Semiconductors and Diodes

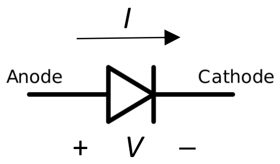
Table 3.1 Summary of Important Equations

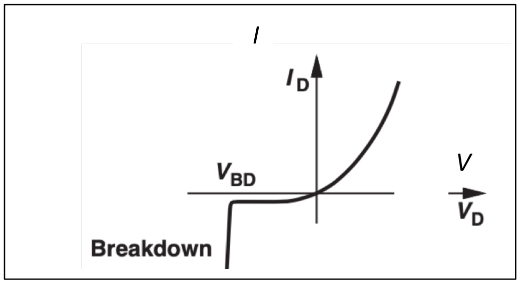
Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Carrier concentration in intrinsic silicon (cm^{-3})	$n_i = BT^{3/2}e^{-E_g/2kT}$	$B = 7.3 \times 10^{15}\ \text{cm}^{-3}\text{K}^{-3/2}$ $E_g = 1.12$ eV $k = 8.62 \times 10^{-5}$ eV/K $n_i = 1.5 \times 10^{10}/\text{cm}^3$
Diffusion current density (A/cm^2)	$J_p = -qD_p \frac{dp}{dx}$ $J_n = qD_n \frac{dn}{dx}$	$q = 1.60 \times 10^{-19}$ coulomb $D_p = 12\ \text{cm}^2/\text{s}$ $D_n = 34\ \text{cm}^2/\text{s}$
Drift current density (A/cm^2)	$J_{\text{drift}} = q(p\mu_p + n\mu_n)E$	$\mu_p = 480\ \text{cm}^2/\text{V} \cdot \text{s}$ $\mu_n = 1350\ \text{cm}^2/\text{V} \cdot \text{s}$
Resistivity ($\Omega \cdot \text{cm}$)	$\rho = 1/[q(p\mu_p + n\mu_n)]$	μ_p and μ_n decrease with the increase in doping concentration
Relationship between mobility and diffusivity	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$	$V_T = kT/q \simeq 25.9$ mV
Carrier concentration in *n*-type silicon (cm^{-3})	$n_{n0} \simeq N_D$ $p_{n0} = n_i^2/N_D$	
Carrier concentration in *p*-type silicon (cm^{-3})	$p_{p0} \simeq N_A$ $n_{p0} = n_i^2/N_A$	
Junction built-in voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	
Width of depletion region (cm)	$\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $W = x_n + x_p$ $= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$	$\epsilon_s = 11.7\epsilon_0$ $\epsilon_0 = 8.854 \times 10^{-14}$ F/cm

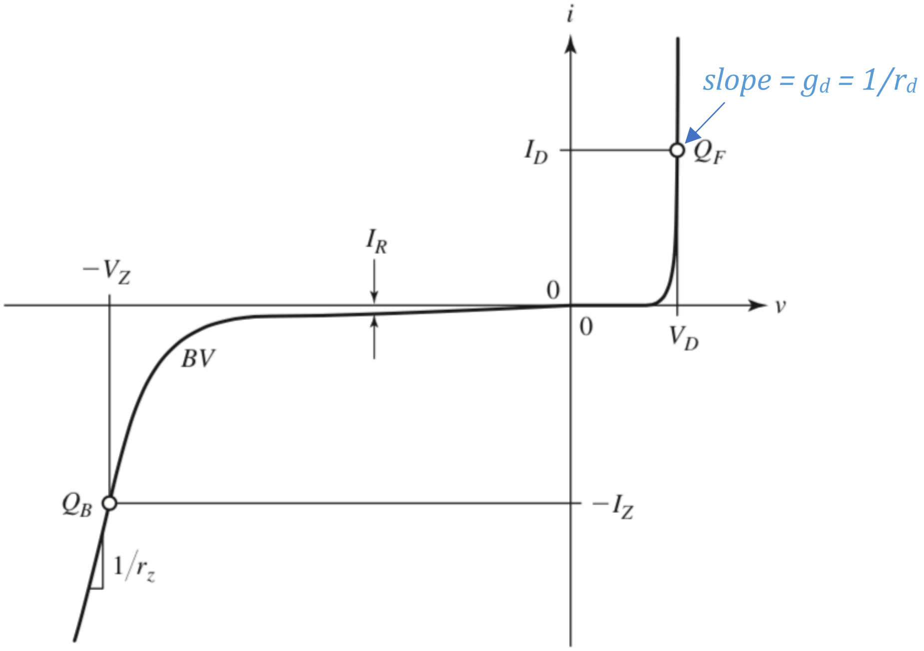
PN junctions
(diodes)

Table 3.1 *continued*

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Charge stored in depletion layer (coulomb)	$Q_J = q\frac{N_A N_D}{N_A + N_D}AW$	
Forward current (A)	$I = I_p + I_n$ $I_p = Aqn_i^2 \frac{D_p}{L_p N_D}\left(e^{V/V_T} - 1\right)$ $I_n = Aqn_i^2 \frac{D_n}{L_n N_A}\left(e^{V/V_T} - 1\right)$	
Saturation current (A)	$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$	
I–V relationship	$I = I_S\left(e^{V/V_T} - 1\right)$	
Minority-carrier lifetime (s)	$\tau_p = L_p^2/D_p \qquad \tau_n = L_n^2/D_n$	$L_p, L_n = 1\ \mu\text{m}$ to $100\ \mu\text{m}$ $\tau_p, \tau_n = 1$ ns to 10^4 ns
Minority-carrier charge storage (coulomb)	$Q_p = \tau_p I_p \qquad Q_n = \tau_n I_n$ $Q = Q_p + Q_n = \tau_T I$	
Depletion capacitance (F)	$C_{j0} = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_0}}$ $C_j = C_{j0}\Big/\left(1 + \frac{V_R}{V_0}\right)^m$	$m = \frac{1}{3}$ to $\frac{1}{2}$
Diffusion capacitance (F)	$C_d = \left(\frac{\tau_T}{V_T}\right)I$	



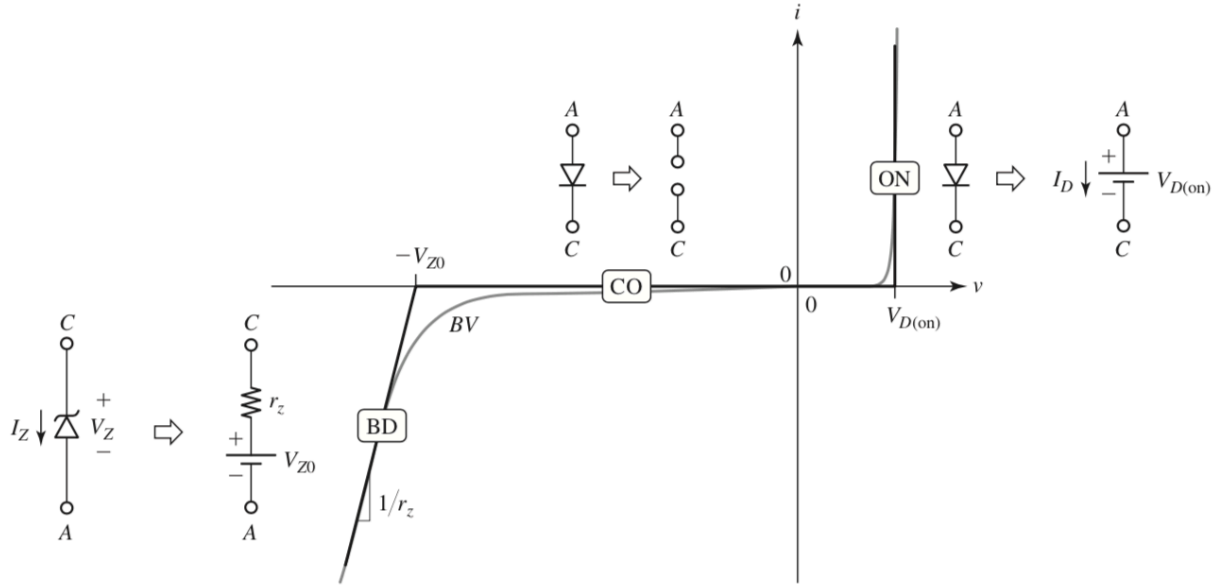
	*I vs. V equation for diode in forward and reverse region:* $I = I_S\left(e^{V/V_T} - 1\right)$ *V is the voltage across the diode from anode to cathode* *I is the current through the diode from anode to cathode* $V_T = \frac{kT}{q}$ V_T = 25.9 mV at room temperature (= 300 degree Kelvin)



slope = $g_d = 1/r_d$

FIGURE 1.46 The complete *i*-*v* characteristic of a *pn* junction.

Approximations of the diode three regions of operation



The *slope* of the diode curve at a given operating current I_D in the forward-bias region is:

$$g_d = \frac{I_D}{V_T}$$

In forward region to cause a *decade* change in I_D we need to change V_D by 60 mV.

Temperature affects the forward region as follows:

$$V_D(T) \cong V_D(T_0) - (2\text{ mV}) \times (T - T_0)$$

and the reverse region as follows:

$$I_R(T) \cong I_R(T_0) \times 2^{(T-T_0)/10}$$

MOSFET in saturation mode ($V_{GS} > V_{TH}$ and $V_{DS} > V_{GS} - V_{TH}$)

$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

Key Small Signal Parameters for the MOSFET in saturation mode:

$$g_m = \mu C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) = \frac{2I_D}{V_{GS} - V_{TH}}$$
$$r_O \approx \frac{1}{\lambda I_D}$$

BIPOLAR in active mode ($V_{BE} \approx 0.7$ V and $V_{CE} > V_{CE,sat} \approx 1$V)

$$I_C \approx I_S e^{V_{BE}/V_T}$$

$$V_T = \frac{kT}{q}$$

V_T = 25.9 mV at room temperature (= 300 degree Kelvin)

$$I_E = I_B + I_C = (\beta + 1)I_B$$

$$\beta = I_C/I_B$$

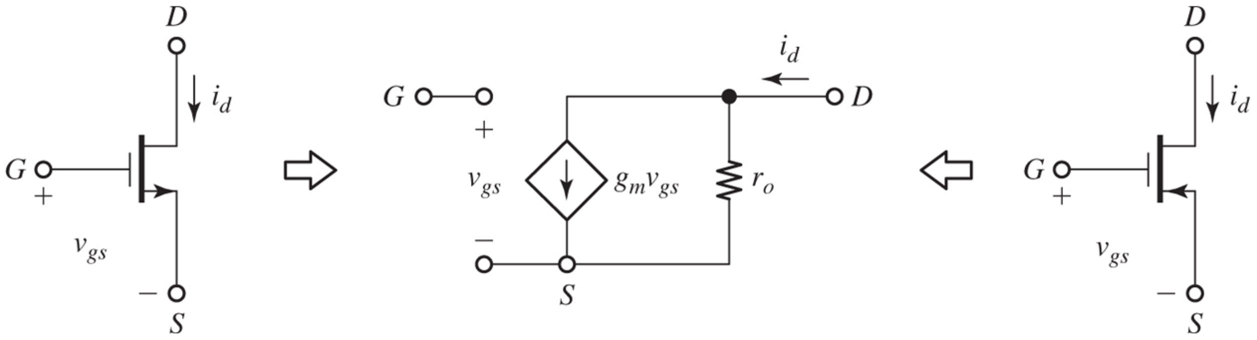
Key Small Signal Parameters for the BIPOLAR in active mode

$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \frac{\beta}{g_m}$$

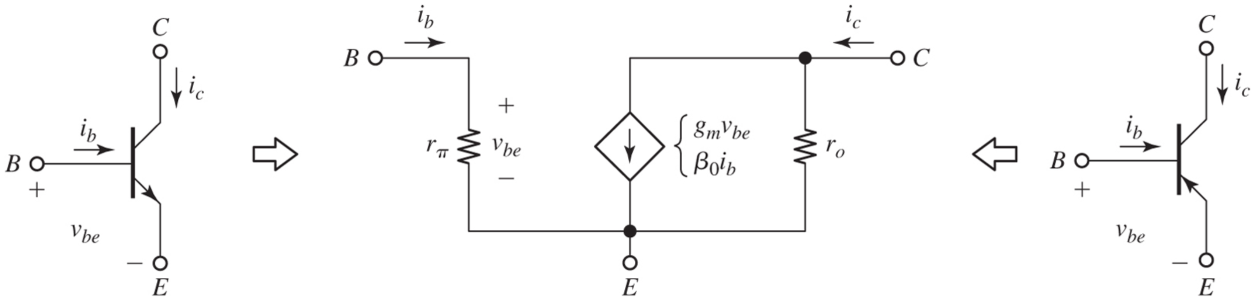
$$r_O = \frac{V_A}{I_C}$$

Small signal model for MOSFET in saturation mode



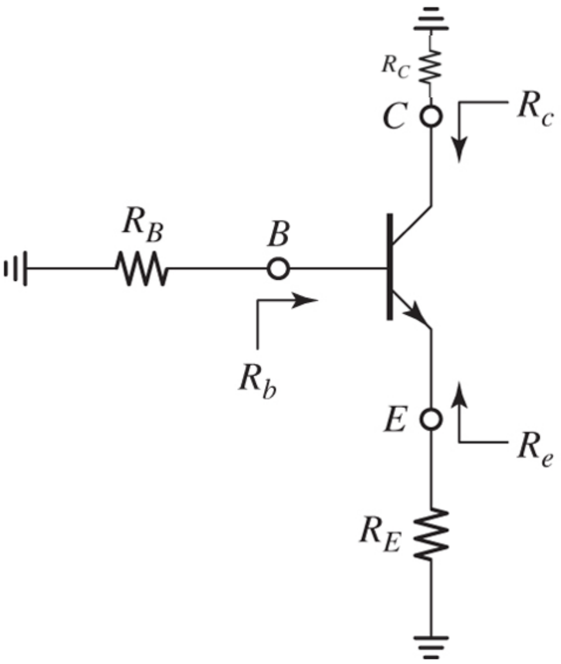
Small-signal MOSFET model. This model applies to both *nMOSFETs* and pMOSFETs

Small signal model for BIPOLAR in active mode



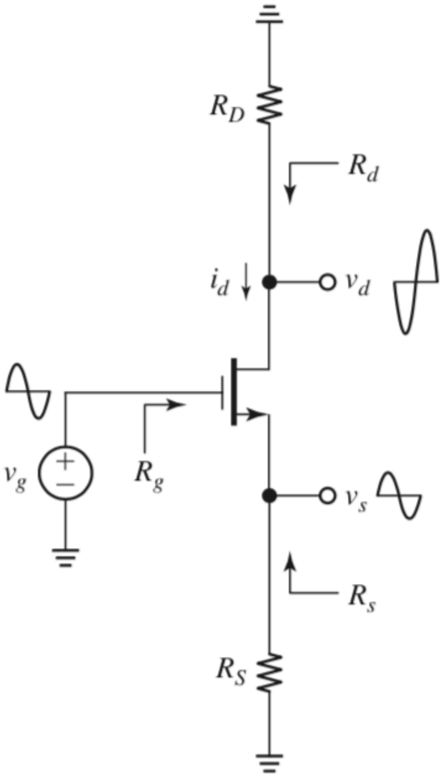
Small-signal BJT model. This model applies to both *npn* and *pnp* BJTs.

Small signal resistances looking into the BJT's terminals



Exact	***Approximate***
$R_b = r_\pi + (\beta+1)R_E\left(\frac{r_o + \frac{R_C}{\beta+1}}{r_o+R_C+R_E}\right)[GHLM]$	$R_b \approx r_\pi + (\beta+1)R_E \quad (r_o \gg R_E, r_o \gg R_C)$
$R_e = \left[\frac{(r_o+R_C)(r_\pi+R_B)}{r_\pi+R_B+\beta r_o}\right] \parallel (r_\pi+R_B)$	$R_e \approx \left[\frac{(r_\pi+R_B)}{\beta+1}\right] \quad (r_o \gg R_C;\ g_m r_o \gg 1; r_o \gg \frac{R_B}{\beta})$
$R_c = r_o\left[1+\frac{g_m r_\pi R_E}{r_\pi+R_B+R_E}\right]+(r_\pi+R_B) \parallel R_E$	$R_c \approx r_o[1+g_m(r_\pi \parallel R_E)] \quad (r_\pi \gg R_B; g_m r_o \gg 1)$

Small signal terminal resistances for the MOSFET

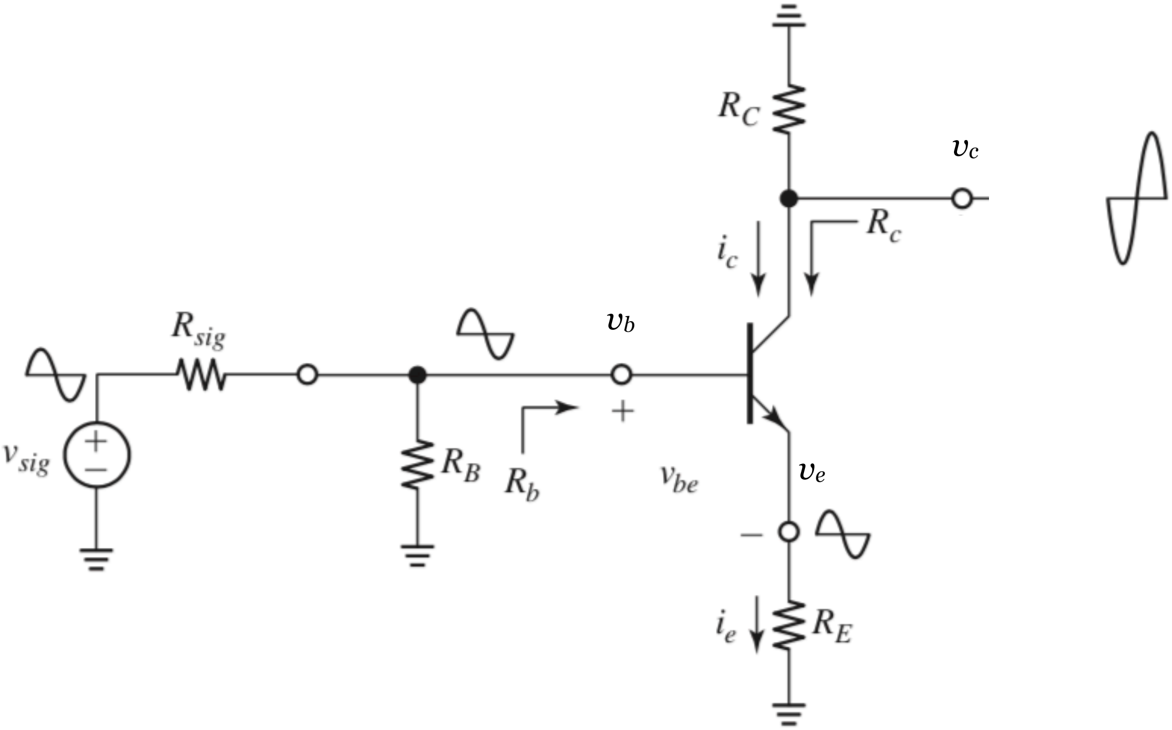


Exact	Approximate
$R_g = \infty$	$R_g = \infty$
$R_s = \left(\frac{1}{g_m}//r_o\right) + \frac{R_D}{1 + g_m r_o}$	$R_s \cong \frac{1}{g_m}$
$R_d = r_o(1 + g_m R_S) + R_S$	$R_d \cong r_o(1 + g_m R_S)$

$$NOTE: \quad R_s = \left(\frac{1}{g_m}||r_o\right) + \frac{R_D}{(1 + g_m r_o)} = \frac{R_D + r_o}{(1 + g_m r_o)}$$

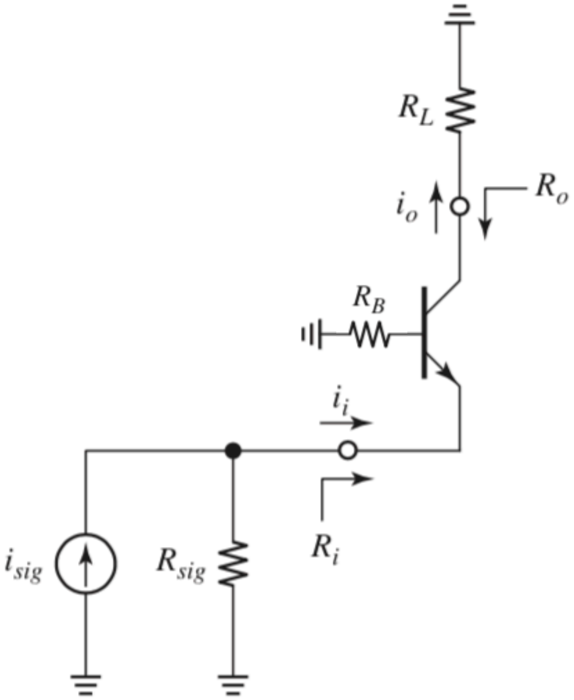
Summary of small signal gains for BJT based topologies

(A)



Exact	Approximate
with $R_E \neq 0$ *[GHLM]:* $\frac{v_c}{v_b} = -\frac{R_C(\beta r_o - R_E)}{R_E[R_C + r_o(\beta + 1)] + r_\pi(R_C + R_E + r_o)}$	*with* $R_E \neq 0$*:* $\frac{v_c}{v_b} = -\frac{g_m R_C}{1 + g_m R_E}$ $r_o \gg R_C + R_E; \beta \gg 1$
with $R_E = 0$*:* $\frac{v_c}{v_b} = -g_m(R_C \vert\vert r_o)$	*with* $R_E = 0$*:* $\frac{v_c}{v_b} = -g_m R_C$ $r_o \gg R_C$
with $R_C \neq 0$ *[GHLM]:* $\frac{v_e}{v_b} = \frac{1}{1 + \frac{r_\pi(R_C + R_E + r_o)}{R_E[R_C + r_o(\beta + 1)]}}$	*with* $R_C \neq 0$*:* $\frac{v_e}{v_b} = \frac{g_m R_E}{g_m R_E + 1}$ $r_o \gg R_C + R_E; \beta \gg 1$
with $R_C = 0$ *[GHLM]:* $\frac{v_e}{v_b} = \frac{(\beta + 1)/r_\pi}{\frac{1}{R_E} + \frac{1}{r_o} + \frac{\beta + 1}{r_\pi}}$	*with* $R_C = 0$ *[GHLM]:* $\frac{v_e}{v_b} = \frac{g_m R_E}{g_m R_E + 1}$ $g_m r_o \gg 1;\ \beta \gg 1$

(B)



$$i_o = \alpha\, i_i = \frac{\beta}{\beta + 1} i_i$$

$$i_i = \frac{R_{sig}}{R_{sig} + R_i} i_{sig}$$

$$v_{be} = -v_e \frac{r_\pi}{r_\pi + R_B}$$

Exact:

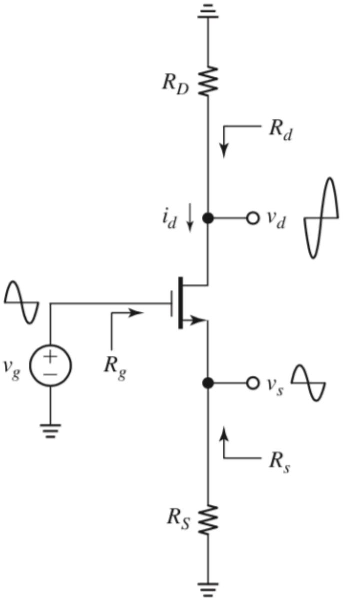
$$\frac{v_c}{v_e} = \frac{g_m R_L \frac{r_\pi}{r_\pi + R_B} + \frac{R_L}{r_o}}{1 + \frac{R_L}{r_o}}$$

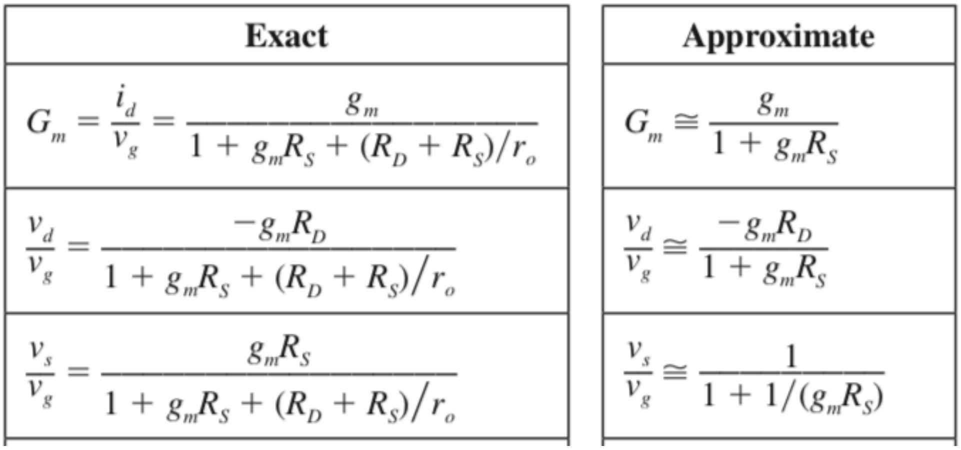
Approximate:

$$\frac{v_c}{v_e} = g_m (R_L || r_o) \qquad r_\pi \gg R_B$$

Summary of small signal gains for MOSFET based topologies

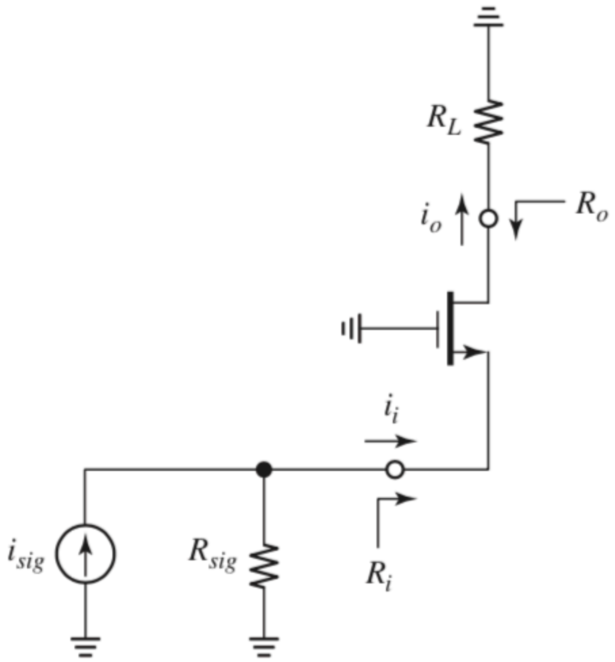
(A)





Exact	Approximate
$G_m = \frac{i_d}{v_g} = \frac{g_m}{1 + g_m R_S + (R_D + R_S)/r_o}$	$G_m \cong \frac{g_m}{1 + g_m R_S}$
$\frac{v_d}{v_g} = \frac{-g_m R_D}{1 + g_m R_S + (R_D + R_S)/r_o}$	$\frac{v_d}{v_g} \cong \frac{-g_m R_D}{1 + g_m R_S}$
$\frac{v_s}{v_g} = \frac{g_m R_S}{1 + g_m R_S + (R_D + R_S)/r_o}$	$\frac{v_s}{v_g} \cong \frac{1}{1 + 1/(g_m R_S)}$

(B)



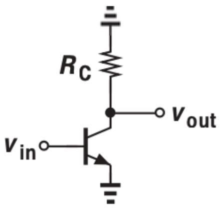
$$i_o = i_i$$

$$\frac{i_o}{i_{sig}} = \frac{1}{1 + \frac{r_o + R_L}{(1 + g_m r_o)R_{sig}}}$$

$$\frac{v_d}{v_s} = +g_m(R_L//r_o)$$

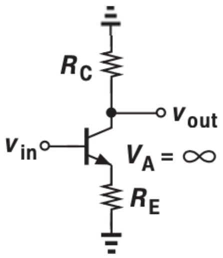
$$\frac{1}{g_m}||r_\pi \approx \frac{1}{g_m}\ (\beta \gg 1)$$

Very Common Cases: Voltage Gain Equations $for\ \beta \gg 1 \rightarrow \left(r_\pi || \frac{1}{g_m}\right) \approx \frac{1}{g_m}$



$$A_v = -g_m(R_C || r_O)$$

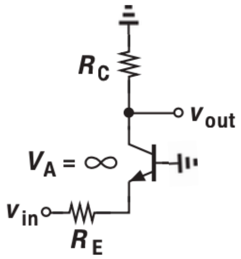
$$v_{out} = -R_c g_m v_{be}$$



$$A_v = -\frac{R_C}{\frac{1}{g_m} + R_E}$$

$$v_{in} = v_{be} + R_E(\beta + 1)\frac{v_{be}}{r_\pi} \approx v_{be} + R_E g_m v_{be}$$

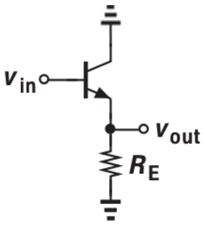
$$v_{out} = R_c g_m v_e$$



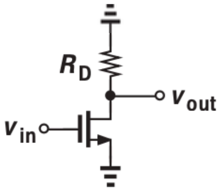
$$A_v = \frac{R_C}{\frac{1}{g_m} + R_E}$$

$$v_e = v_{in}\frac{\left(r_\pi || \frac{1}{g_m}\right)}{R_E + \left(r_\pi || \frac{1}{g_m}\right)} \approx v_{in}\frac{1}{R_E g_m + 1}$$

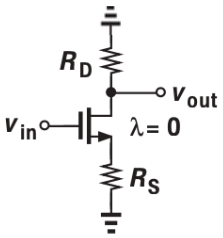
*



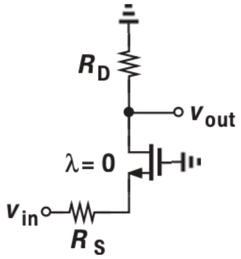
$$A_v = \frac{R_E || r_O}{\frac{1}{g_m} + R_E || r_O}$$



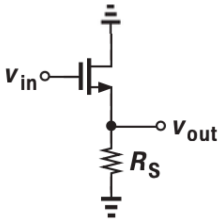
$$A_v = -g_m(R_D || r_O)$$



$$A_v = -\frac{R_D}{\frac{1}{g_m} + R_S}$$



$$A_v = \frac{R_D}{\frac{1}{g_m} + R_S}$$



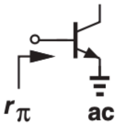
$$A_v = \frac{R_S || r_O}{\frac{1}{g_m} + R_S || r_O}$$

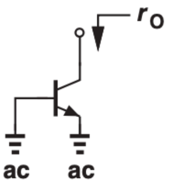
*

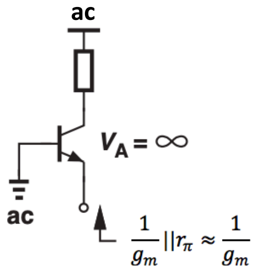
$$v_{out} = (R_E||r_o) g_m v_{be} + (R_E||r_o)\frac{v_{be}}{r_\pi} =$$
$$= \frac{R}{r_\pi} v_{be}(\beta + 1) \approx g_m R v_{be}$$

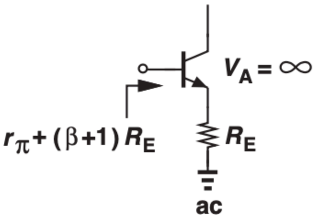
$$v_{in} = v_{be} + v_{out} \approx$$
$$\approx v_{be}(1 + g_m R)$$

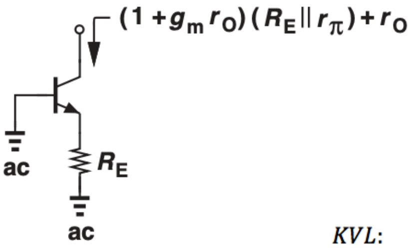
Very Common Cases: Input and Output Impedances $for\ \beta \gg 1 \rightarrow \left(r_\pi || \frac{1}{g_m}\right) \approx \frac{1}{g_m}$

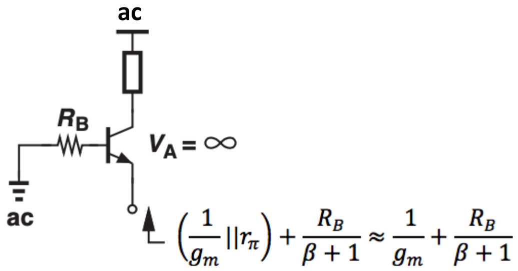












KVL:

$v_t = r_\pi i_t + R_E i_t(\beta + 1)$

R_E can be "moved" in base as $R_B\,(\beta$+1) and E becomes ground

KVL:

$v_e = -r_\pi i_b - R_B i_b = -r_\pi i_b - R_B \frac{i_e}{\beta + 1}$

R_B can be "moved" in emitter as $R_E / (\beta$+1) and B becomes ground