allowable value of V_{b1}?

(b) With such a value chosen for V_{b1}, what is the value of V_X?

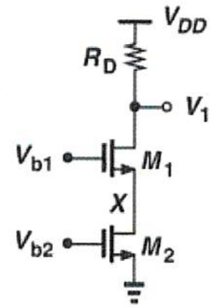


Figure 9.49

(a) V_{b1}(max) =	
(b) V_X =	

(b) $V_{B1} = 1.7V$

$$I_{BIAS} = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)_1 \left(\underbrace{\underbrace{V_{B1} - V_X}_{V_{GS1}} - V_{TH}}_{V_{OV1}}\right)^2$$

$$V_{B1} - V_X - V_{TH} = V_{OV1}$$

$$V_X = V_{B1} - V_{OV1} - V_{TH} = 1.7 - 424m - 0.4 \cong 0.88\,V$$