

Name: Fall 2017 - Solution

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*EE303 - Final Exam*

**Closed Book:**

Two 8.5"x11" sheet of handwritten notes permitted  
Calculator permitted

**Important Notes:**

- Read each problem completely and thoroughly
- Summarize all your answers in the boxes provided on these exam sheets
- Make sure to mark the units on your answers!
- Do all your work on the exams sheets provided. If you use any additional sheets, please turn them in, so we can consider all work for partial credit
- Do not forget to put your name in the space above

<b>Problem #</b>	<b>Points</b>	<b>Score</b>
1	20	
2	20	
3	20	
4	20	
<b>TOTAL</b>	<b>80</b>	

In the following problems unless otherwise stated, always assume:

$\mu_n C_{ox} = 200 \mu\text{A}/\text{V}^2$ ,  $\mu_p C_{ox} = 100 \mu\text{A}/\text{V}^2$ ,  $\lambda = 0 \text{ V}^{-1}$ ,  $V_{TH} = 0.4 \text{ V}$  for NMOS devices and  $V_{TH} = -0.4 \text{ V}$  for PMOS devices.

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Problem 1 [20 pts=5,10,5]

Consider the circuit of figure 1 with  $R_{D1} \neq R_{D2}$  (i.e. the node P is not at AC ground). Assume the transistors have the same  $g_m$  and channel length modulation is negligible. Calculate the following voltage gains:  $v_x/v_{in}$ ,  $v_p/v_{in}$ ,  $v_y/v_p$

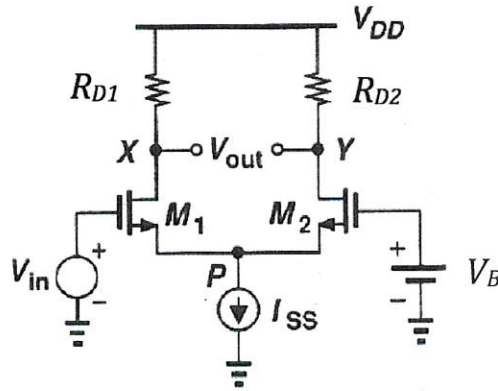
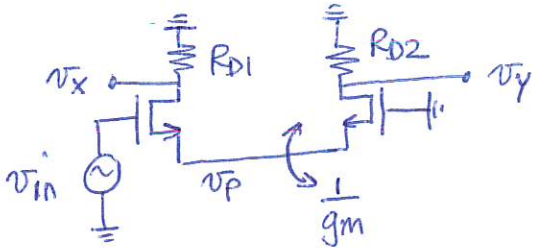


Figure 1

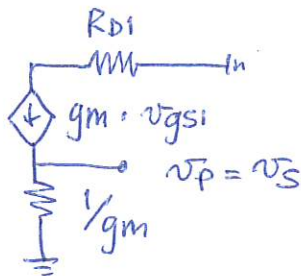
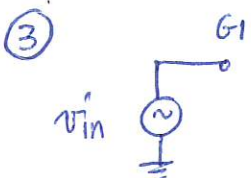
$v_x/v_{in} =$	
$v_p/v_{in} =$	
$v_y/v_p =$	



①  $\frac{v_x}{v_{in}} = - \frac{g_m R_{D1}}{1 + g_m \frac{1}{g_m}} = - \frac{g_m}{2} R_{D1}$

common source with degeneration  $1/g_m$

②  $\frac{v_y}{v_p} = g_m R_{D2} \leftarrow \text{common gate}$



source follower

$$v_{in}' = \cancel{v_{gs1}} v_{gs1} + v_p = \cancel{v_{gs1}} v_{gs1} + \frac{1}{g_m} \cdot g_m v_{gs1} = 2 v_{gs1}$$

$$v_p = \frac{1}{g_m} \cdot g_m v_{gs1} = \frac{1}{g_m} \cdot g_m \cdot \frac{v_{in}'}{2}$$

$$\rightarrow \frac{v_p}{v_{in}'} = \frac{1}{2}$$

↓ out of curiosity

$$\frac{v_y}{v_{in}'} = \frac{v_y}{v_p} \cdot \frac{v_p}{v_{in}'} = \frac{g_m R_{D2}}{2}$$

$$\frac{v_y - v_x}{v_{in}'} = A_v = \frac{g_m}{2} (R_{D1} + R_{D2})$$

Name: \_\_\_\_\_

Problem 2 [20 pts=5,10,5]

Calculate the differential voltage gain of the circuits depicted in Figure 2.  $V_b$  is a DC bias voltage. Assume perfect symmetry, transistors having the same  $g_m$  and  $r_o$ , and  $g_m r_o \gg 1$ . State clearly any assumption you make.

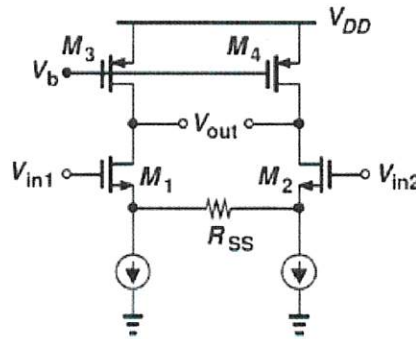
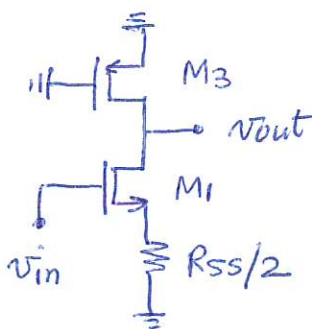


Figure 2

$$A_{dm} = \frac{v_{out2} - v_{out1}}{v_{in1} - v_{in2}}$$

$R_{out} =$	
$G_m =$	
$A_{dm} = -G_m R_{out}$	



$M_1 + R_{ss}/2$  is a "super" transistor with:

$$G_m = \frac{g_{m1}}{1 + g_{m1} \frac{R_{ss}}{2}} \quad \text{and} \quad R_o \approx g_{m1} r_{o1} \frac{R_{ss}}{2}$$

Therefore:

$$A_v = -G_m \cdot \underbrace{(R_o \parallel r_{o3})}_{= R_{out}}$$

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Problem 3 [20 pts]

Determine the output impedance of the circuits shown in Figure 3. Assume  $\lambda \neq 0$ .

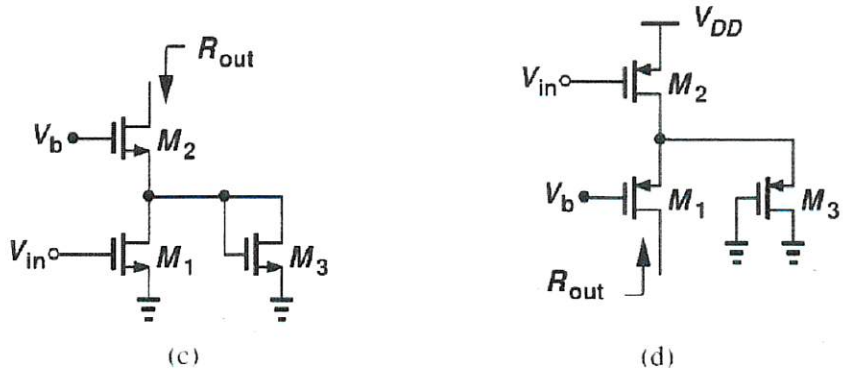
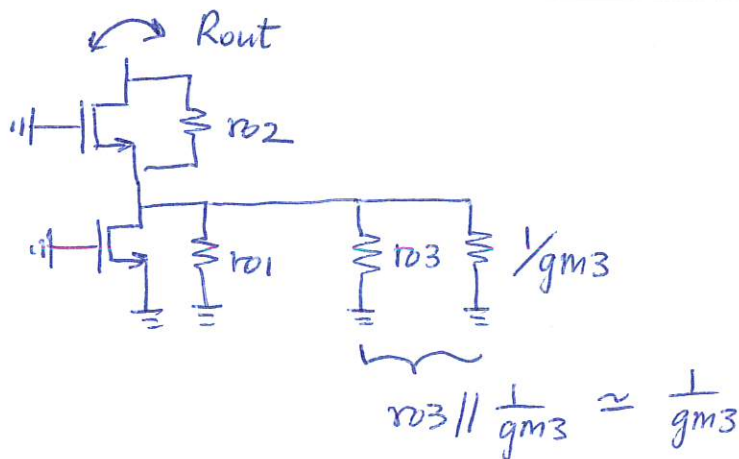


Figure 3

$V_b$  is a DC bias voltage.

c. $R_{out} =$	
d. $R_{out} =$	

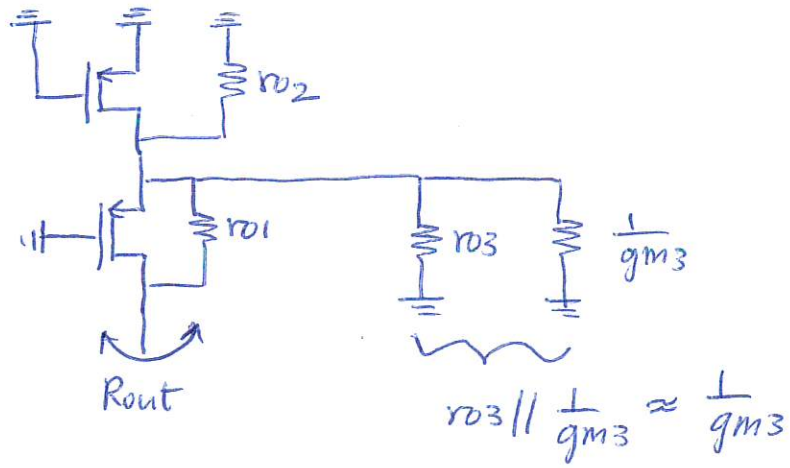
(c.)



$$R_{out} \approx g_{m2} r_{o2} \cdot R_s$$

$$\text{with } R_s = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m3}}$$

d)



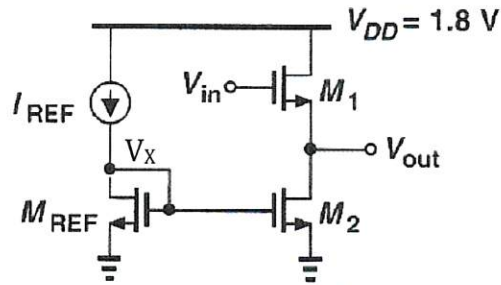
$$R_{out} \approx g_{m1} r_{o1} R_S$$

$$\text{with } R_S = r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m3}}$$

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Problem 4 [20 pts]

The source follower of figure 4 must achieve a voltage gain of 0.85 and an output impedance of 100  $\Omega$ . Assuming  $V_X = 1V$ ,  $\lambda_n = 0.1 V^{-1}$ ,  $I_{REF} = 0.1 I_{D1}$  design the circuit. (Hint:  $A_v = g_{m1} R_{out}$ )



$\mu_n C_{ox} = 200 \mu A/V^2$   
 $V_{TH} = 0.4V$

Figure 4.

$g_{m1} =$	
$r_{o1} = r_{o2} = r_o =$	
$I_{D1} = I_{D2} = I_D =$	
$(W/L)_2 =$	
$(W/L)_{REF} =$	

Since the <sup>DC</sup> current flowing through M1 and M2 is the same  
 $r_{o1} = r_{o2} \cong \frac{1}{\lambda I_D} \triangleq r_o$

$$R_{out} = r_{o2} \parallel (r_{o1} \parallel \frac{1}{g_{m1}}) = \frac{1}{\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m1}} =$$

$$= \frac{1}{g_{m1} + \frac{2}{r_o}}$$

$$A_v = g_{m1} \cdot R_{out} \rightarrow g_{m1} = \frac{A_v}{R_{out}} = \frac{0.85}{100} = 8.5 \text{ mS}$$

$$R_{out} = \frac{1}{g_{m1} + \frac{2}{r_o}} \iff \frac{2}{r_o} R_{out} + g_{m1} R_{out} = 1 \iff$$

$$\frac{1}{r_o} = \frac{1 - g_{m1} R_{out}}{2 R_{out}} \implies$$

$$r_o = \frac{2 R_{out}}{1 - g_{m1} R_{out}} = \frac{200}{1 - 0.85} \approx 1333 \Omega$$

$$I_D \approx \frac{1}{\lambda_n r_o} = 7.5 \text{ mA}$$

$$\left(\frac{W}{L}\right)_2 = \frac{2 \cdot I_D}{\mu_n C_{ox} (V_x - V_{TH})^2} \approx 208$$

$$I_{REF} = 0.1 \times I_D = 0.75 \text{ mA}$$

$$\left(\frac{W}{L}\right)_{REF} = \left(\frac{W}{L}\right)_2 \cdot \frac{I_{REF}}{I_D} = \frac{208}{10} \approx 21$$