EE 304 Electronics II

Lecture 9

# Oscillators

Plan:

- conditions under which sustained oscillations are obtained
- frequency of the oscillations
- circuits for implementing sine-wave oscillators = linear oscillators
  - op-amp RC oscillators
    - Wien-Bridge oscillator
    - Phase-Shift oscillator
    - (active-filter-tuned oscillator)
  - LC and crystal oscillators
- non-linear circuits to control the amplitude of the sine-wave
- circuits for implementing triangular and square wave form oscillators
  - multivibrators
  - 555 timer ← example
  - (ring oscillators)

TO DO AY 2021 →

- LC oscillators
  S&S 7/e 18.3 pp. 1396–1404
  Razavi (AIC) 14.3 pp. 495–509
  Horenstein - pp. 851–855
- Ring Oscillators
  Razavi (AIC) - 14.2 pp. 484–495
  J. Baker 4/e pp. 354–356,
- original HP oscillator circuit

## General Considerations

- In the design of electronic systems (e.g. communication systems) there is a frequent need for signals that have standard shapes
  - sinusoidal
  - square
  - triangular
  - pulse

- Types of oscillators
  - → Linear oscillators (waveform generated: - sinusoid)
  - → Non Linear oscillators (a.k.a. function generator) (waveforms generated: - square - triangular - pulse)

Basic Building Block for linear oscillators
↓
Positive FB loop ≡ amplifier and an RC or LC frequency selective feedback network

Basic Building Block for non linear oscillators
↓
multivibrator circuit
- bistable
- astable
- monostable

It is also possible to build sinusoidal waveforms by using a function generator (that produces a square or triangular waveform) followed by a waveform shaping circuit (= filter)

{function generator + waveform shaping circuit} ⟷ It provides a huge number of possibilities

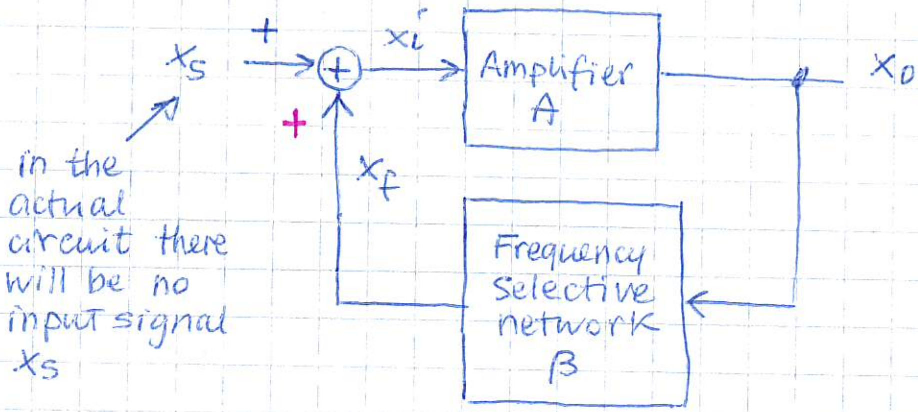
↓

example: 555 one of the most popular IC chips of all time

## Basic Principles behind oscillation mechanism

- an oscillator is a FB circuit in which the stability condition is intentionally violated as a means of creating oscillation

- The basic structure of an oscillator consists of an amplifier and a frequency-selective network connected in a positive FB loop




$$A_f = \frac{A(s)}{1 - A(s)\beta(s)} = \frac{A(s)}{1 - L(s)}$$

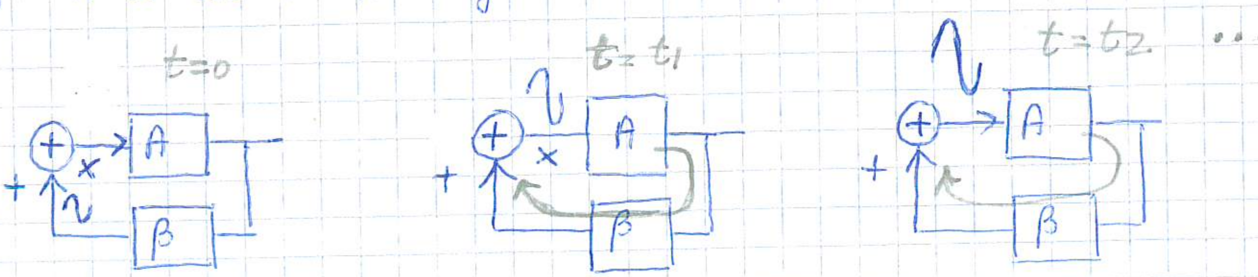
$$L(s) \equiv A(s)\beta(s) \equiv \text{loop gain}$$

- The oscillation criterion ≡ Barkhausen criterion

If at a specific frequency $\omega_0$ the loop-gain $L = A \cdot \beta$ is equal to unity $\Rightarrow$ $A_f$ will be $\infty$, that is at the frequency $\omega_0$ the circuit will have a finite output for zero input signal $\Leftrightarrow$ the circuit oscillates

$$\boxed{\begin{array}{c} L(j\omega_0) = A(j\omega_0)\beta(j\omega_0) = 1 \\ \Updownarrow \\ |L(j\omega_0)| = 1 \\ \angle L(j\omega_0) = 0 \end{array}} \leftarrow \text{Barkhausen criterion}$$

Under the Barkhausen condition the circuit amplifies its own noise component at $\omega_0$ indefinitely




evolution of oscillatory system with time

added to what comes out of it as it goes through the system

$$|V_x| = |V_N| + \underbrace{|V_N|\,|L(j\omega_0)| + |V_N|\,|L(j\omega_0)|^2 + |V_N|\,|L(j\omega_0)|^3 + \dots}$$

initial noise component (at $\omega_0$)

the larger the signal the more evident is the N.L. behavior of any system, but → In practice all systems are N.L.

$$\frac{|V_x|}{|V_N|} = 1 + \sum_{K=1}^{\infty} x^K = \sum_{K=0}^{\infty} x^K$$

If $|L(j\omega_0)| \geq 1$ the above summation diverges, whereas if $|L(j\omega_0)| < 1$ the summation converge to a finite value:

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{for } |x| < 1$$

In practice, in order to ensure oscillation in the presence of temperature and process variation, we typically chose the magnitude of the loop-gain ($\rightarrow |L(j\omega_0)|$) to be $2\times \div 3\times$ the required value ($|L(j\omega_0)| = 1$)

↓

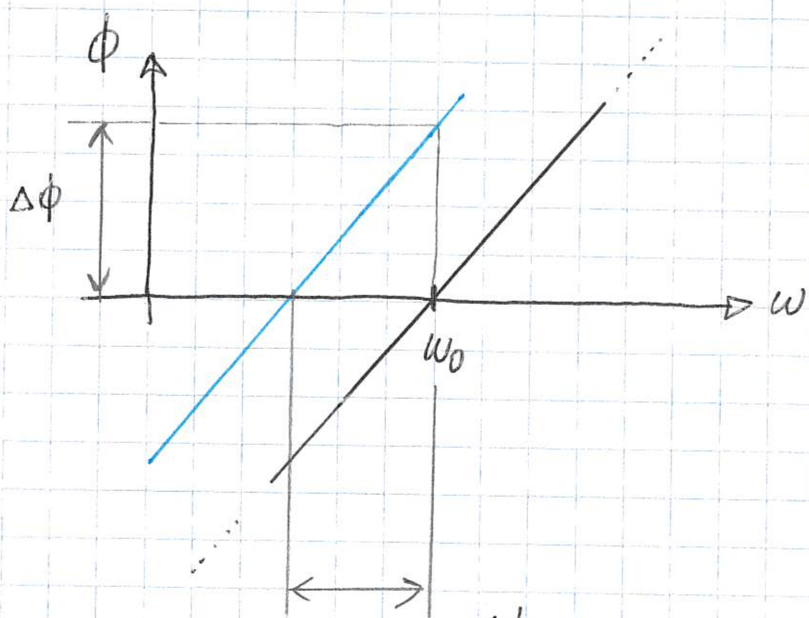
$$\boxed{\begin{array}{c} |L(j\omega_0)| \geq 1 \\ \phi(\omega_0) = 0^\circ \end{array}}$$

Barkhausen criteria

It is important to notice that the stability of the frequency of oscillation $w_0$ it is determined solely by the phase characteristics of the loop-gain (→ the magnitude of the loop-gain determine only if the oscillations are sustained) → The system oscillates at the frequency for which the phase of the loop gain is $0^\circ$ (or equivalently $360^\circ$)

↓

The stability of the frequency of oscillation is determined by the manner in which the phase $\phi(w) = \angle L(jw)$ of the loop varies with frequency




$$\Delta w = \frac{\Delta\phi}{d\phi/dw}$$

"A steep" function $\phi(w)$ results in a more stable frequency

→ imagine a change in phase $\Delta\phi$ due to a change in one of the circuit component →

if $d\phi/dw$ is large (very steep slope) the resulting change in $w_0$ is small

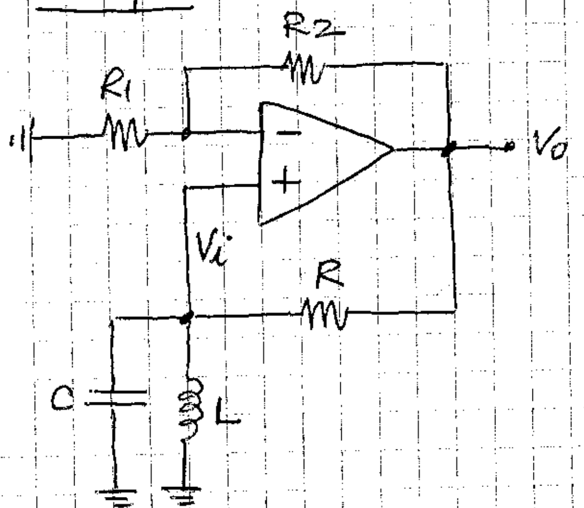
## Analysis of oscillator circuits

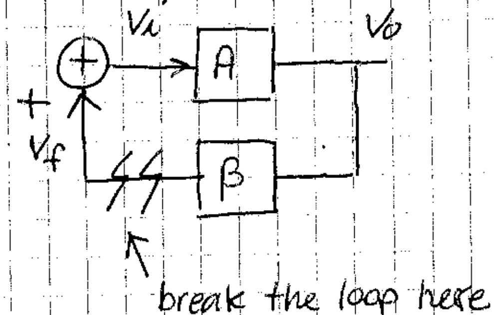
1. Break the FB loop to determine the loop gain $L(s) = A(s)\beta(s)$
2. The oscillation frequency $\omega_0$ is found as the frequency for which the phase angle of the loop gain is
$\angle L(j\omega_0) \triangleq \phi(\omega_0) = 0°$ (or alternatively $360°$)
3. The condition for the oscillations to start is found from

$$|A(j\omega_0)\beta(j\omega_0)| \triangleq |L(j\omega_0)| \geq 1$$

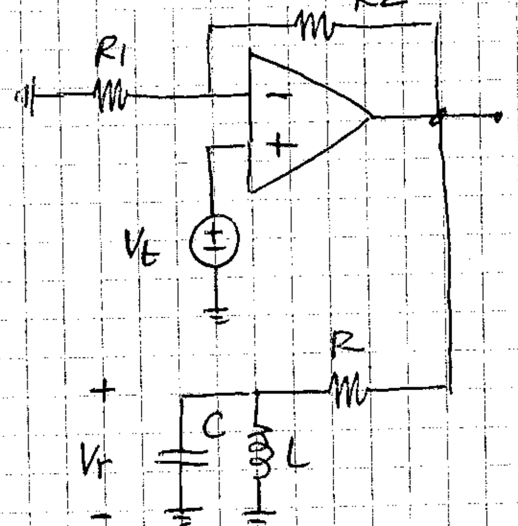
(making the loop-gain magnitude slightly greater than unity ensures that oscillation will start)

**Example**




$$A(s) = \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$ ← assume ideal op-amp.




Band-Pass Filter

$$\beta(s) = \frac{V_r}{V_o} = \frac{s/(RC)}{s^2 + \frac{s}{CR} + \frac{1}{LC}}$$

$$A(s)\beta(s) \equiv L(s) = \left(1+\frac{R2}{R1}\right) \frac{s/(RC)}{s^2+\frac{s}{RC}+\frac{1}{LC}}$$

substituting $s=j\omega$ (physical frequency):

$$A(j\omega)\beta(j\omega) \equiv L(j\omega) = \left(1+\frac{R2}{R_1}\right) \frac{\frac{j\omega}{RC}}{-\omega^2+\frac{1}{LC}+\frac{j\omega}{RC}}$$

the frequency that makes the phase of $L(j\omega)$ zero is the value of $\omega$ that makes the real part of the denominator equal to zero

$\downarrow$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

at this frequency the magnitude of the loop gain is:

$$A(j\omega_0)\beta(j\omega_0) = \left(1+\frac{R2}{R1}\right) \underbrace{\frac{\cancel{j\omega_0/(RC)}}{0+\cancel{j\omega/(RC)}}}_{=1}$$

therefore for the oscillation to start we need:

$$\frac{R2}{R1} \geq 0$$

NOTE: careful to blindly apply "math" $\rightarrow$ selecting the value of $R2=0$ puts the "$-$" terminal of the op-amp at physical gnd so the circuit cannot work.
if we want to make the condition $R2/R1 = 0$ the proper choice is to take off $R_1$ ($\rightarrow R_1 = \infty$)

## Oscillation Amplitude Control

- As can be observed from the time evolution of an oscillatory system all oscillators (despite the name) are "heavily" non-linear circuits → strictly speaking Laplace Analysis is no longer applicable (→ when we apply it we do so just up to the point we start the "oscillation")
- the amplitude of the oscillation is set an controlled using a non-linear mechanism implemented either with a separate circuit or using the nonlinearities of the amplifying device itself.

- Suppose we work hard to make $A\beta = 1$ at $\omega = \omega_0$

  ↓

  then the temperature changes and:
  - * $|A\beta|$ becomes $< 1$ → oscillations will die out
  - * $|A\beta|$ becomes $> 1$ → oscillations will grow in amplitude

  ⇓

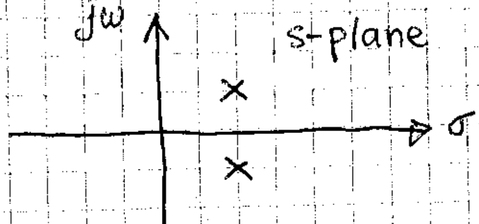
  we need some mechanism for forcing $|A\beta| = 1$ at the desired value of output amplitude

  ↓

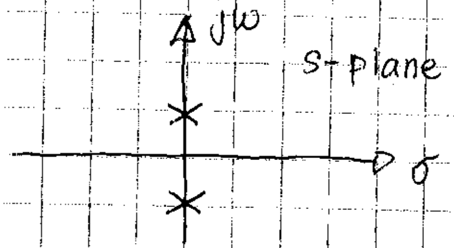
  we need a circuit for gain control

- Gain-control mechanism operation

  ① First, to ensure that the oscillations will start, one design the circuit such that $|A\beta|$ is sligthly $> 1$ → this correspond to designing the circuit so that the poles are in the RHP



(2) when the amplitude reaches the desired level the gain control circuit kicks-in and causes the loop-gain to be reduced to exactly $|A\beta| = 1 \rightarrow$ this correspond to pull back the poles to the $j\omega$ axis.




$$A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)} = \frac{A(s)}{s^2 + \omega_0^2}$$

If for some reason the loop gain $|A\beta|$ is reduced below 1, the non-linear gain control circuit will detect that the amplitude of the output sine wave is diminished and will cause the loop gain to increase back exactly to unity.

- amplitude-stabilization appraches

(a.) Limiter circuit

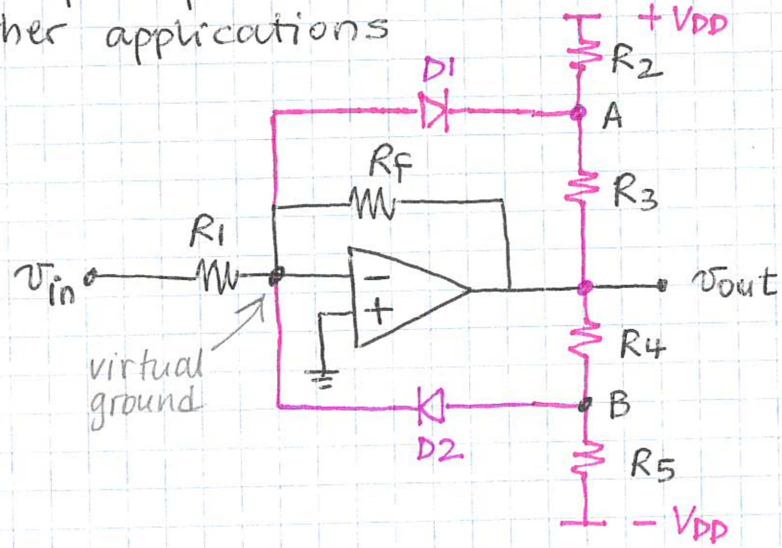
oscillations are allowed to grow until the amplitude reaches the level to which the limiter is set
The limiter should be "smooth" (soft limiter) to do not cause excessive N.L. distorsion.
However the n.L. distorsion is reduced by the filtering action of the frequency-selective FB. network.

(b) the other approach for amplitude control uses an element whose resistance can be controlled by the amplitude of the output (diodes and MOSFETs operated in triode region are commonly employed to implement the controlled resistance element)

By placing this element in the FB network so that its resistance determines the loop gain the circuit can be designed to ensure that the loop gain reaches unity at the desired output amplitude.

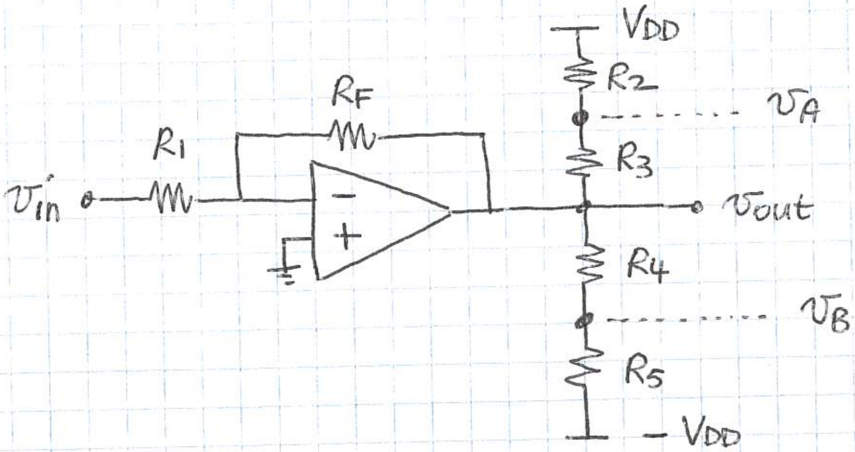
# → Amplitude control via limiter circuit (detailed analysis) ←

the limiter circuit is very popular for controlling the amplitude of op-amp based oscillators, but also in a variety of other applications



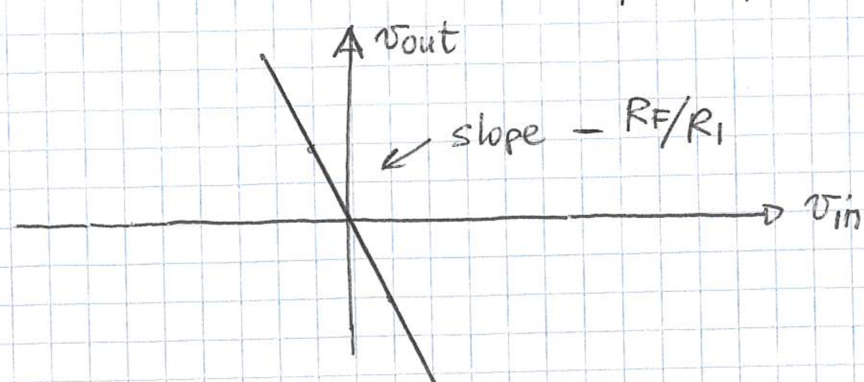

NOTE: no matter how the diodes operate the limiter circuit is wrapped around the op-amp in negative FB!

(1.) first consider the case of a small (close to zero) $v_{in}$ and a small $v_{out}$ → $v_A > 0$ and $v_B < 0$ → D1 and D2 are OFF

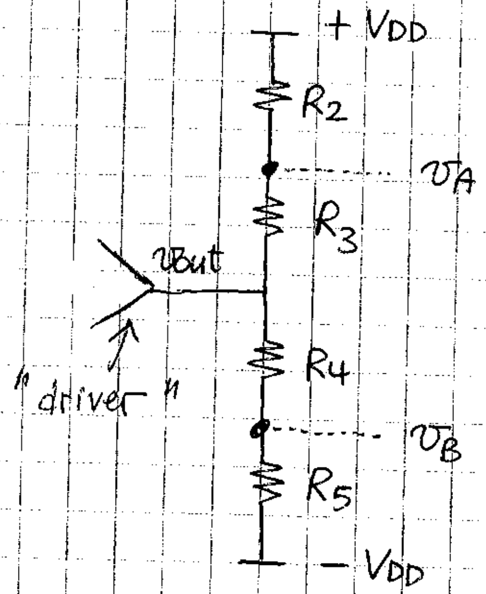



$$v_{out} = -\frac{R_F}{R_1} \cdot v_{in}$$

← This is the linear portion of the limiter circuit transfer function



as long as we are in "linear" operation we can use superposition to find the voltages $v_A$ and $v_B$ in terms of $\pm V_{DD}$ and $v_{out}$




$\pm V_{DD}$ only ($v_{out} = 0$)

$$v_A' = V_{DD} \frac{R_3}{R_2 + R_3}$$

$$v_B' = -V_{DD} \frac{R_4}{R_4 + R_5}$$

$v_{out}$ only (and $\pm V_{DD} = 0$)

$$v_A'' = \frac{R_2}{R_2 + R_3} v_{out}$$

$$v_B'' = \frac{R_5}{R_4 + R_5} v_{out}$$

$$\Downarrow$$

$$v_A = v_A' + v_A'' = V_{DD} \frac{R_3}{R_2 + R_3} + v_{out} \frac{R_2}{R_2 + R_3}$$

$$v_B = v_B' + v_B'' = -V_{DD} \frac{R_4}{R_4 + R_5} + v_{out} \frac{R_5}{R_4 + R_5}$$

(2.) As $v_{in}$ goes positive $\Rightarrow$ $v_{out}$ goes negative
$\left( v_{out} = -\frac{R_f}{R_1} v_{in} \right)$

$\Downarrow$

* $v_B$ becomes more negative $\rightarrow$ D2 keep staying OFF
* $v_A$ becomes less positive $\rightarrow$ as $v_{in}$ continue to increase eventually we will reach a negative value of $v_{out}$ at which $v_A$ becomes $-0.7V$ $\rightarrow$ D1 turns ON

the value of $v_{out}$ at which $v_A = -0.7V \triangleq -V_{D_{ON}}$ is:

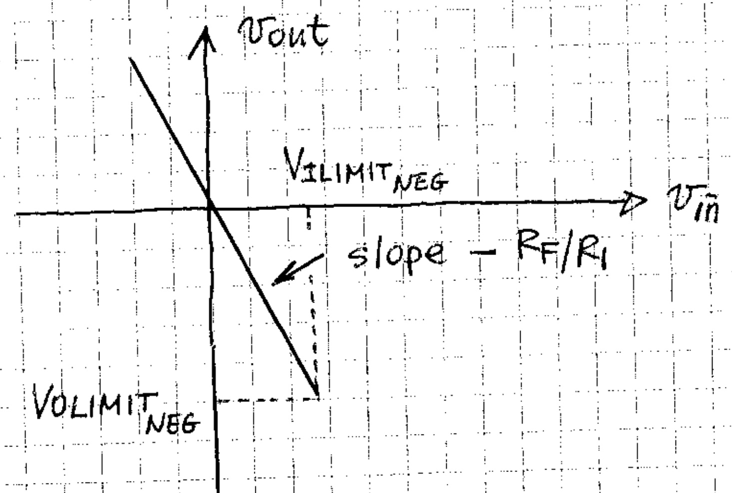
$$-V_{D_{ON}} = V_{DD}\frac{R_3}{R_2+R_3} + V_{OLIMIT_{NEG}}\frac{R_2}{R_2+R_3}$$

$$\Updownarrow$$

$$V_{OLIMIT_{NEG}} = \left[-V_{D_{ON}} - V_{DD}\frac{R_3}{R_2+R_3}\right]\frac{R_2+R_3}{R_2}$$

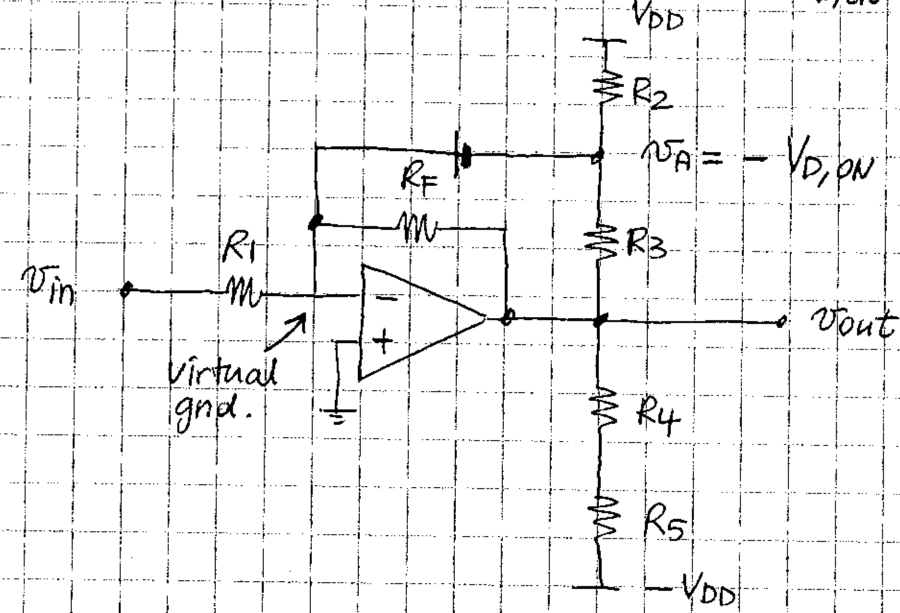
$$\Updownarrow$$

$$V_{OLIMIT_{NEG}} = -V_{D_{ON}}\left(1+\frac{R_3}{R_2}\right) - V_{DD}\frac{R_3}{R_2}$$

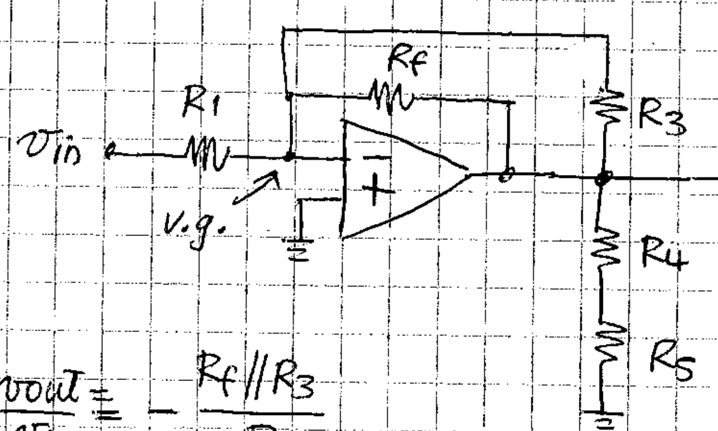



$$V_{ILIMIT_{NEG}} = \frac{V_{OLIMIT_{NEG}}}{-R_F/R_1}$$

once D1 turns ON ( $V_A$ remains fixed at $-V_{D,ON}$ $\rightarrow$

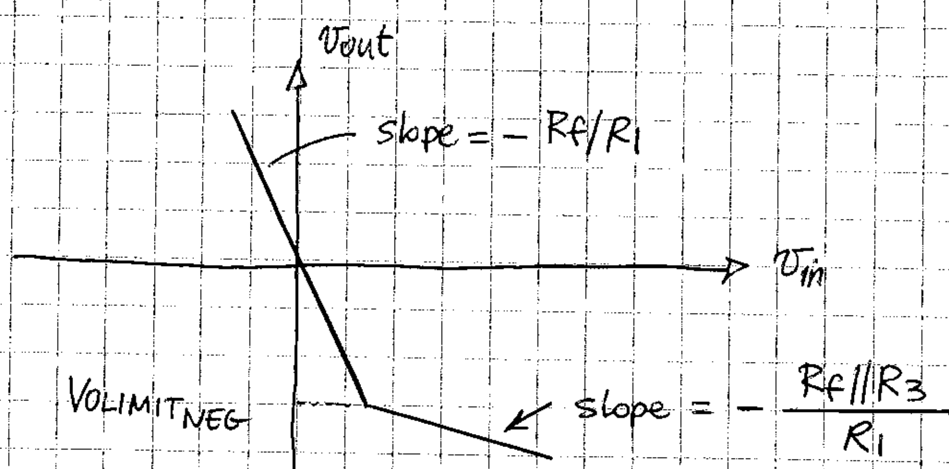



$\rightarrow$ therefore as $v_{in}$ keep increasing the additional current $\frac{v_{in}}{R_1}$ can only flow in $R_3$ and $R_F$ ( $\rightarrow$ the voltage across $R_2$ is constant at $V_{DD} - V_A = V_{DD} + V_{D,ON}$ $\rightarrow$ no AC current flows through R2)




AC circuit

$$\frac{v_{out}}{v_{in}} = -\frac{R_f \| R_3}{R_1}$$



(3) As $v_{in}$ goes negative $\Rightarrow$ $v_{out}$ goes positive

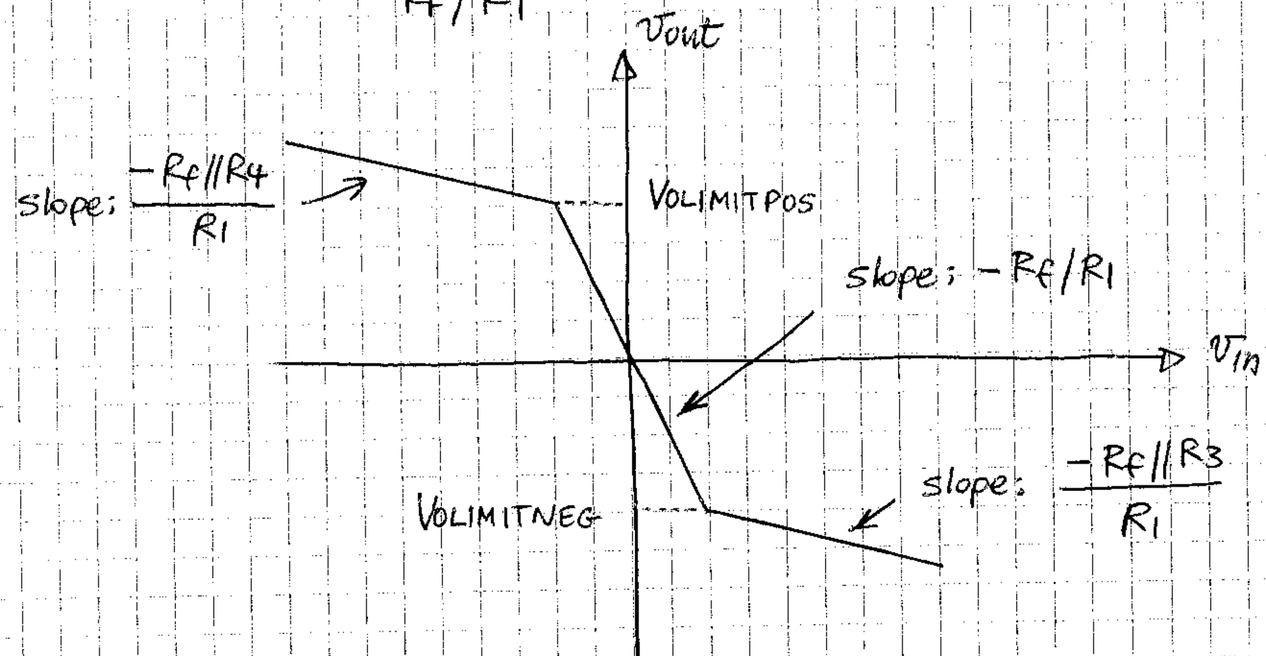
$$\left(v_{out} = -\frac{R_F}{R_1} \cdot v_{in}\right)$$

* $v_A$ stays positive (becomes more positive) $\rightarrow$ D1 keep staying OFF
* $v_B$ becomes more positive $\rightarrow$ as $v_{in}$ continue to decrease eventually we will reach a positive value of $v_{out}$ at which $v_B$ becomes $+0.7V$ $\rightarrow$ D2 turns ON

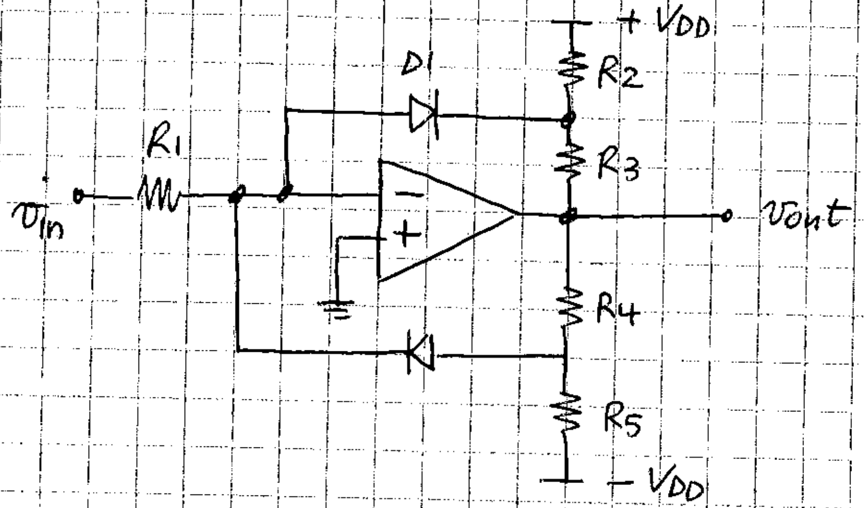
the value of $v_{out}$ at which $v_B = +0.7 = V_{D,ON}$ is:

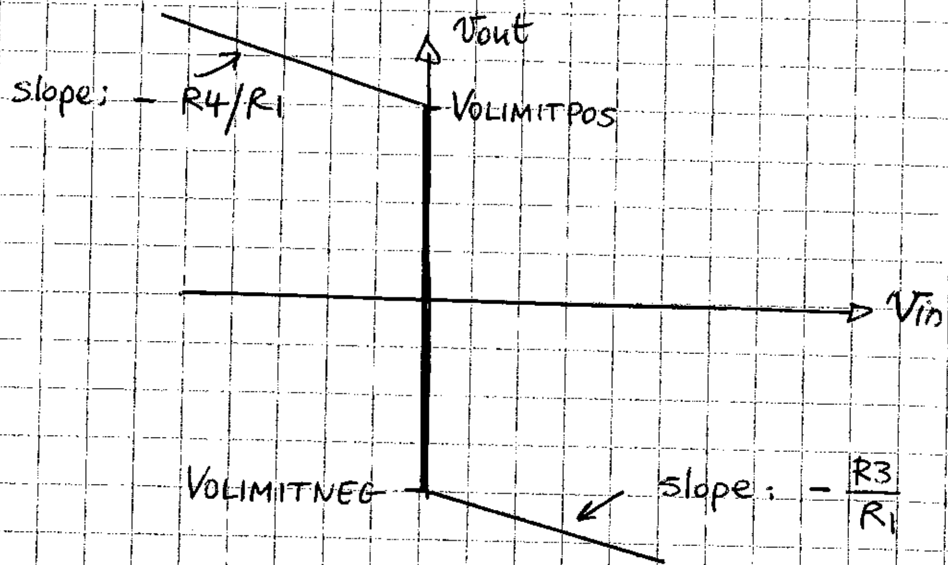
$$V_{OLIMITPOS} = V_{D,ON}\left(1+\frac{R_4}{R_5}\right) + V_{DD}\,\frac{R_4}{R_5}$$

$$V_{ILIMITPOS} = \frac{V_{OLIMITPOS}}{-R_F/R_1}$$



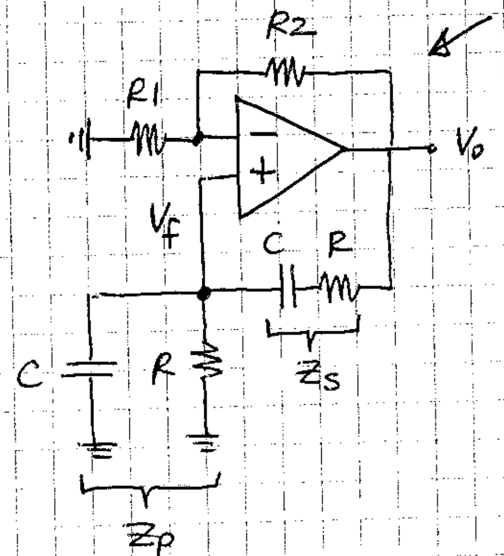
④ If we remove $R_f$ ($R_f = \infty$) the limiter turns into a comparator with the following characteristics:

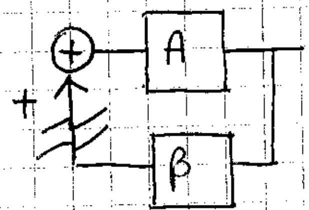


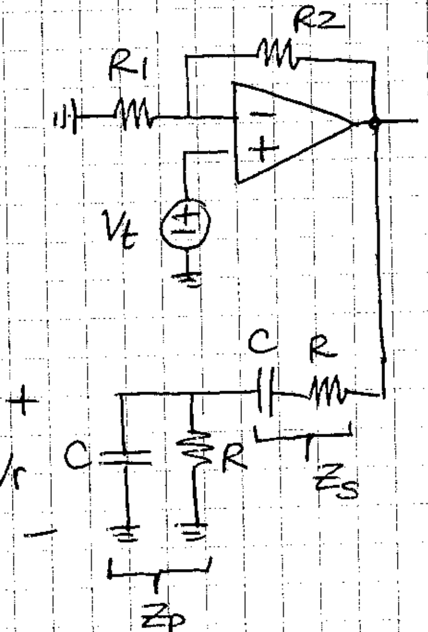



Wien-Bridge Oscillator ($\subset$ opamp-RC oscillators $\subset$ linear oscillators)

Figure. Wien Bridge oscillator (w/o amplitude stabilization)




$$A(s) = \frac{V_o}{V_t} = 1 + \frac{R_2}{R_1}$$ ← Assume ideal op-amp

$$\beta(s) = \frac{V_r}{V_o} = \frac{Z_p}{Z_s + Z_p}$$

$$L(s) = A(s)\beta(s) = \left(1 + \frac{R_2}{R_1}\right)\frac{1/Y_p}{Z_s + Z_p} = \left(1 + \frac{R_2}{R_1}\right)\frac{1}{1 + Z_s \cdot Y_p} =$$

$$= \left(1 + \frac{R_2}{R_1}\right)\frac{1}{1 + \left(R + \frac{1}{sC}\right)\left(\frac{1}{R} + sC\right)} =$$

$$= \left(1 + \frac{R_2}{R_1}\right)\frac{1}{1 + 1 + \frac{1}{sRC} + sRC + 1} =$$

$$= \left(1 + \frac{R_2}{R_1}\right)\frac{1}{3 + \frac{1}{sRC} + sRC}$$

For $s = j\omega$:

$$L(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\frac{1}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}$$

- the loop-gain has phase of $0°$ at the freq. $\omega_0$ for which the imaginary part of the denominator is zero:

$$\omega_0 RC - \frac{1}{\omega_0 RC} = 0$$

$$\Updownarrow$$

$$\omega_0^2 R^2 C^2 - 1 = 0 \quad \Leftrightarrow \quad \omega_0 = \frac{1}{RC}$$

- to obtained substained oscillations at $\omega_0$ one has to set the magnitude of the loop gain at $\omega_0$ such that:

$$|L(j\omega_0)| = 1$$

$$\Updownarrow$$

$$|L(j\omega_0)| = \frac{1 + R2/R1}{3} = 1 \rightarrow \frac{R2}{R1} = 2$$

- In practice to ensure that oscillations start for sure one chooses $R2/R1 = 2+\delta$ (where $\delta$ is a relatively small quantity)

$$|L(j\omega_0)| \geq 1$$

$$\Updownarrow$$

$$\frac{1 + R2/R1}{3} \geq 1 \Leftrightarrow \frac{R2}{R1} \geq 2$$

- With $\frac{R2}{R1} = 2+\delta$ the roots of the characteristic eqn. $1 - L(s) = 0$ are in the R.H.P. ($\rightarrow$ so the oscillations starts for sure!)

$$L(s) = \frac{1 + R2/R1}{3 + sRC + \frac{1}{sRC}}$$

$$\Downarrow$$

$$1 - L(s) = 1 - \frac{1 + 2 + \delta}{3 + sRC + \frac{1}{sRC}} =$$

$$= \frac{3 + sRC + 1/sRC - 3 - \delta}{3 + sRC + \frac{1}{sRC}} =$$

$$= \frac{s^2R^2C^2 + \cancel{3sRC} - \cancel{3sRC} - \delta sRC + 1}{3 + sRC + \frac{1}{sRC}} = 0$$

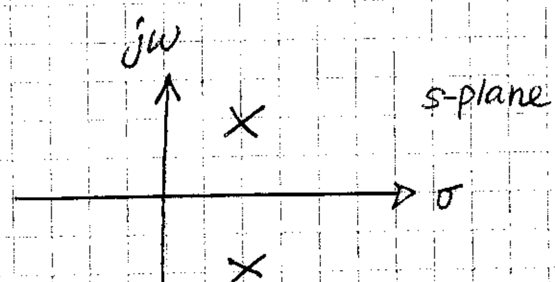
$$\Updownarrow$$

$$s^2(RC)^2 - s\,\delta RC + 1 = 0$$

$$\Updownarrow$$

$$s_{1,2} = \frac{\delta RC \pm \sqrt{\delta^2R^2C^2 - 4R^2C^2}}{2R^2C^2} =$$

$$= \frac{\delta \pm \sqrt{\delta^2 - 4}}{2RC}$$

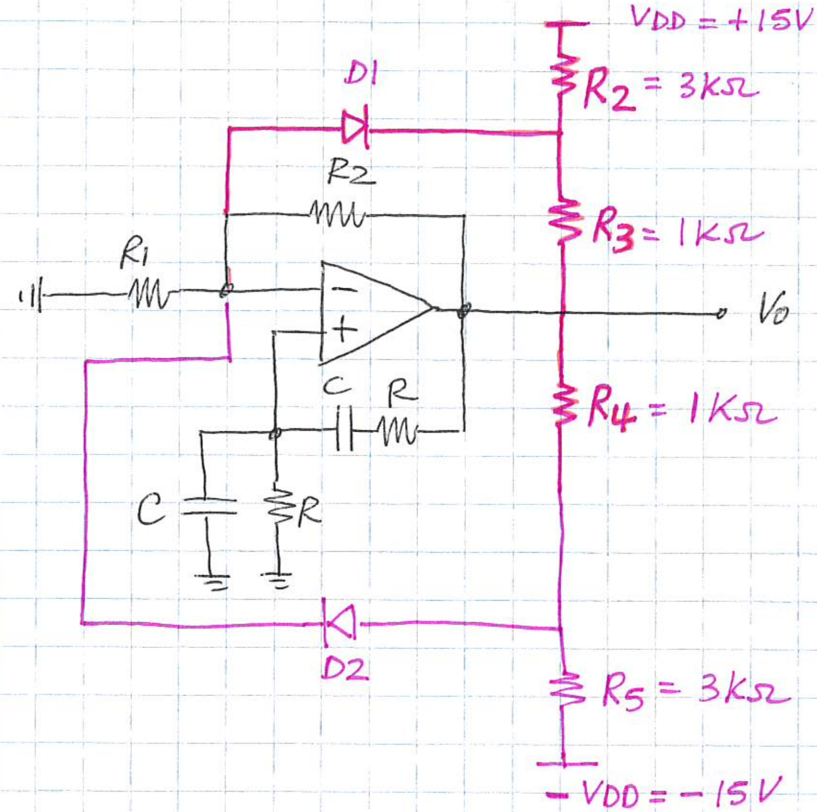



For the oscillations to start $\delta$ must be a small positive quantity $< 2$

NOTE: the condition $\frac{R2}{R1} > 2$ is necessary but not sufficient!!

$$2 < \frac{R2}{R1} < 4$$

The amplitude of oscillation can be stabilized using the limiter circuit we have analyzed earlier



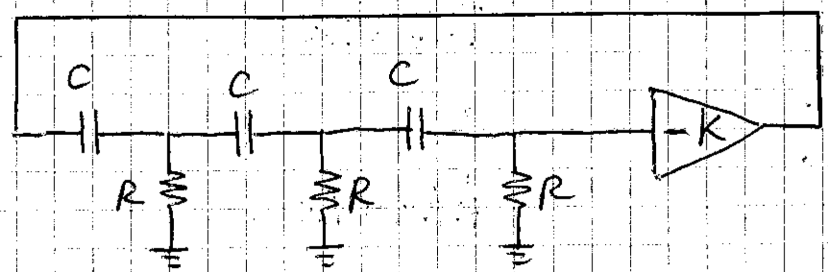

- Limiter
- Wien-Bridge oscillator

$R = 10\ K\Omega$
$C = 16\ nF$ $\rightarrow f_0 \simeq 995\ Hz$

$R_2 = 20.3\ K\Omega$
$R_1 = 10\ K\Omega$ $\rightarrow \frac{R_2}{R_1} = 2.3$

## Phase shift oscillator ($\subset$ opamp-RC oscillators $\subset$ linear oscillators)

- The basic structure of the phase-shift oscillator consists of a negative gain amplifier $(-K)$ with a three-section (third order) RC-ladder network in the FB.




- The circuit will oscillate at the frequency for which the phase-shift of the RC network is $180° \rightarrow$ only at this frequency the total phase shift of the loop be $0°$ (or $360°$).
(the reason for using a 3-section RC network is that 3 is the minimum number of sections that is capable of producing a phase shift of $180°$ at a finite frequency).

- For the oscillations to be sustained the value of $K$ should be equal (in practice slightly higher) to the inverse of the magnitude of the RC network transfer function at the frequency of oscillation ($\rightarrow$ unity loop gain condition)

$$|A(j\omega_0)\,\beta(j\omega_0)| \geq 1 \quad \leftrightarrow \quad |A(j\omega_0)| \geq \frac{1}{|\beta(j\omega_0)|} \quad \leftrightarrow \quad K \geq \frac{1}{|\beta(j\omega_0)|}$$

$$\downarrow$$

the analysis of the phase-shift network $\beta(j\omega)$ is rather tedious due to the loading between the successive stages. Despite the tedious algebra it is possible to show that for the following practical implementation (Fig. 13.35 p. 850 Horenstein):

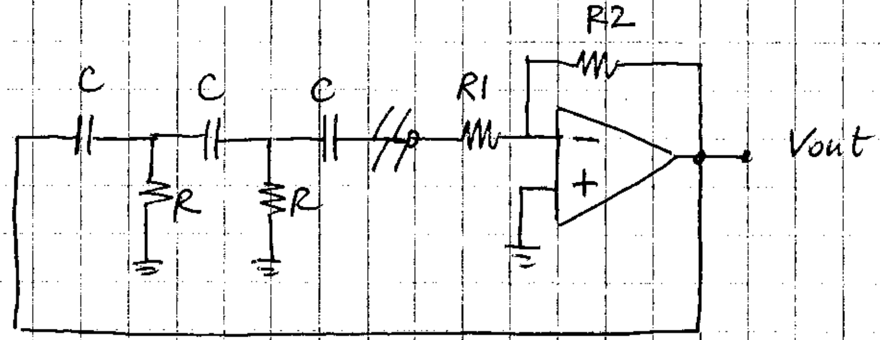



Fig. 13.35
Horenstein

$$\beta(j\omega) = \frac{(j\omega RC)(j\omega RC)^2}{[1-6(\omega RC)^2] + j\omega RC[5-(\omega RC)^2]} =$$

$$= \frac{-j(\omega RC)^3}{[1-6(\omega RC)^2] + j\omega RC[5-(\omega RC)^2]}$$

oscillation occurs where $\beta(j\omega)$ becomes purely real $\rightarrow$
$\omega_0 = 1/(\sqrt{6}\,RC)$

at $\omega_0$ the loop-gain $A(j\omega_0)\beta(j\omega_0)$ is:

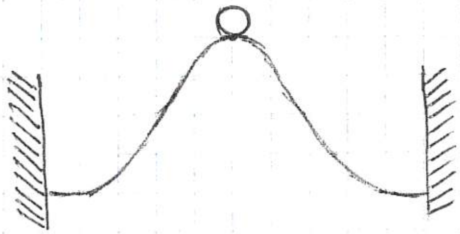
$$K \cdot \frac{(\omega_0 RC)^2}{5-(\omega_0 RC)^2} = K \cdot \frac{1/6}{5-1/6} = K \cdot \frac{1}{29}$$

therefore the condition to obtain sustained oscillations is:

$$K \cdot \frac{1}{29} \geq 1 \iff K \geq 29 \iff \frac{R_2}{R_1} \geq 29$$

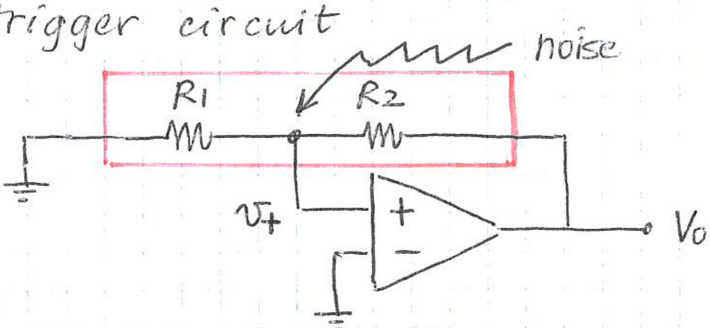
## The Bistable multivibrator

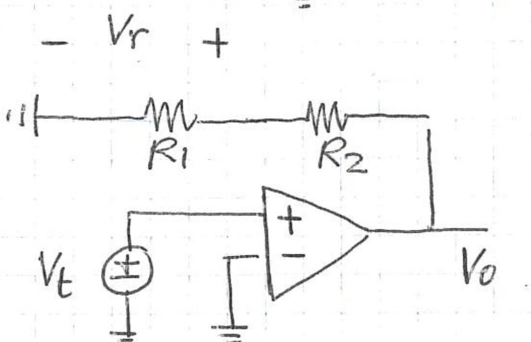
- A bistable multivibrator has two stable states. The circuit can remain in either stable state indefinitely and moves to the other stable state only when appropriately triggered

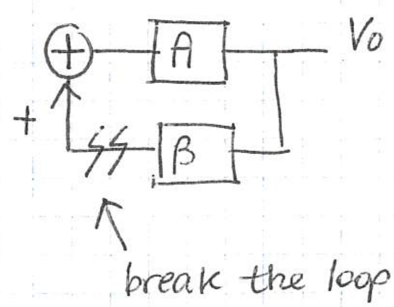


Physical analogy for the operation of the bistable. The ball cannot remain at the top of the hill; the inevitably present disturbance will cause the ball to fall to one side or the other where it can remain indefinetely.

- A possible way to obtain bistability is to connect a dc amplifier in a positive feedback with loop-gain greater than unity → this implementation is a.k.a. Schmitt trigger circuit




- assume that the electric noise that is inevitably present in every electronic circuit causes a small positive value ~~increment~~ in the voltage $V_+$
This incremental signal will be amplified by the large open-loop gain $A_0$ of the op-amp → with the result

that a much greater signal will appear at the op-amp output $\rightarrow$ $v_O = A_O \cdot v_+$

the voltage divider $R_1$, $R_2$ will feed a fraction $\beta = \frac{R_1}{R_1+R_2}$ of the output signal back to the positive terminal of the op amp.

If $A\beta \left(= A_O \frac{R_1}{R_1+R_2}\right)$ is greater than unity (as usually is) the signal fed back will be greater than the original increment in $v_+$ $\rightarrow$ this regenerative process continues until eventually the opamp saturates with its output voltage at the positive saturation level $V_{POS}$ $\rightarrow$

when this happen the positive input terminal $V_+$ becomes $V_+ = V_{POS} \frac{R_1}{R_1+R_2}$ and the opamp is kept in positive saturation.

$\rightarrow$ this is one of the two stable states of the circuit

- If we had assumed the equally probably situation of a negative noise voltage at $V_+$ $\rightarrow$ the opamp would have ended up saturated in the negative direction with $V_O = V_{NEG}$ and $V_+ = V_{NEG} \frac{R_1}{R_1+R_2}$

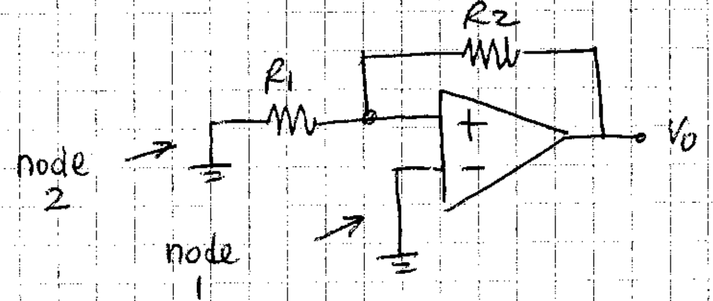
This circuit can stay in either of these two states indefinitely !!

$\downarrow$

How can we make a bistable circuit change state ?

## Transfer characteristic of the Bistable

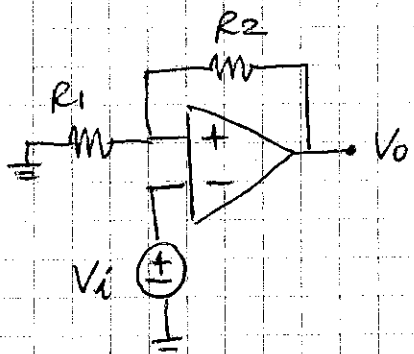
- How can we make the bistable change state?
  We must inject a voltage capable to counteract the regenerative process started by the noise and that has pushed the output voltage to one of the two (either $V_{POS}$ or $V_{NEG}$) saturation levels.
- Either of the two nodes that are connected to ground can serve as an input terminal to inject voltage



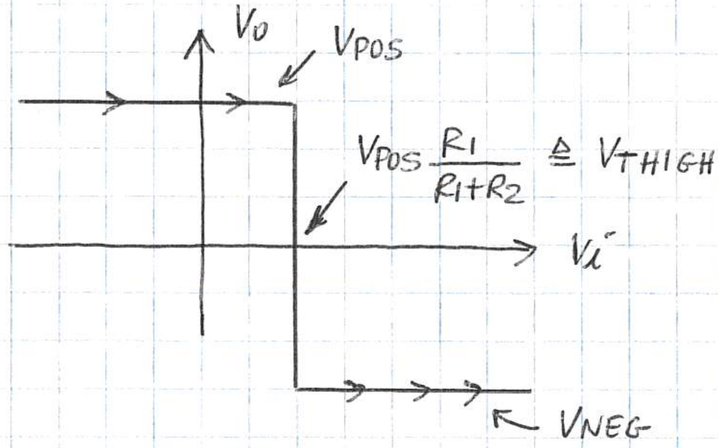

let's investigate both possibilities.

① Schmitt trigger with Inverting transfer characteristic

- assume $V_o$ is at one of its two possible levels, say $V_{POS}$ and therefore $V_+ = V_{POS}\,\beta = V_{POS}\,\dfrac{R_1}{R_1+R_2}$

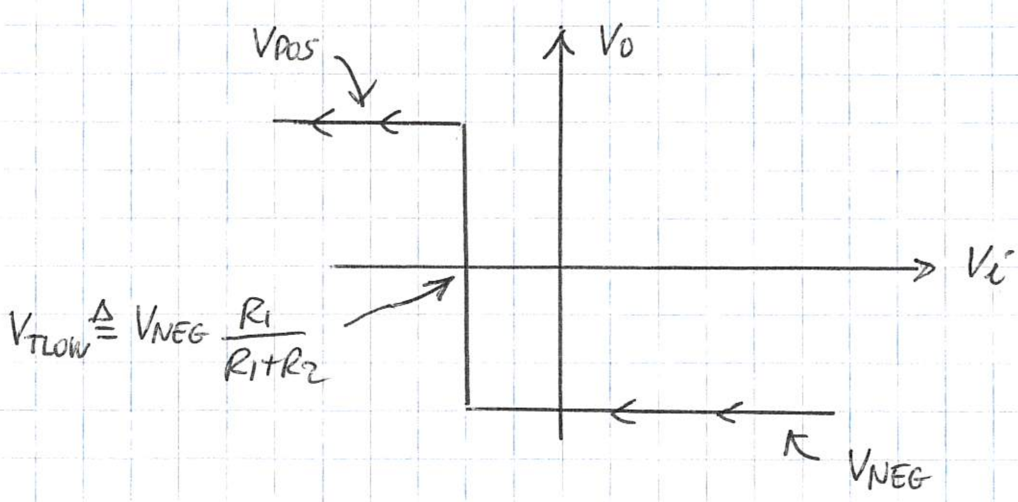



As $V_i$ is increased from 0V on, nothing happens until $V_i$ reaches a value equal to $V_{POS}\dfrac{R_1}{R_1+R_2}$. As $V_i$ begins to exceed this value, a net negative voltage develops between the input terminals of the opamp.
$\rightarrow$ $V_o$ goes negative

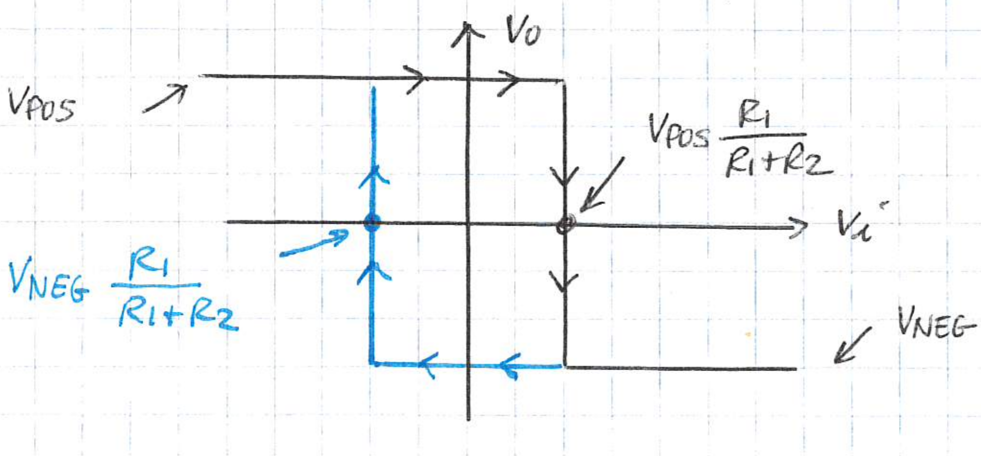



- let's now consider what happens as $V_i$ is decreased, as we are in the level $V_{NEG}$ (and therefore $V_+ = V_{NEG} \cdot \frac{R_1}{R_1+R_2}$)

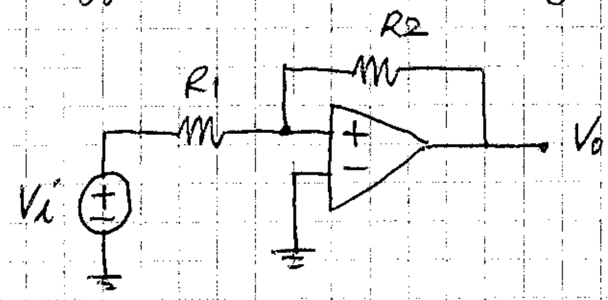
The circuit remains in the negative saturation state until $V_i$ goes negative to the point that it equals $V_{NEG} \frac{R_1}{R_1+R_2}$. As $V_i$ goes below this value, a net positive voltage appears between the input terminals of the opamp $\rightarrow$ $V_o$ goes positive.




The transfer characteristic has hysteresis:



② Schmitt Trigger with non-inverting transfer characteristic




$$V_+ = V_i \frac{R_2}{R_1+R_2} + V_o \frac{R_1}{R_1+R_2}$$

superposition

NOTE: careful with positive FB we cannot use "virtual ground"

- assume $V_o$ is in the positive stable state $V_{POS}$
  Positive values for $V_i$ do not have any effect on the state of the output.
  To trigger the circuit into the $V_{NEG}$ state, $V_i$ must be made negative and of such a value as to make $V_+$ decrease below zero

$$V_+ \equiv V_i \frac{R_2}{R_1+R_2} + V_{POS} \frac{R_1}{R_1+R_2} \leq 0$$

$$\Updownarrow$$

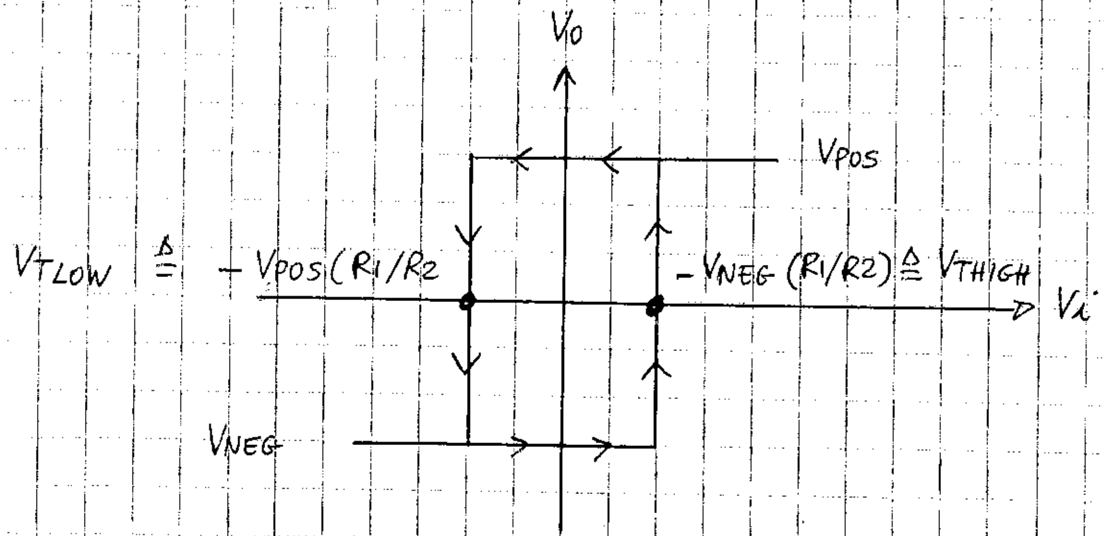
$$V_{T_{LOW}} = - V_{POS} \frac{R_1}{R_2}$$

- similarly if we assume $V_o$ is in the negative stable state $V_{NEG}$ negative values of $V_i$ do not have any effect on the state of the output.
  To trigger the circuit into the $V_{POS}$ state, $V_i$ must be made positive and of of such a value as to make $V_+$ increase above zero

$$V_+ \equiv V_i \frac{R_2}{R_1+R_2} + V_{NEG} \frac{R_1}{R_1+R_2} \geq 0$$

$$\Updownarrow$$

$$V_{T_{HIGH}} = - V_{NEG} \frac{R_1}{R_2}$$

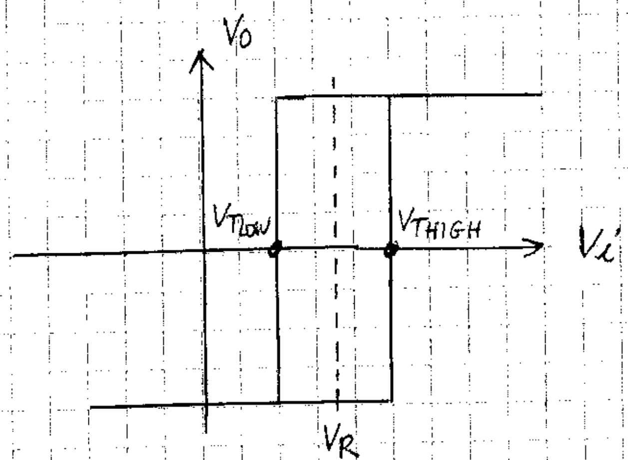


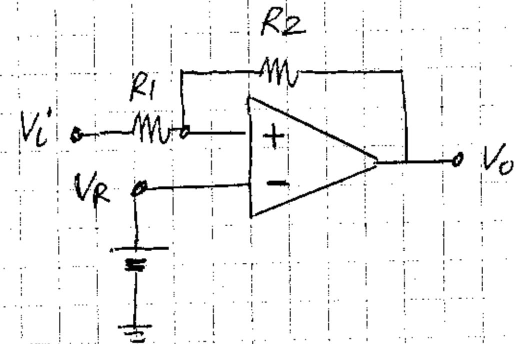

- NOTE : it is important to note that to make a bistable change state the input $V_i$ needs to merely initiates (= trigger) the regeneration process, once the regeneration process is started we can remove $V_i$ → The input signal $V_i$ can be simply a pulse of short duration (→ thus, the input signal $V_i$ is referred as a trigger signal, or simply a trigger)

## Applications of bistable circuits

- comparator with hysteresis
- as basic building block to build astable multivibrators and monostable vibrators

## Comparator with hysteresys




$$V_{TLOW} = -V_{POS}\frac{R_1}{R_2} + V_R$$

$$V_{THIGH} = -V_{NEG}\frac{R_1}{R_2} + V_R$$

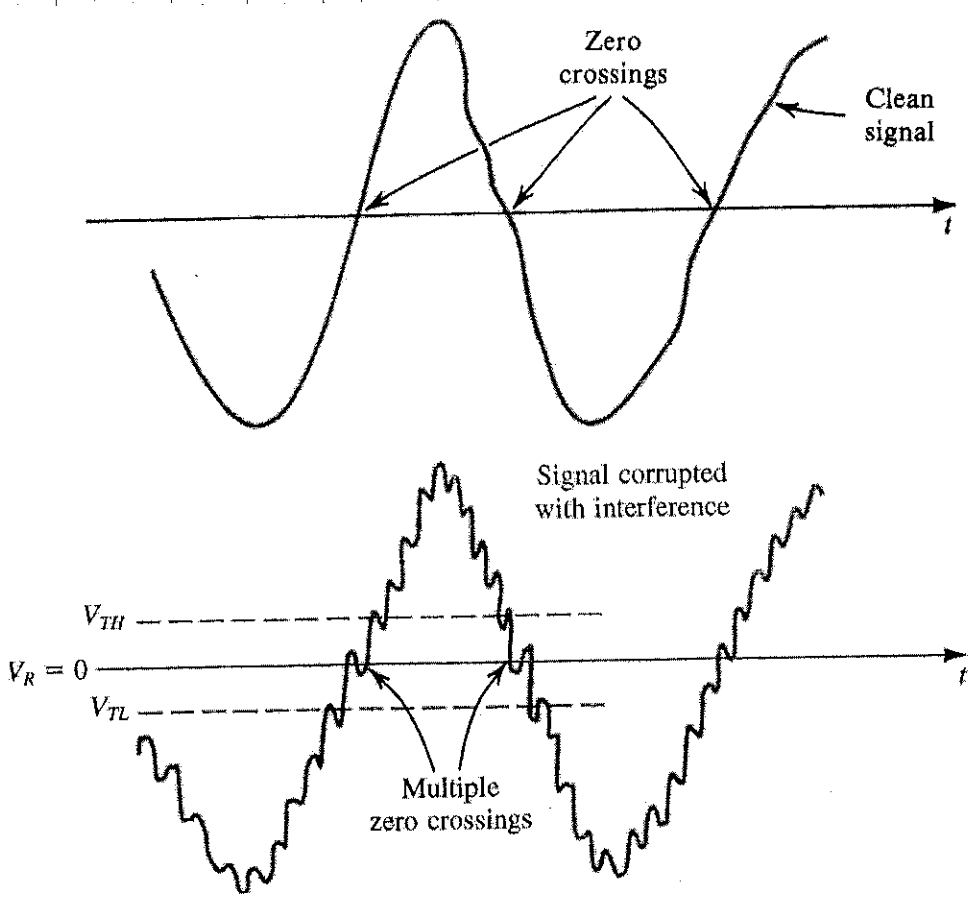
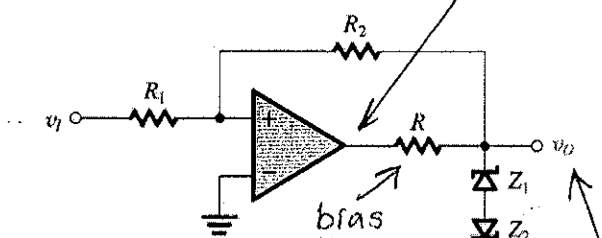



**Figure 18.24** Illustrating the use of hysteresis in the comparator characteristic as a means of rejecting interference.

## Making the output levels of the bistable more precise

Typically we do not know the precise value of the saturation voltages at the output of the opamp. To obtain more precise output levels (and consequently more precise $V_{TLOW}$ and $V_{THIGH}$) for the bistable we can use limiters circuits.

output of the opamp: either $V_{POS}$ or $V_{NEG}$



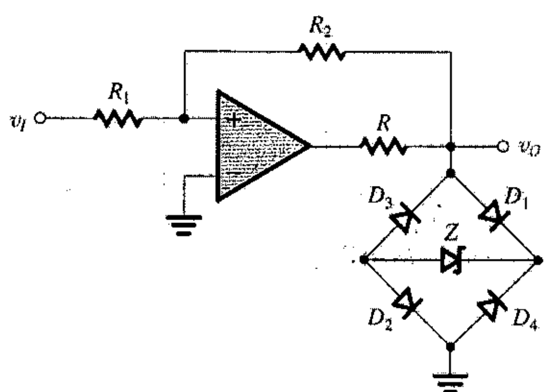

bias the diodes

(a)

$$+V_{SAT} = V_{Z1} + 0.7V$$

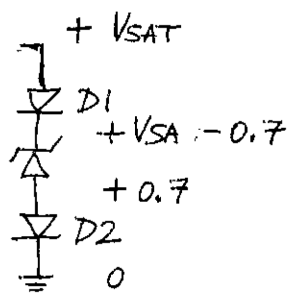
$$-V_{SAT} = -V_{Z2} - 0.7V$$

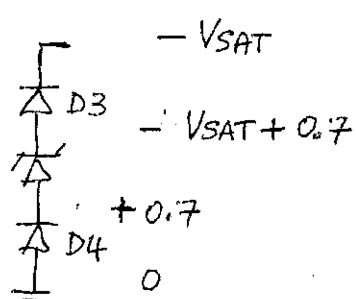
output of the bistable: either $+V_{SAT}$ or $-V_{SAT}$




(b)

With this solution we have only one value for $V_Z$ (no need to rely on how well the two zeners are matched like in the previous circuit)

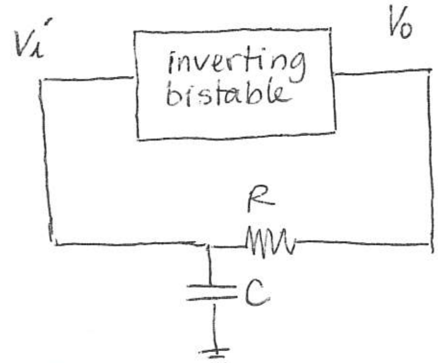


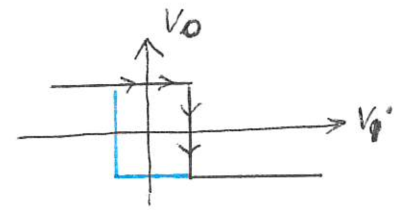




$$+V_{SAT} = V_Z + 1.4V \qquad -V_{SAT} = -V_Z - 1.4V$$

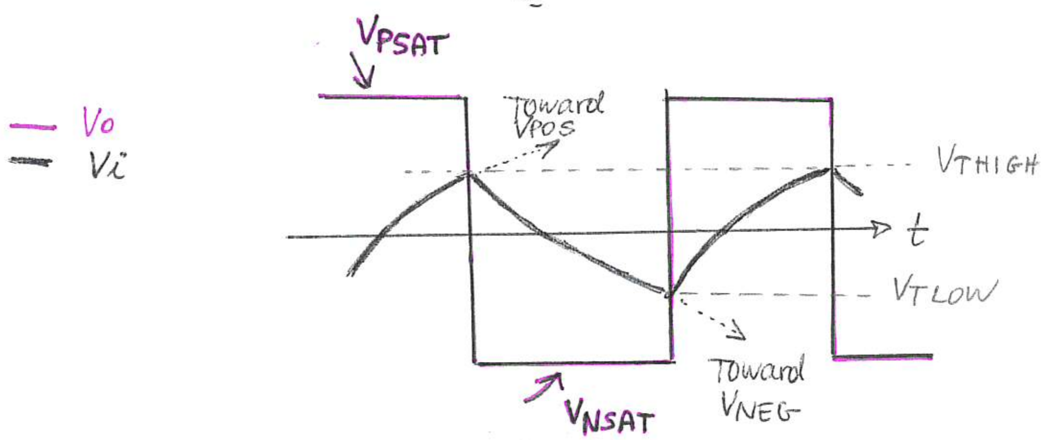
# Astable multivibrator square wave oscillator ($\subset$ non-linear oscillators)

- A square-wave can be generated by making a bistable multivibrator switch states periodically → since the resulting circuit has no stable states is named an astable multivibrator.
- To make the bistable switch states periodically we connect it in a feedback loop with a simple RC network




T.F. of inverting bistable




the charging and discharging of the capacitance has time constant $\tau = RC$

↓

a capacitor $C$ that is charging or discharging through a resistor $R$ toward a final voltage $V_\infty$ has a voltage

$$v(t) = V_\infty - (V_\infty - V_{0^+})\, e^{-t/\tau}$$

Where $V_{0+}$ is the voltage across the cap. at time $t = 0^+$ and $\tau = RC$ is the time constant

⇓

$$\Downarrow$$

charging phase

$$V_{THIGH} = V_{PSAT} - (V_{PSAT} - V_{TLOW})\, e^{-T_1/\tau}$$

$$\Updownarrow$$

$$-\frac{V_{THIGH} - V_{PSAT}}{V_{PSAT} - V_{TLOW}} = e^{-T_1/\tau}$$

$$\Updownarrow$$

$$T_1 = \tau \ln \frac{V_{PSAT} - V_{TLOW}}{-V_{THIGH} + V_{PSAT}}$$

discharging phase:

$$V_{TLOW} = V_{NSAT} - (V_{NSAT} - V_{THIGH})\, e^{-T_2/\tau}$$

$$\Updownarrow$$

$$-\frac{V_{TLOW} - V_{NSAT}}{V_{NSAT} - V_{THIGH}} = e^{-T_2/\tau}$$

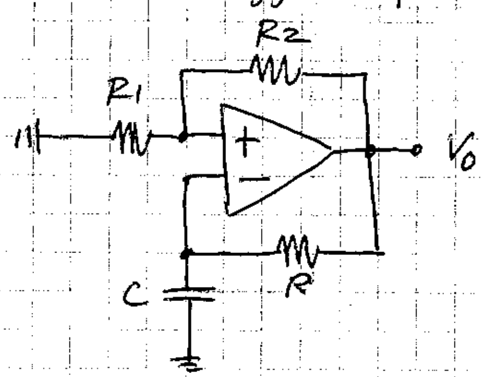
$$\Updownarrow$$

$$T_2 = \tau \ln \frac{V_{THIGH} - V_{NSAT}}{V_{TLOW} - V_{NSAT}}$$

Therefore the period $T$ is:

$$T = T_1 + T_2 = \tau \ln \frac{(V_{PSAT} - V_{TLOW})(V_{THIGH} - V_{NSAT})}{(V_{PSAT} - V_{THIGH})(V_{TLOW} - V_{NSAT})}$$

For the Schmitt-trigger implementation:




$$\beta = \frac{R_1}{R_1 + R_2}$$

$$V_{THIGH} = V_{PSAT} \cdot \beta$$

$$V_{TLOW} = V_{NSAT} \cdot \beta$$

$$T_1 = \tau \ln \frac{V_{PSAT} - V_{NSAT}\beta}{V_{PSAT} - V_{PSAT}\cdot\beta} = \tau \ln \frac{1-\beta(V_{NSAT}/V_{PSAT})}{1-\beta}$$

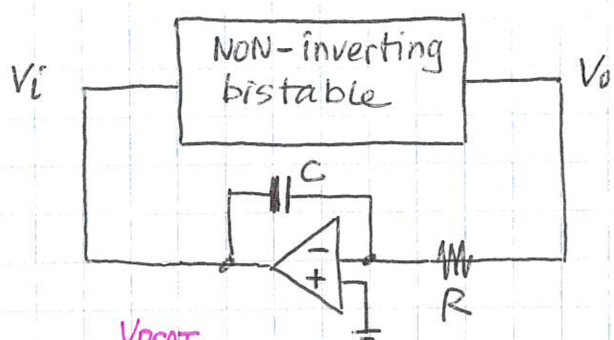
$$T_2 = \tau \ln \frac{\beta V_{PSAT} - V_{NSAT}}{\beta V_{NSAT} - V_{NSAT}} = \tau \ln \frac{1-\beta(V_{PSAT}/V_{NSAT})}{1-\beta}$$

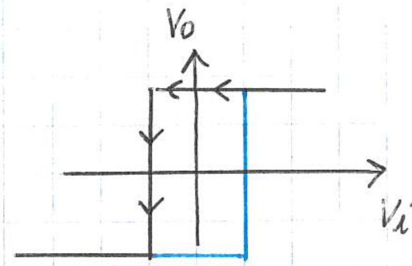
Normally $V_{PSAT} = -V_{NSAT}$ so the square-wave is symmetric and has period:

$$T = 2\tau \cdot \ln \frac{1+\beta}{1-\beta}$$

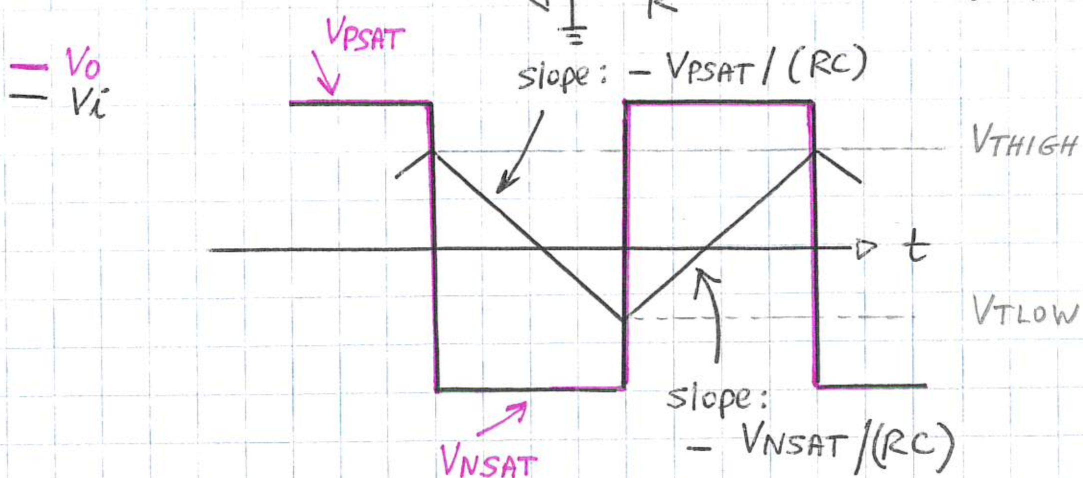
<u>Astable multivibrator triangular wave oscillator</u> ($\subset$ non-linear osc.)

- The exponential waveform generated by the low-pass RC network in the FB loop of the bistable can be changed to triangular by replacing the low-pass RC circuit with an integrator.
- Because the integrator is inverting, it is necessary to invert the T.F. of the bistable used → Thus the bistable circuit required here is of the non inverting type.

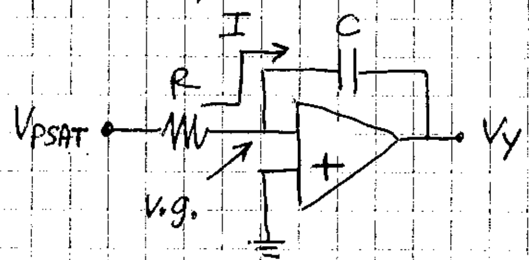


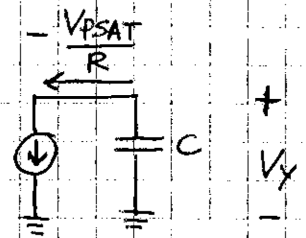




T.F. of non-inv. bistable

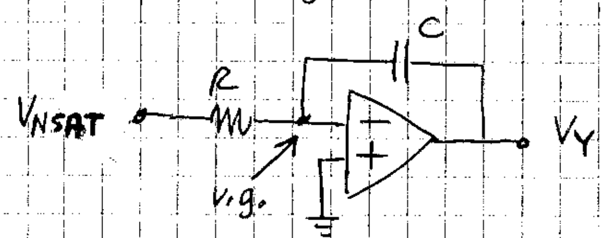


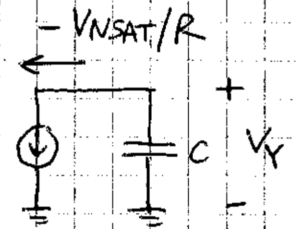
When the output is $V_{PSAT}$ a constant current equal to $V_{PSAT}/R$ will flow into the resistor and is pumped in the capacitor C causing the voltage at the output of the integrator to decrease linearly with slope $-\frac{V_{PSAT}}{RC}$




When the output is $V_{NSAT}$ a constant current equal to $V_{NSAT}/R$ flows into the resistor and causes the voltage at the output of the integrator to increase linearly with slope $-\frac{V_{NSAT}}{RC}$




Therefore deriving an expression for the period T of the square and triangular waveform is relatively easy

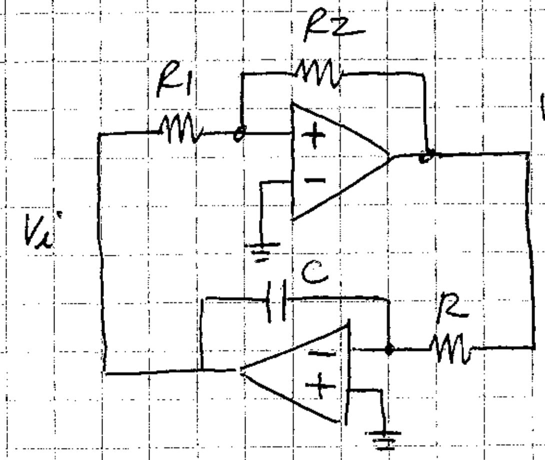
$$\frac{V_{PSAT}}{RC} = \frac{V_{THIGH} - V_{TLOW}}{T_{LOW}} \longrightarrow T_{LOW} = RC\,\frac{V_{THIGH} - V_{TLOW}}{V_{PSAT}}$$

$$\underbrace{\frac{-V_{NSAT}}{RC}}_{\text{slope}} = \frac{\overbrace{V_{THIGH} - V_{TLOW}}^{\Delta V}}{\underbrace{T_{HIGH}}_{\Delta T}} \longrightarrow T_{HIGH} = RC\,\frac{V_{THIGH} - V_{TLOW}}{-V_{NSAT}}$$

$$T = T_{HIGH} + T_{LOW} = RC\,\frac{V_{THIGH} - V_{TLOW}}{V_{PSAT}} + RC\,\frac{V_{THIGH} - V_{TLOW}}{-V_{NSAT}} =$$

$$= RC\,(V_{THIGH} - V_{TLOW})\left(\frac{1}{V_{PSAT}} - \frac{1}{V_{NSAT}}\right)$$

For the schmitt-Trigger implementation:




$$V_{THIGH} = -V_{NSAT} \cdot \gamma$$

$$V_{TLOW} = -V_{PSAT} \cdot \gamma$$

$$\gamma = \frac{R_1}{R_2}$$

$$T_{HIGH} = RC \, \frac{-V_{NSAT}\gamma + V_{PSAT} \cdot \gamma}{V_{PSAT}} =$$

$$= RC \quad \gamma \left(1 - V_{NSAT}/V_{PSAT}\right)$$

$$T_{LOW} = RC \, \frac{-V_{NSAT} \cdot \gamma + V_{PSAT} \cdot \gamma}{-V_{NSAT}} =$$

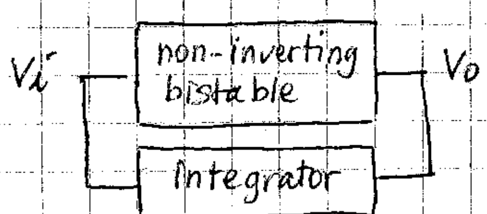
$$= RC \quad \gamma \left(+1 - V_{PSAT}/V_{NSAT}\right)$$

$$T = T_{HIGH} + T_{LOW} = RC \cdot \gamma \cdot \left(2 - \frac{V_{NSAT}}{V_{PSAT}} - \frac{V_{PSAT}}{V_{NSAT}}\right)$$

Normally $V_{PSAT} = -V_{NSAT}$ so the triangular and the square waveforms are symmetrical and have period

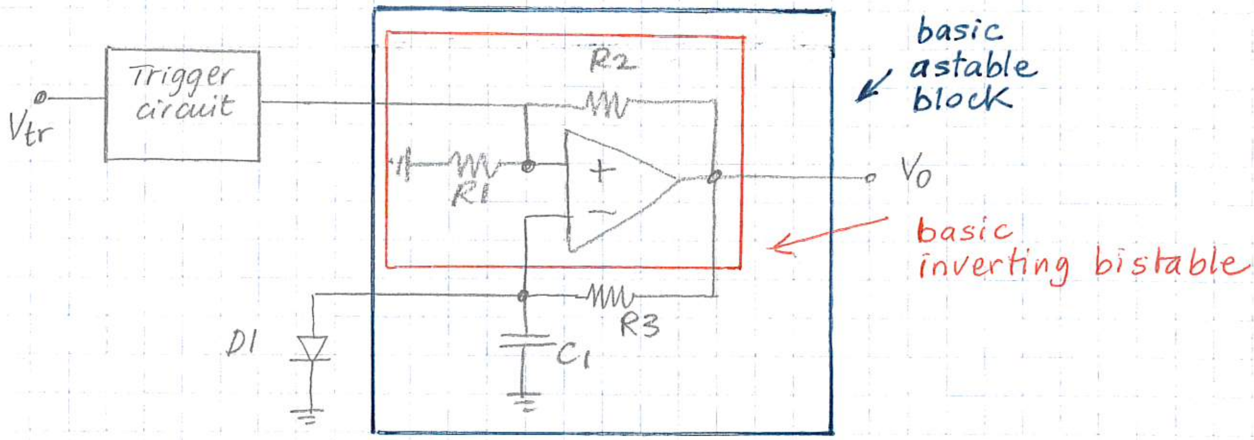
$$T = 4 RC \gamma$$

- NOTE: <u>this</u> astable multivibrator generates both triangular and square waveforms !!



# Monostable multivibrator → generation of a standard pulse

- In some application there is a need for a pulse of known height and width generated in response to a trigger signal
- The monostable vibrator has one stable state in which it can remain indefinitely ↔ a.k.a. one-shot
- The monostable is an augmented form of the astable circuit
  - a clamping diode D1 is added across the capacitor $C_1$ and
  - a trigger circuit composed of capacitor $C_2$, resistor $R_4$ and diode D2 is connected to the non-inverting input terminal of the opamp.




- The circuit operates as follows:

  ① let's start analyzing the circuit operation assuming the "trigger circuit" does not inject any signal

  Theoretically the astable can be either in the state $V_{NSAT}$ or $V_{PSAT}$

  - Let' assume we're in the state $V_{NSAT}$:

$$V_+ = V_{NSAT} \frac{R_1}{R_1+R_2} = V_{NSAT} \cdot \beta$$

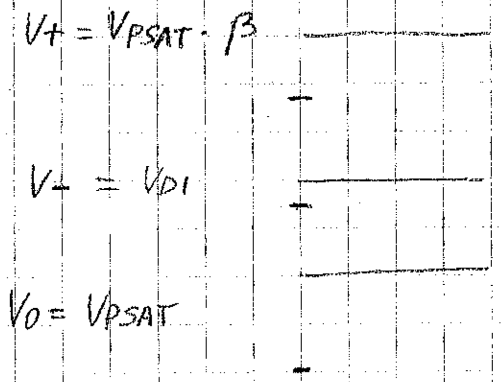
the node $V_-$ starts to charge toward $V_{NSAT}$ (that is a negative value → D1 is OFF) → as soon $V_-$ becomes more negative than $V_+$ the state of the output flips and becomes $V_{PSAT}$

- in the state $V_{PSAT}$ we have that

$$V_+ = V_{PSAT} \frac{R_1}{R_1+R_2} = V_{PSAT} \cdot \beta$$

and the node $V_-$ starts to charge toward $V_{PSAT}$ → however due to the addition of $D_1$ the node $V_-$ cannot charge above $V_{D1}$ → therefore since the circuit is designed such that $V_{PSAT} \cdot \beta > V_{D1}$

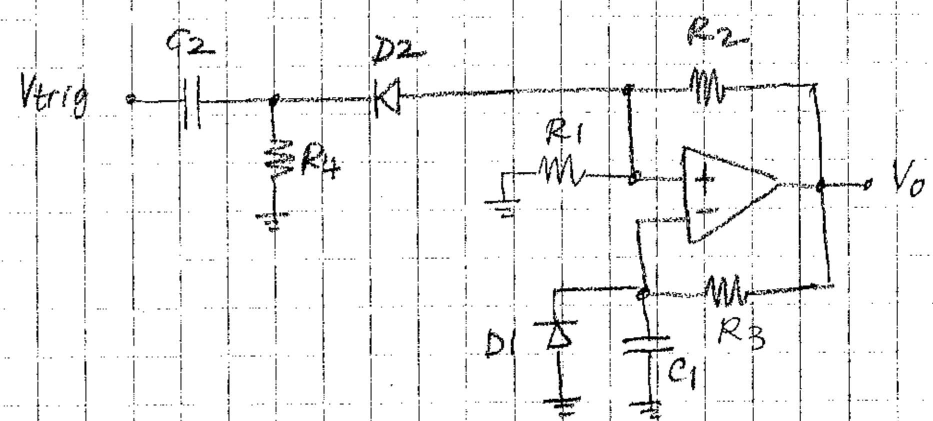
→ in absence of triggering the output of the monostable is always stuck at $V_{PSAT}$ !




② Let's now think about what can we do to make the output of the circuit leave the $V_{PSAT}$ state and go in the $V_{NSAT}$ state for a short amount of time (→ in other words what can we do to create a pulse)

↓

the answer is easy we need to temporally cause $V_+$ to become smaller than $V_- = V_{D1}$



- If we keep $V_{trig}$ constant $C_2$ acts as an open and D2 is ON $\rightarrow$ as long as we make $R_4 >> R_1$ the trigger circuit does not have any effect on the behavior of the multivibrator:

$$V_+ \simeq \frac{R_1 // R_4}{R_1 // R_4 + R_2} V_{PSAT} \approx \frac{R_1}{R_1 + R_2} V_{PSAT} = \beta V_{PSAT}$$

- now consider the application of a negative going step at the trigger input

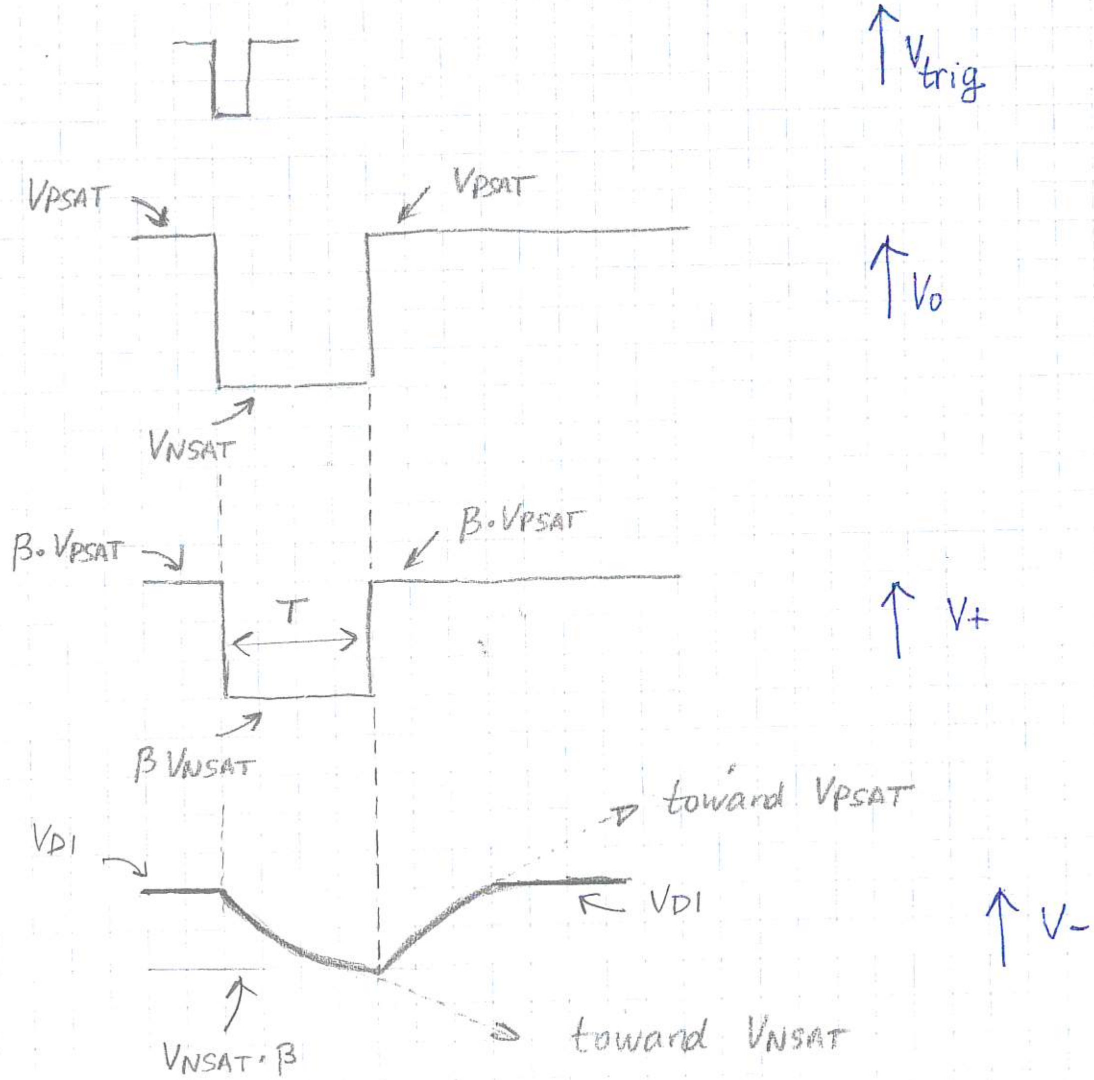
Vtrig 

the capacitance $C_2$ experience a surge of current that to be sustained requires D2 to conduct more heavily ($\rightarrow$ since the current through D2 comes from the output node by flowing through R2, the extra current current through D2 is "stolen" away from R1 $\rightarrow$ the voltage at the node $V_+$ is pulled down

If the trigger signal is of sufficient height to cause $V_+$ to go below $V_-$ ($= V_{D1}$) the opamp will switch its output to $V_{NSAT}$. $\rightarrow$ this will in turn causes $V_+$ to go negative ($V_+ = V_{NEG} \cdot \beta$) and causes D2 to cut-off ($\Rightarrow$ thus isolating the trigger circuit and any further changes at the input trigger terminal

The negative voltage $V_{NSAT}$ at the output of the opamp causes D1 to cut-off, and $C_1$ begins to "discharge" from $V_{D1}$ toward $V_{NSAT}$ with time constant $R_3 C_1$

The monostable multivibrator stays in its state $V_{NSAT}$ (quasi-stable state) until $V_-$ goes below the voltage $\beta V_{NSAT}$.




The duration $T$ of the output pulse is determined by the exponential

$$v_-(t) = V_\infty - (V_\infty - V_{0+})\, e^{-t/\tau}$$

$$\Updownarrow$$

$$V_{NSAT} \cdot \beta = V_{NSAT} - (V_{NSAT} - V_{D1})\, e^{-T/\tau}$$

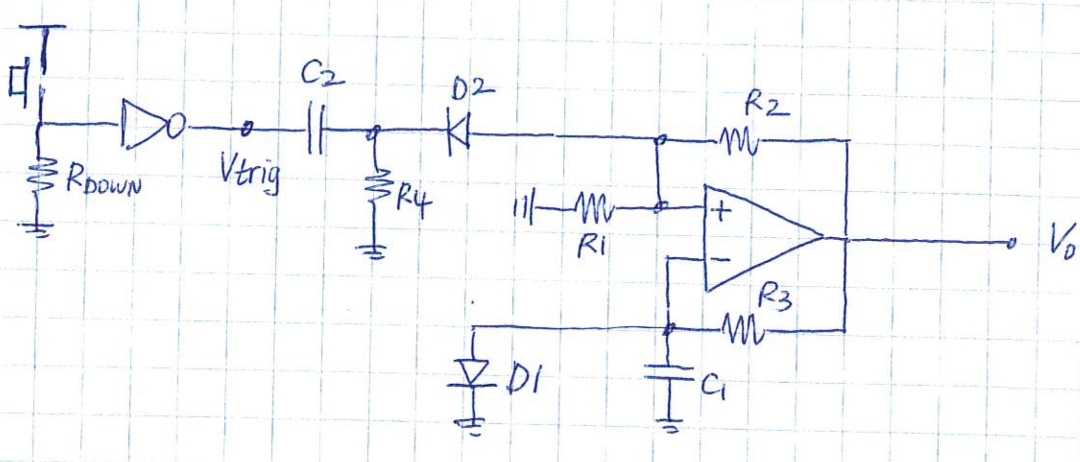
$$\Updownarrow$$

$$-\frac{V_{NSAT} \cdot \beta - V_{NSAT}}{V_{NSAT} - V_{D1}} = e^{-T/\tau} \longleftrightarrow \frac{V_{NSAT} \cdot \beta - V_{NSAT}}{V_{D1} - V_{NSAT}} = e^{-t/\tau}$$

$$\Downarrow$$

$$T = \tau \ln \frac{V_{D1} - V_{NSAT}}{V_{NSAT}(\beta - 1)} \approx R_3 C_1 \ln\left(\frac{1}{1-\beta}\right)$$

$$|V_{NSAT}| \gg V_{D1}$$




Note : the monostable circuit should not be triggered again until the capacitor $C_1$ has been recharged to $V_{D1}$ otherwise the resulting output pulse will be shorter than normal → This recharging time is known as the recovery period

$$V_{D1} = V_{PSAT} - \left( V_{PSAT} - V_{NSAT} \cdot \beta \right) e^{-T_{rec}/\tau}$$

$$\Updownarrow$$

$$-\frac{V_{D1} - V_{PSAT}}{V_{PSAT} - V_{NSAT} \cdot \beta} = e^{-T_{rec}/\tau}$$

$$\Updownarrow$$

$$\ln \frac{V_{PSAT} - V_{D1}}{V_{PSAT} - V_{NSAT} \cdot \beta} = -\frac{T_{rec}}{\tau}$$

$$\Updownarrow$$

$V_{PSAT} \gg V_{D1}$

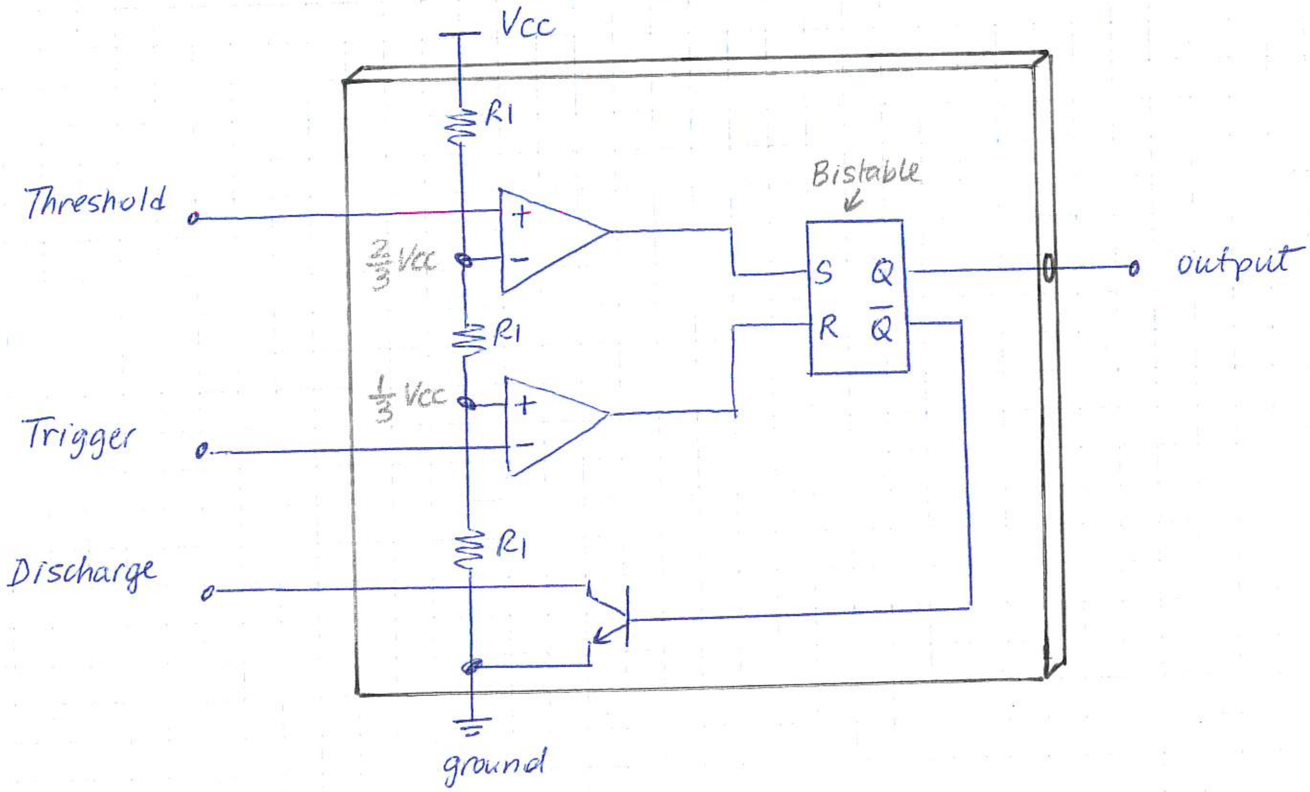
$$T_{rec} = \tau \ln \frac{V_{PSAT} - V_{NSAT} \cdot \beta}{V_{PSAT} - V_{D1}} \approx \tau \ln \frac{V_{PSAT} - V_{NSAT}\beta}{V_{PSAT}} =$$

$$= \tau \ln \left( 1 - \beta \frac{V_{NSAT}}{V_{PSAT}} \right)$$

for the case $V_{PSAT} = -V_{NSAT} \rightarrow T_{rec} \approx \tau \ln (1 + \beta)$

# The 555 Integrated Circuit

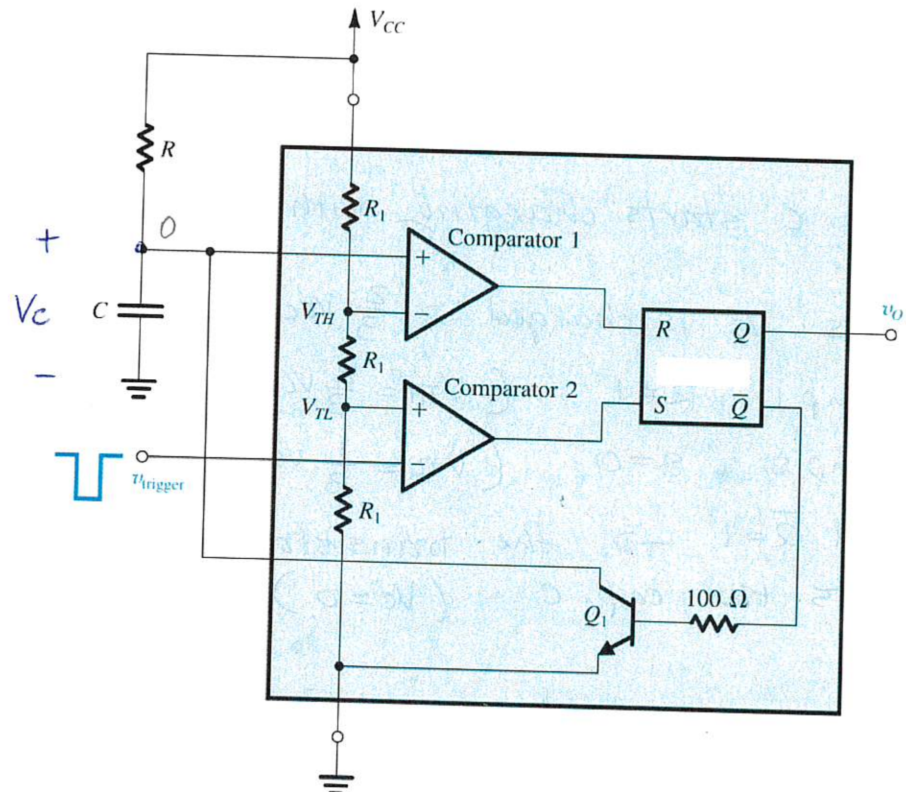
The 555 timer circuit is a very common IC that allows to implement monostable and astable multivibrators with precise characteristics



# Example : Monostable multivibrator using the 555

## ① start-up

C is uncharged, $V_{trigger} = V_{cc}$, "guess" $\overline{Q} = 0$
($V_c = 0$)




(a)

output cmp. 1 $\quad R = 0 \quad$ ($V_{TH} = \frac{2}{3}V_{cc}$ $\quad V_c = 0$)
output cmp 2 $\quad S = 0 \quad$ ($V_{TL} = \frac{1}{3}V_{cc}$ $\quad V_{tr} = V_{cc}$)
$\overline{Q} = 0$ → the transistor $Q_1$ is switched OFF → the cap. C start charging with time constant RC → as soon as $V_c > \frac{2}{3}V_{cc}$ →

→ output cmp. 1 $\quad R = 1 \quad$ ($V_{TH} = \frac{2}{3}V_{cc}$ $\quad V_c > \frac{2}{3}V_{cc}$)
output cmp 2 $\quad S = 0 \quad$ ($V_{TL} = \frac{1}{3}V_{cc}$ $\quad V_{tr} = V_{cc}$)
$Q = 0$ and $\overline{Q} = 1$ → the transistor $Q_1$ is switched ON and shorts the cap. C (→ $V_c = 0$)

→ output cmp 1 $\quad R = 0 \quad$ ($V_{TH} = \frac{2}{3}V_{cc}$ $\quad V_c = 0$)
output cmp 2 $\quad S = 0 \quad$ ($V_{TL} = \frac{1}{3}V_{cc}$ $\quad V_{tr} = V_{cc}$)
$Q = 0$ and $\overline{Q} = 1$ → the transistor $Q_1$ stay switched ON and shorts the cap C

↓

$V_o$ is stable at $V_{cc}$ ( $V_o = Q$ of the S-R latch)

② Vtrigger goes below $V_{TL} = \frac{1}{3} V_{cc}$

output cmp 1 $R = 0$ ( $V_{TH} = \frac{2}{3} V_{cc}$ $V_c = 0$ )
output cmp 2 $S = 1$ ( $V_{TL} = \frac{1}{3} V_{cc}$ $V_{tr} < \frac{1}{3} V_{cc}$ )
$Q = 1$ and $\overline{Q} = 0$ → the transistor Q1 turns off

so the cap. C starts charging with time const. RC

as soon as $V_c$ is charged $> \frac{2}{3} V_{cc}$

output cmp 1 $R = 1$ ( $V_{TH} = \frac{2}{3} V_{cc}$ $V_c > \frac{2}{3} V_{cc}$ )
output cmp 2 $S = 0$ ( $V_{TL} = \frac{1}{3} V_{cc}$ $V_{tr} = V_{cc}$ )
$Q = 0$ and $\overline{Q} = 1$ → the transistor Q1 turns ON

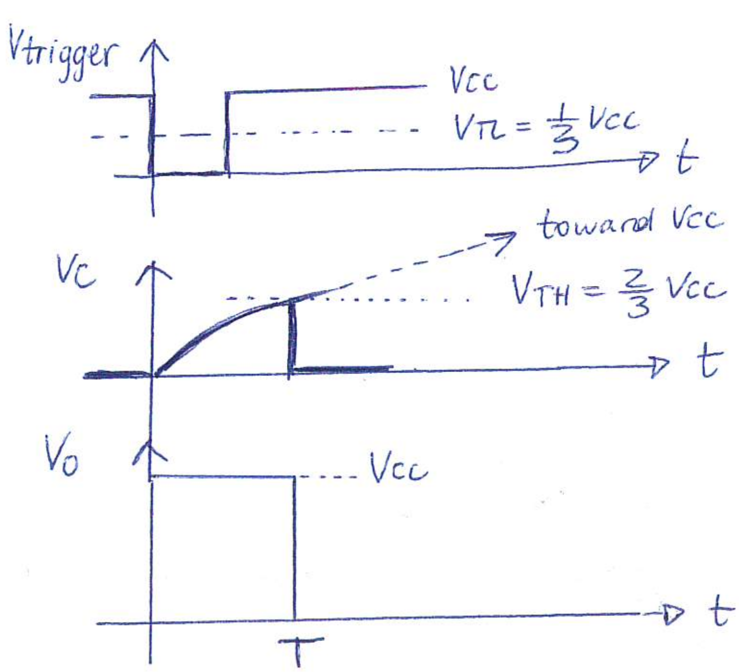
and shorts the cap. C ( $V_c = 0$ )

→ output cmp 1 $R = 0$ ( $V_{TH} = \frac{2}{3} V_{cc}$ $V_c = 0$ )
output cmp 2 $S = 0$ ( $V_{TL} = \frac{1}{3} V_{cc}$ $V_{tr} = V_{cc}$ )
$Q = 0$ and $\overline{Q} = 1$

the trigger signal is shorter than the time it takes the cap to charge up to $\frac{2}{3} V_{cc}$

↓

$V_o$ is back to the stable state $V_{cc}$




$$\frac{2}{3} V_{cc} = V_{cc}\left(1 - e^{-T/\tau}\right)$$

↕

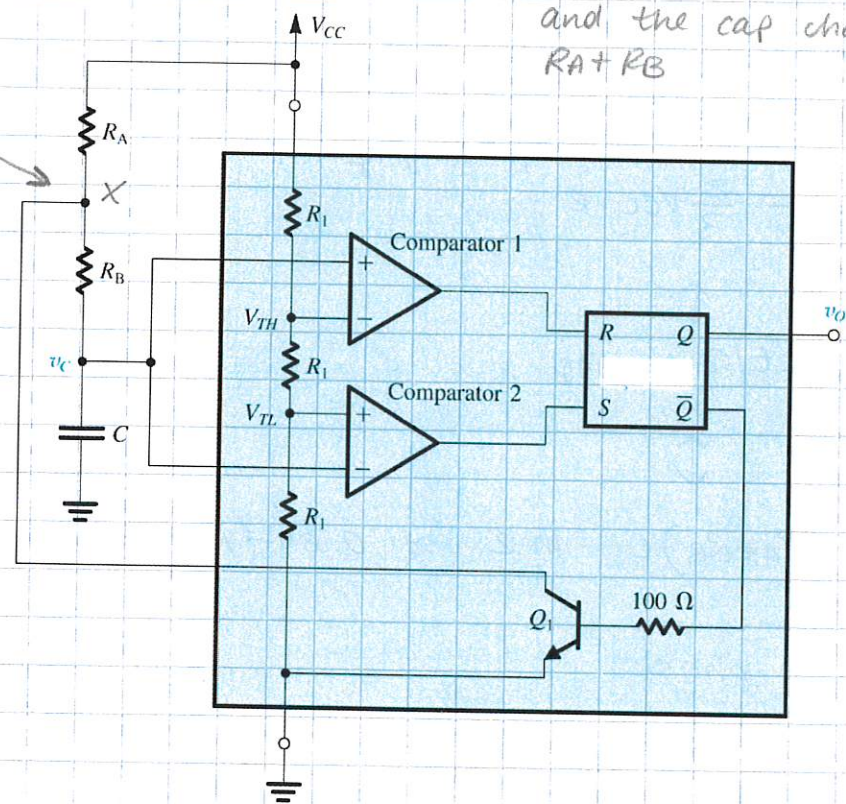
$$T = RC \cdot \ln 3 \approx 1.1 \times RC$$

## Example : Astable multivibrator using the 555

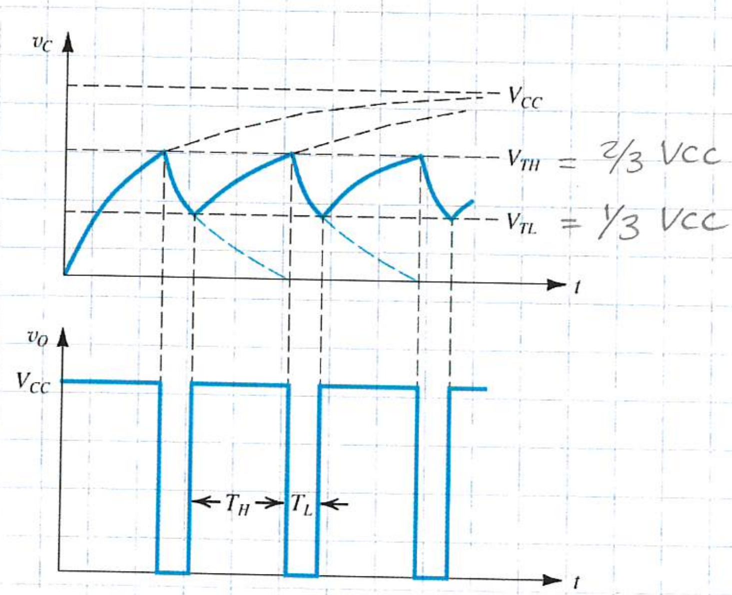
Assume that initially C is discharged and S-R latch is set $(Q=1, \bar{Q}=0)$.

if $\bar{Q}=1$ this node is at gnd (then the cap. discharges on $R_B$)

if $\bar{Q}=0$ the node x is floating and the cap charges through $R_A + R_B$




(a)




(b)

recall:

$$v_C(t) = V_\infty - (V_\infty - V_{0+})\,e^{-t/\tau}$$

- charging phase

$$\frac{2}{3}V_{CC} = V_{CC} - \left(V_{CC} - \frac{1}{3}V_{CC}\right) e^{-T_H/(R_A+R_B)C}$$

$$\Updownarrow$$

$$-\frac{1}{3}V_{CC} = -\frac{2}{3}V_{CC}\, e^{-T_H/\tau_{charging}}$$

$$\Updownarrow$$

$$1/2 = e^{-/\tau_{charging}}$$

$$\Updownarrow$$

$$T_H = (R_A+R_B)C \cdot \ln 2 \approx 0.69\,(R_A+R_B)C$$

- discharging phase

$$T_L = R_B \cdot C \cdot \ln 2 \cong 0.69\, R_B \cdot C$$

$$\Downarrow$$

$$T = T_H + T_L = 0.69C\,(R_A + 2R_B)$$

$$\text{duty cycle} \equiv \frac{T_H}{T} = \frac{R_A+R_B}{R_A+2R_B}$$

the duty cycle will always be greater than 50%

it approaches 0.5 for $R_A \ll R_B$ (at expense of increased supply current)