PN Junction and MOS structure

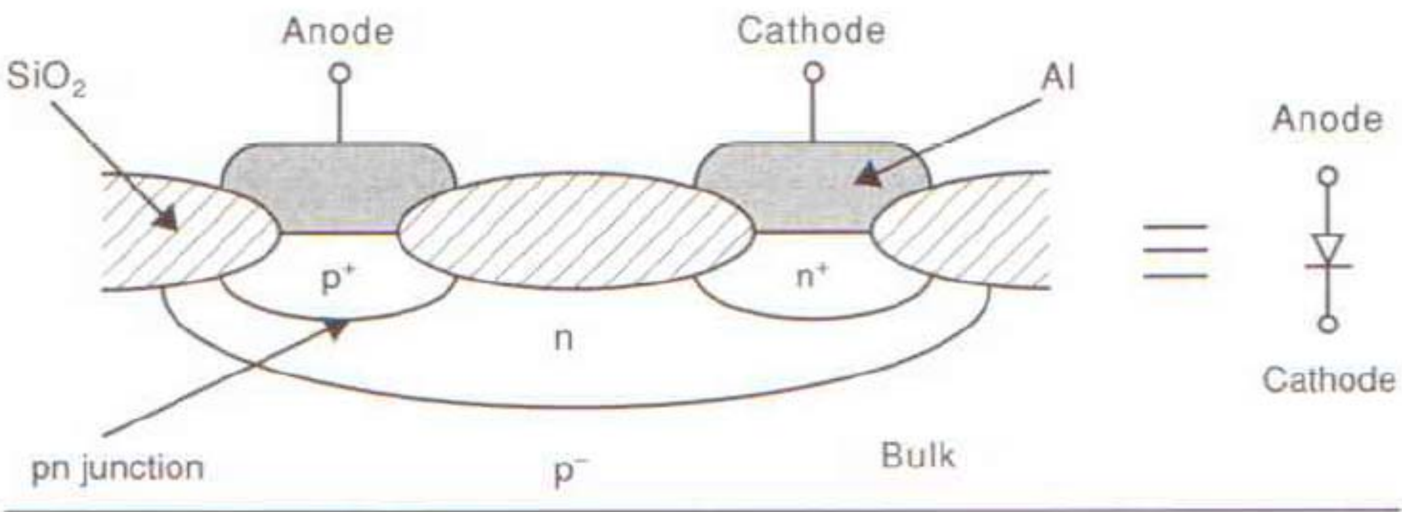


Fig. 1.1 A cross section of a pn diode.

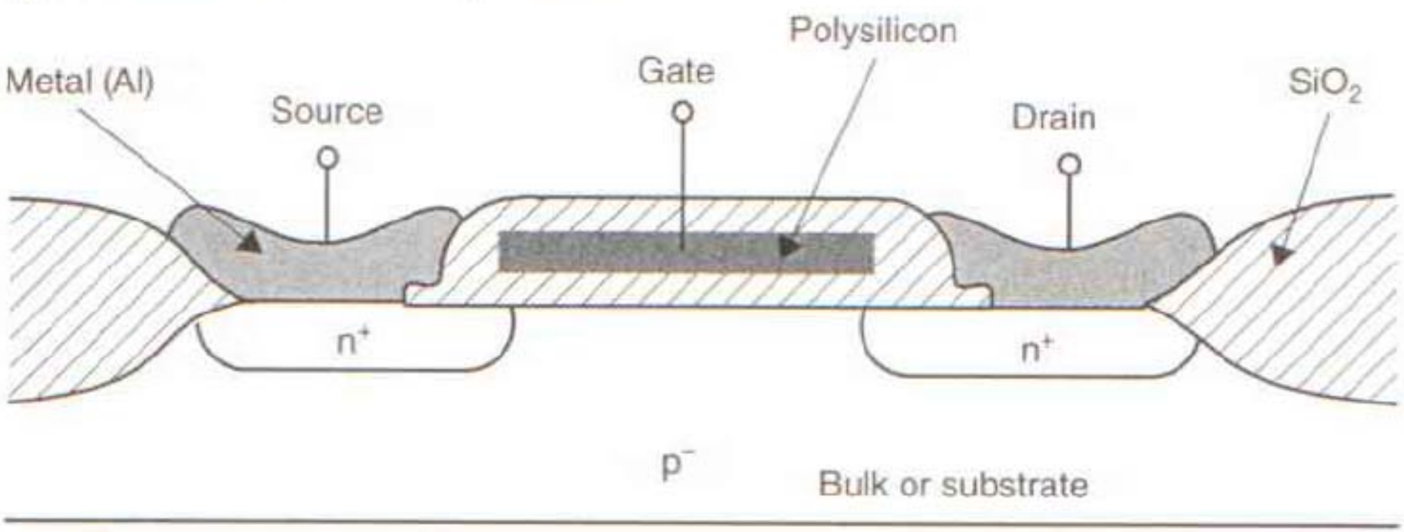


Fig. 1.6 A cross section of a typical n-channel transistor.

Basic electrostatic equations

- We will use simple one-dimensional electrostatic equations to develop insight and basic understanding of how semiconductor devices operate

  – Gauss's Law

  – Potential Equation

  – Poisson's Equation

It puts together Gauss's Law and the potential equation

Gauss's Law

E = electric field [V/m]

$$\boxed{\frac{dE}{dx}=\frac{\rho}{\epsilon}}$$

charge density [C/m^3]

permittivity [F/m]

charge per area in the interval from x_a to x [C/m^2]

$$\int_{x_a}^{x} d\underbrace{[\epsilon E(x)]}=\epsilon_x E(x)-\epsilon_a E(x_a)=\int_{x_a}^{x}\rho(x)\,dx=Q_S(x)$$

The possibility of a change in permittivity due to a material interface has been accounted for by keeping the permittivity together with the field

Potential Equation

reference point

$$\phi(x) - \phi(x_R) = \int_{x_R}^{x} -E(x)\,dx$$

$$\boxed{E(x) = -\frac{d\,\phi(x)}{dx}}$$

Poisson's Equation

- It directly links the potential with the charge distribution (there is no need to go through the field)

$$\begin{cases} \dfrac{dE}{dx} = \dfrac{\rho}{\epsilon} \\ E(x) = -\dfrac{d\phi}{dx} \end{cases} \longrightarrow \boxed{\dfrac{d^2\phi(x)}{dx^2} = -\dfrac{dE(x)}{dx} = -\dfrac{\rho}{\epsilon}}$$

Boundary Conditions

- Electronic devices are made of layers of different materials
- We need conditions for Φ and E at the boundary between two materials

Potential at a boundary

- An abrupt jump of Φ (“along x”) would lead to an infinite electric field at the boundary

$$E(x) = -\frac{d\phi}{dx}$$

- Infinite electric fields are not possible (they would tear the material apart)
- Therefore Φ(x) must be continuous:

$$\boxed{\phi(0^-) = \phi(0^+)}$$

Where the boundary is located at x = 0

Electric Field at a boundary

- The electric field usually jumps at a boundary

$$\int_{-\Delta}^{+\Delta} d[\epsilon E(x)] = \epsilon_2 E(x=+\Delta) - \epsilon_1 E(x=-\Delta) = \int_{-\Delta}^{+\Delta} \rho(x)\, dx$$

- By letting $\Delta \rightarrow 0$:

$$\int_{-\Delta}^{+\Delta} \rho(x)\, dx = 0$$

$$\epsilon_2 E(x=0^+) - \epsilon_1 E(x=0^-) = 0$$

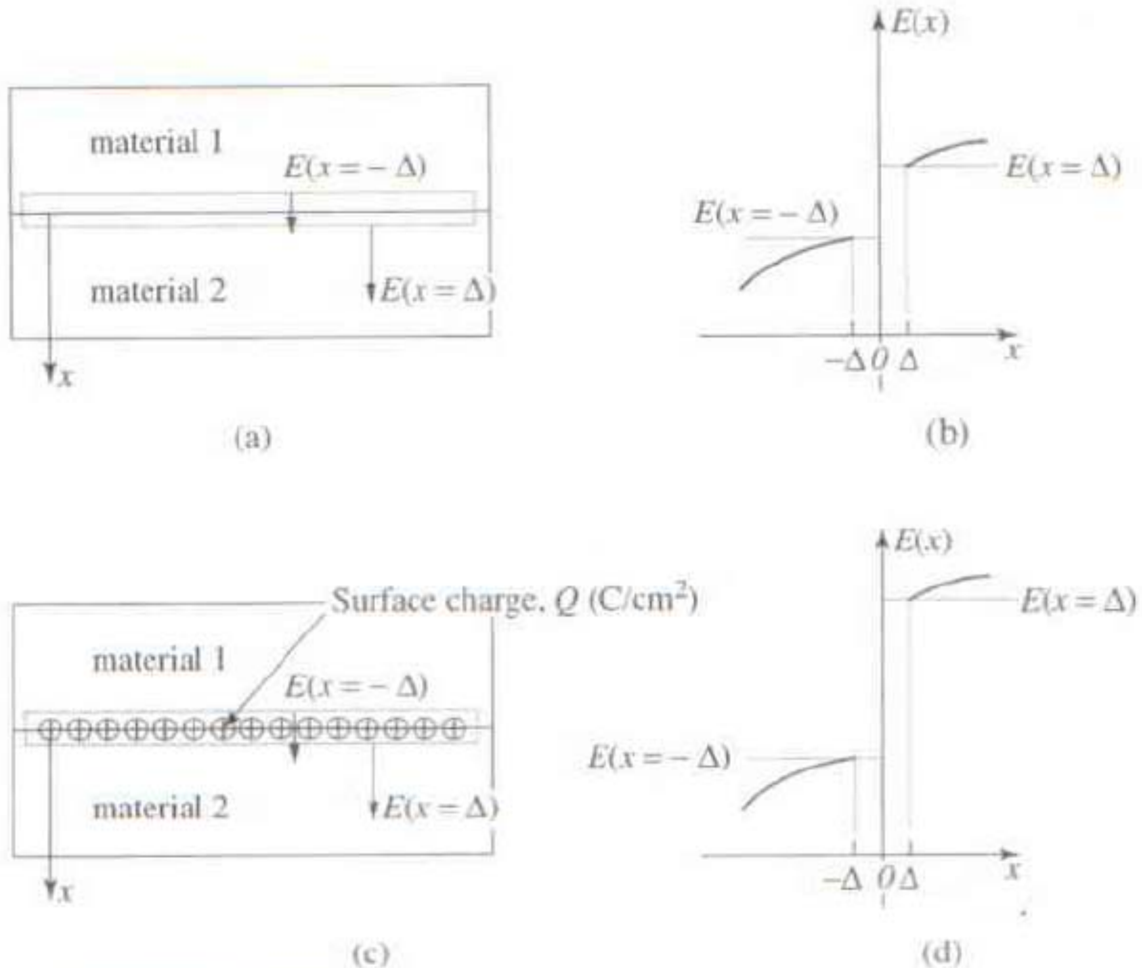
There can be a sheet of charge at the boundary

$$\int_{-\Delta}^{+\Delta} \rho(x)\, dx = Q_S$$

$$\epsilon_2 E(x=0^+) - \epsilon_1 E(x=0^-) = Q_S$$

A sheet of charge is an Infinite amount of charge all distributed on the boundary surface (that is a Dirac function)

Boundary between materials



➤ **Figure 3.4** (a) Boundary between materials 1 and 2 with permittivities $\varepsilon_1 > \varepsilon_2$ and (b) resulting jump in the electric field. (c) Boundary, with a surface charge Q located at the interface and (d) resulting jump in the electric field.

Boundary conditions

$$\boxed{\phi(x=0^{+})=\phi(x=0^{-})}$$

continuity of potential at a boundary

$$\int_{-\Delta}^{+\Delta} \rho(x)\,dx=0$$

$$\boxed{E(x=0^{+})=\frac{\epsilon_1}{\epsilon_2}E(x=0^{-})}$$

electric field jump for charge free boundary

$$\int_{-\Delta}^{+\Delta} \rho(x)\,dx=\mathbb{Q}_S$$

$$\boxed{E(x=0^{+})=\frac{\epsilon_1}{\epsilon_2}E(x=0^{-})+\frac{\mathbb{Q}_S}{\epsilon_2}}$$

electric field jump for charged boundary

Oxide-Silicon interface

- Example of very common interface in microelectronic devices

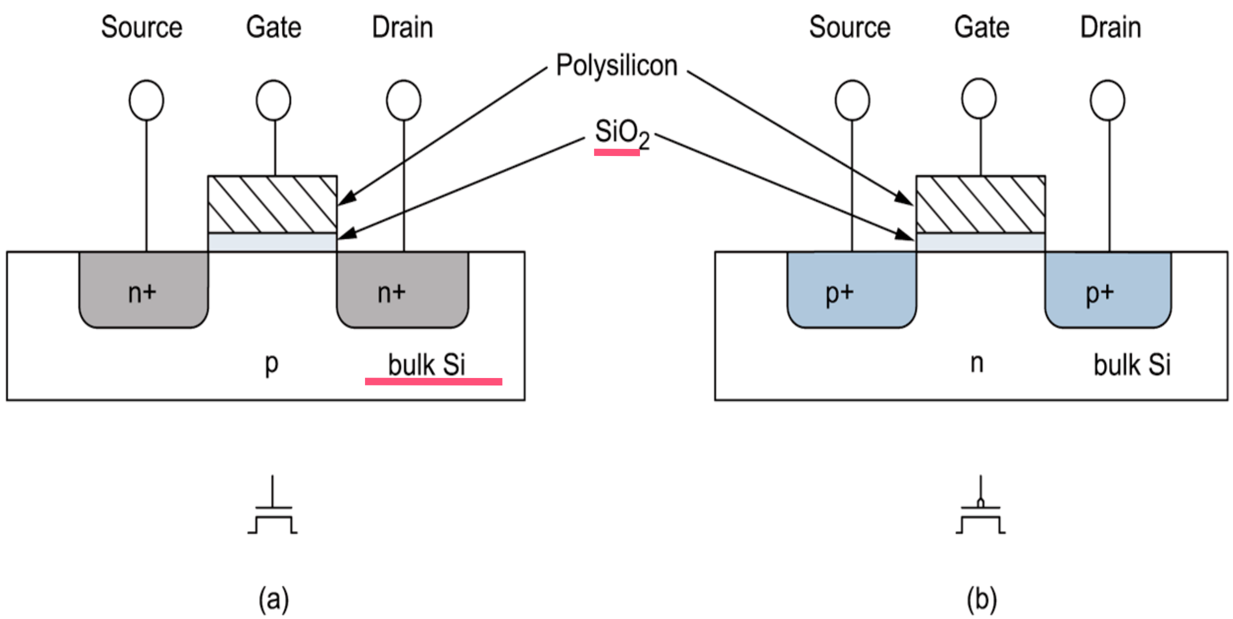


FIG 1.8 nMOS transistor (a) and pMOS transistor (b)

Permittivity of vacuum

$$\epsilon_{si} = 11.7 \cdot \epsilon_0$$

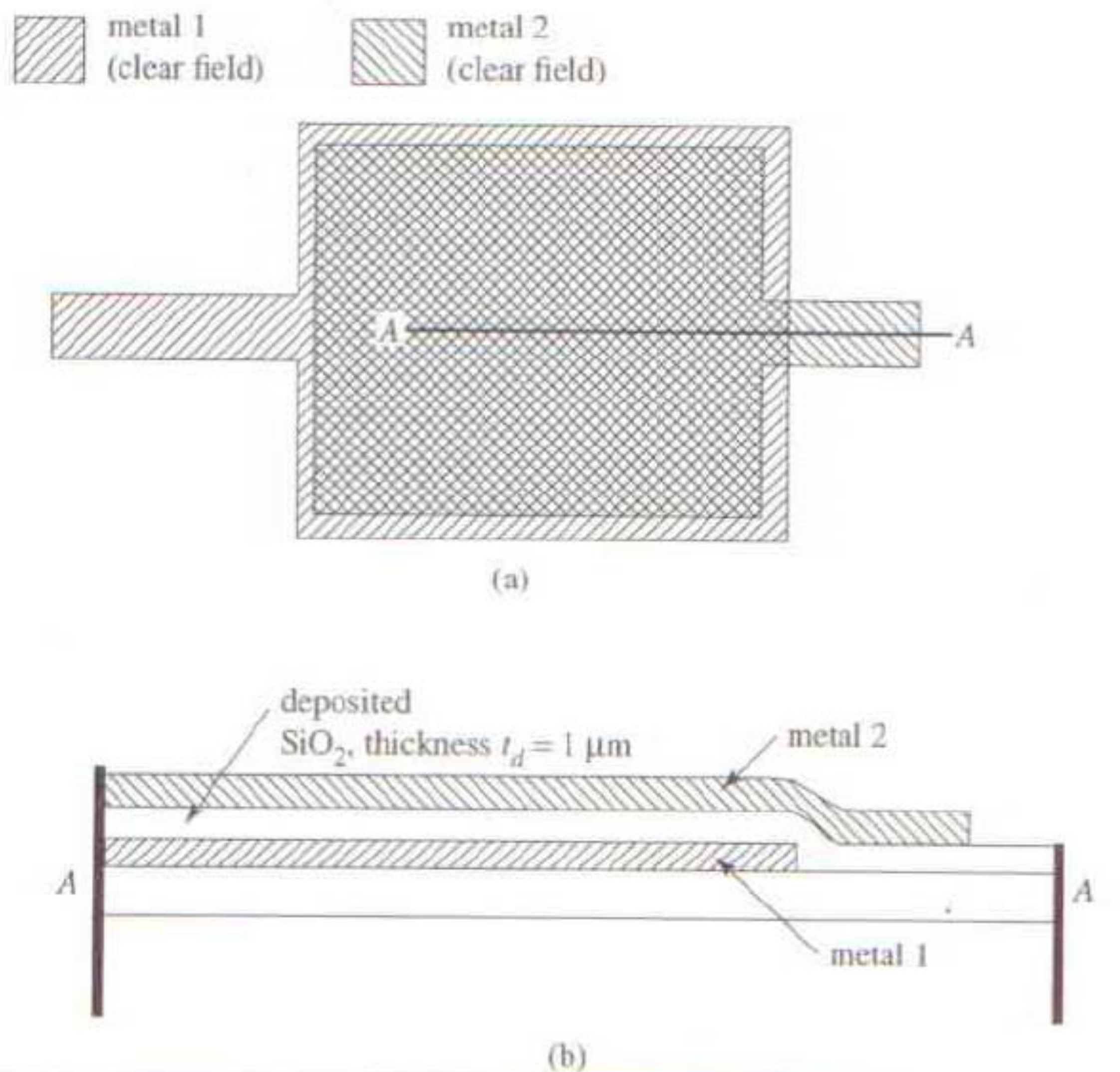
$$\epsilon_{ox} = 3.9 \cdot \epsilon_0$$

$$\epsilon_0 = 8.85 \times 10^{-12} [F/m]$$

$$E_{si}(0^+) = \frac{\epsilon_{ox}}{\epsilon_{si}} E_{ox}(0^-) \approx \frac{E_{ox}(0^-)}{3.0}$$

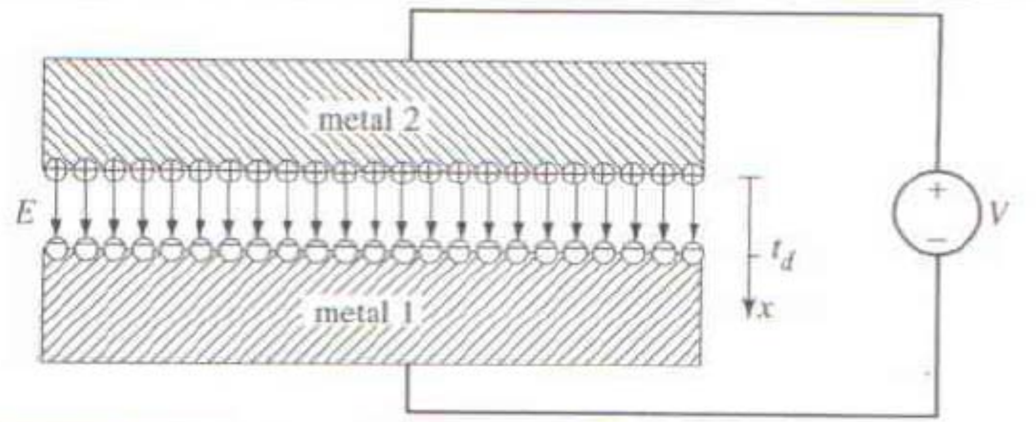
Metal – Metal Capacitor

- In many IC processes there are two or more levels of metal separated by silicon oxide



➤ **Figure Ex3.1A** Metal-metal IC capacitor: a) layout and (b) cross section.

Metal – Metal Capacitor



➤ **Figure Ex3.1B** Close-up of metal-metal capacitor with applied voltage, showing surface charges on top and bottom metal plates and electric field lines.

Metal – Metal Capacitor

$\rho = 0$

$$\frac{dE}{dx} = \frac{\rho}{\epsilon}$$

- Since there is no charge present in the oxide
  - the electric field in the oxide is constant
- Since the electric field in the oxide is constant and the voltage drop across the gap is V
  - it follows that: $E_{ox} = \frac{V}{t_d}$

$$\begin{cases} \phi(t_d) - \phi(0) = \int_0^{t_d} -E_{ox}\,dx \\ V = \phi(0) - \phi(t_d) \end{cases}$$

$$E_{ox} = \frac{\epsilon_{metal}}{\epsilon_{ox}} \cancel{E_{metal}} + \frac{\mathbb{Q}_S}{\epsilon_{ox}} \Rightarrow \frac{V}{t_d} = \frac{\mathbb{Q}_S}{\epsilon_{ox}} \Rightarrow \mathbb{C}_S = \frac{d\,\mathbb{Q}_S}{dV} = \frac{\epsilon_{ox}}{t_d}$$

No electric field inside metals

Capacitance per unit area [F/m^2]

Example

- Find potential, electric field and charge distribution for a metal-metal capacitor with t_d =1 μm and an applied voltage of 1V

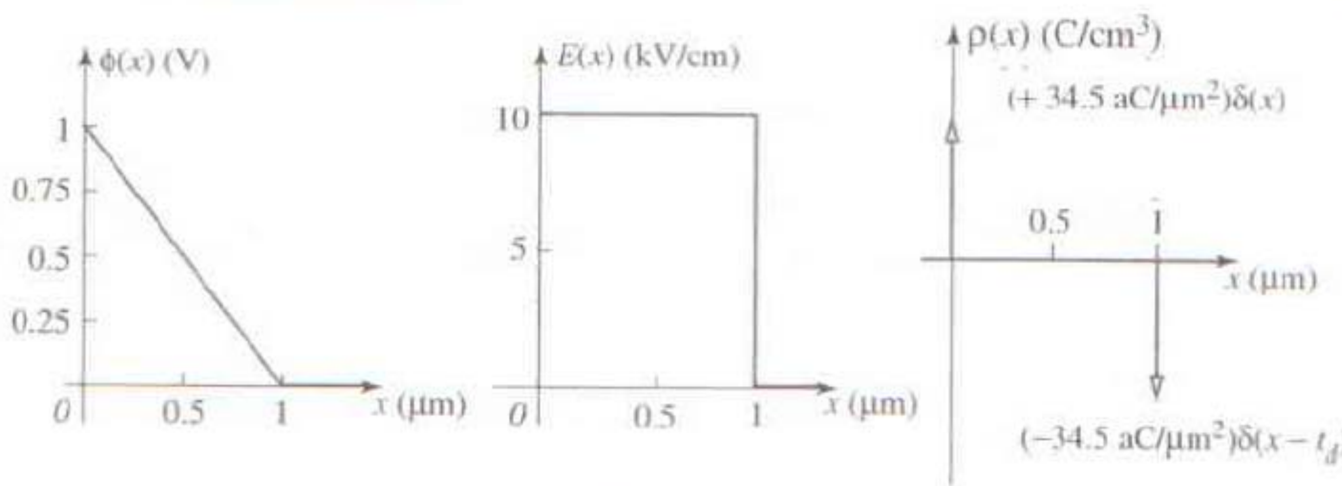
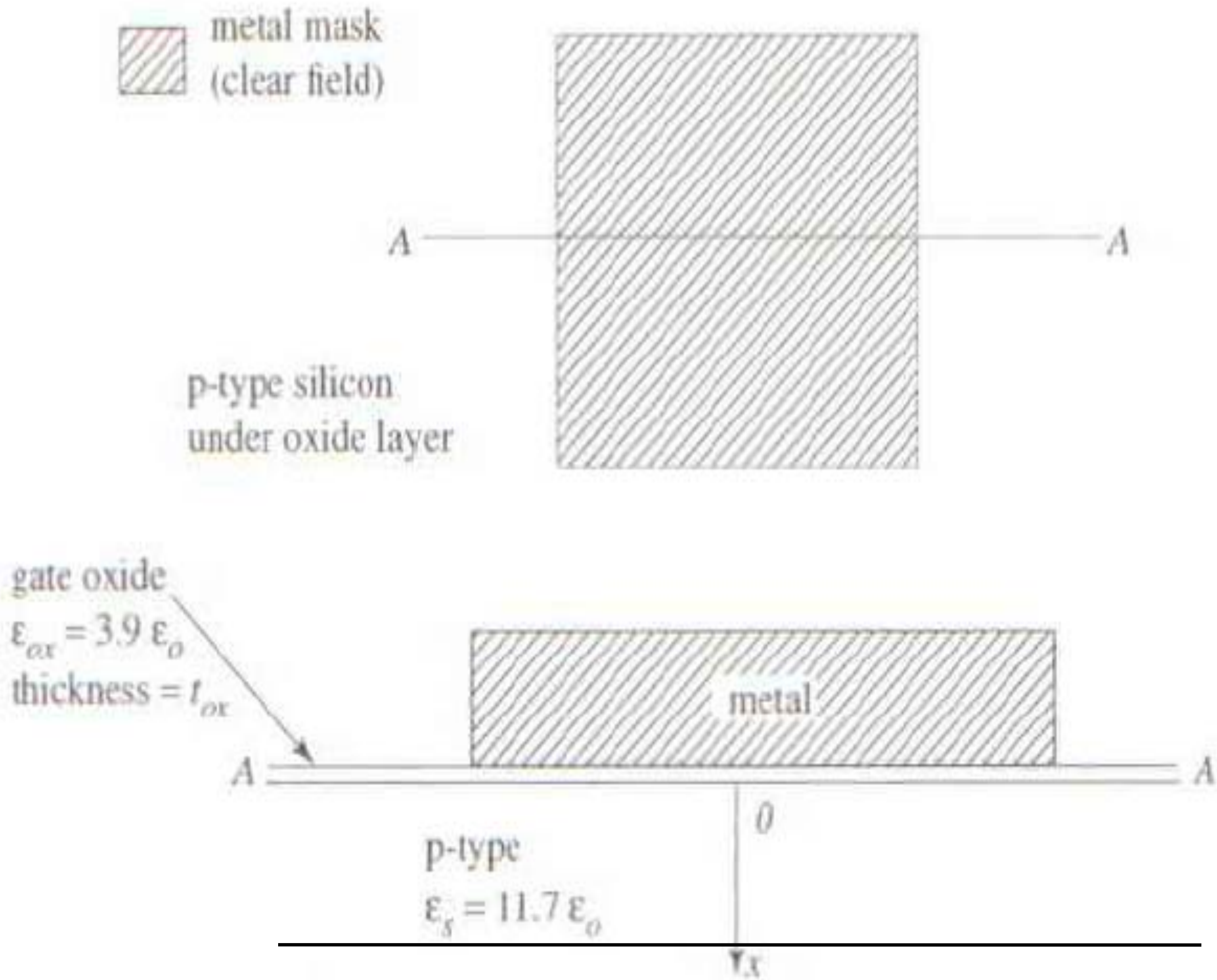


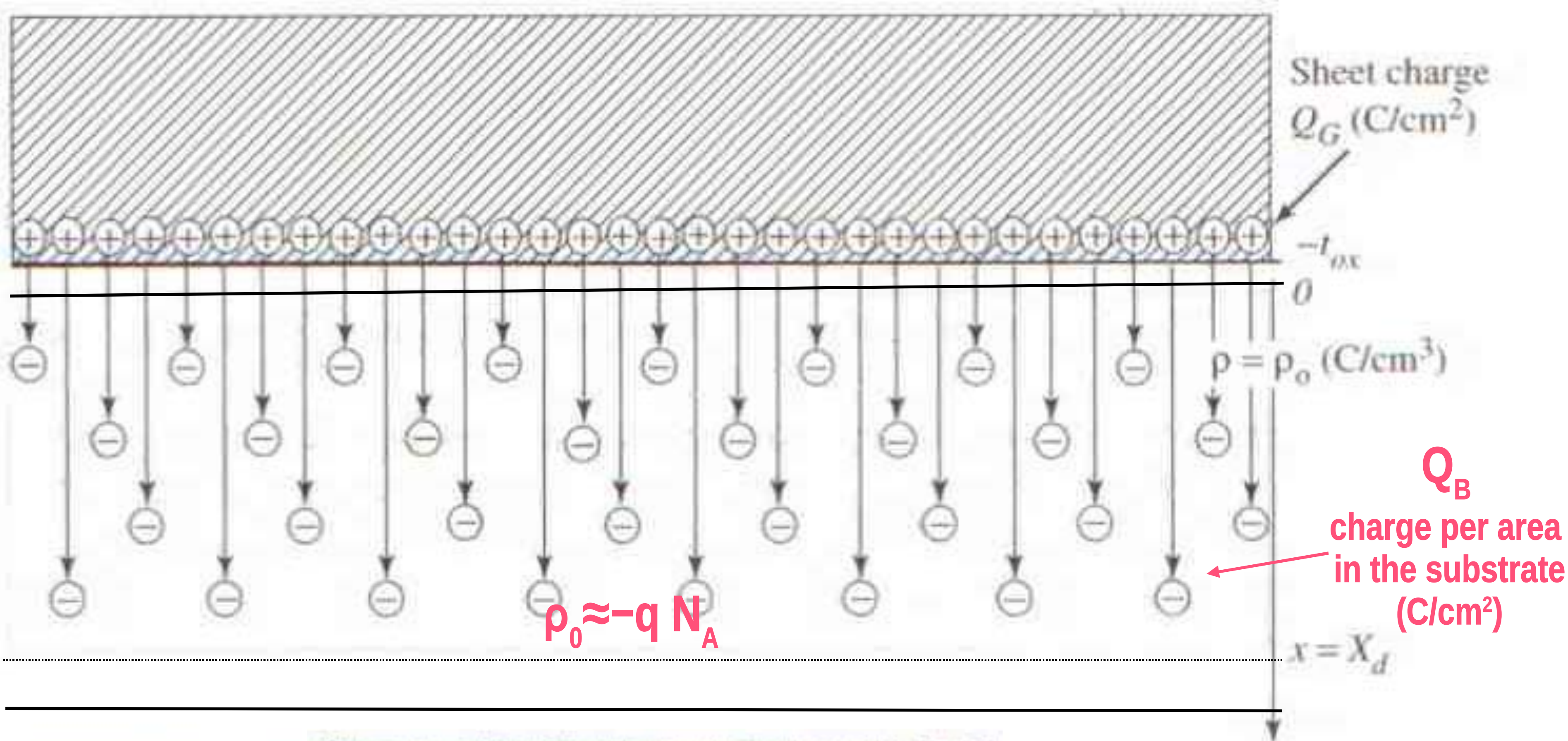
Figure Ex3.1C Potential, electric field, and charge density for the capacitor with $t_d = 1$ μm.

M-O-S Capacitor



➤ **Figure Ex3.3A** Metal-oxide-silicon capacitor: (a) layout and (b) cross section.

M-O-S charge distribution



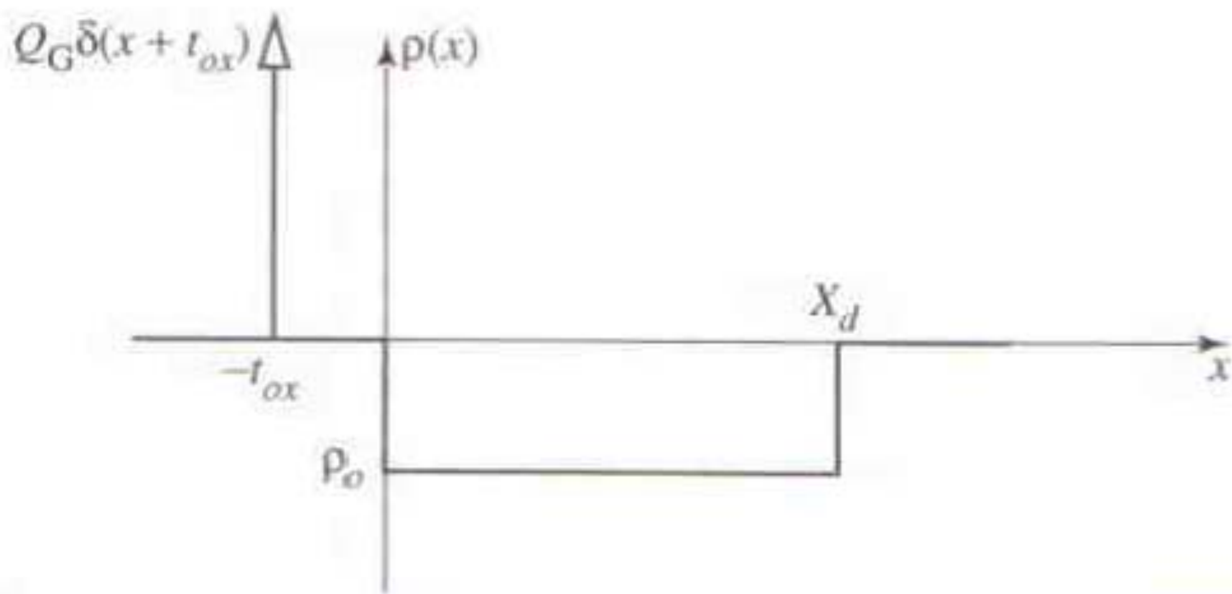
Charge distribution and electric field

$$\mathbb{Q}_B = \rho_0 \cdot X_d = -\mathbb{Q}_G$$

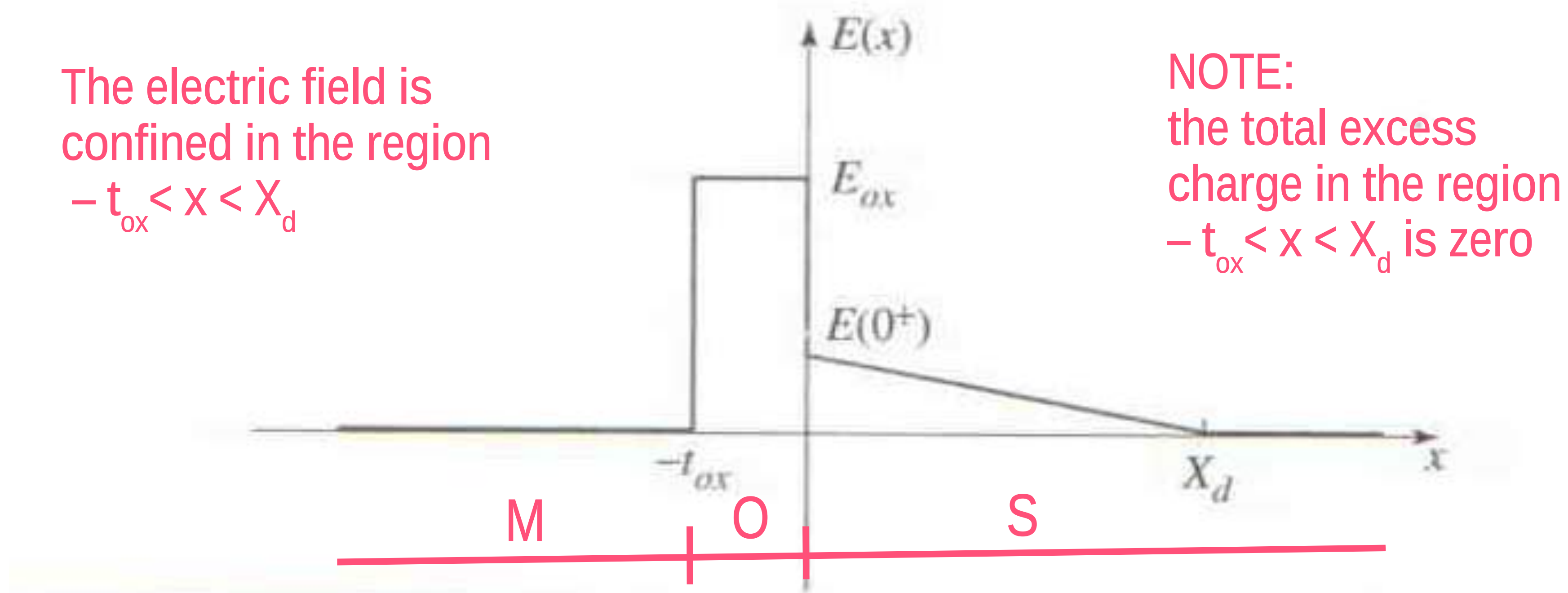
$$\mathbb{Q}_G = \mathbb{C}_{ox} \cdot V = \frac{\epsilon_{ox}}{t_{ox}} V$$

$$X_d = -\frac{\epsilon_{ox} V}{t_{ox} \rho_0} \approx \frac{\epsilon_{ox} V}{t_{ox}\, q\, N_A}$$

M-O-S charge density profile



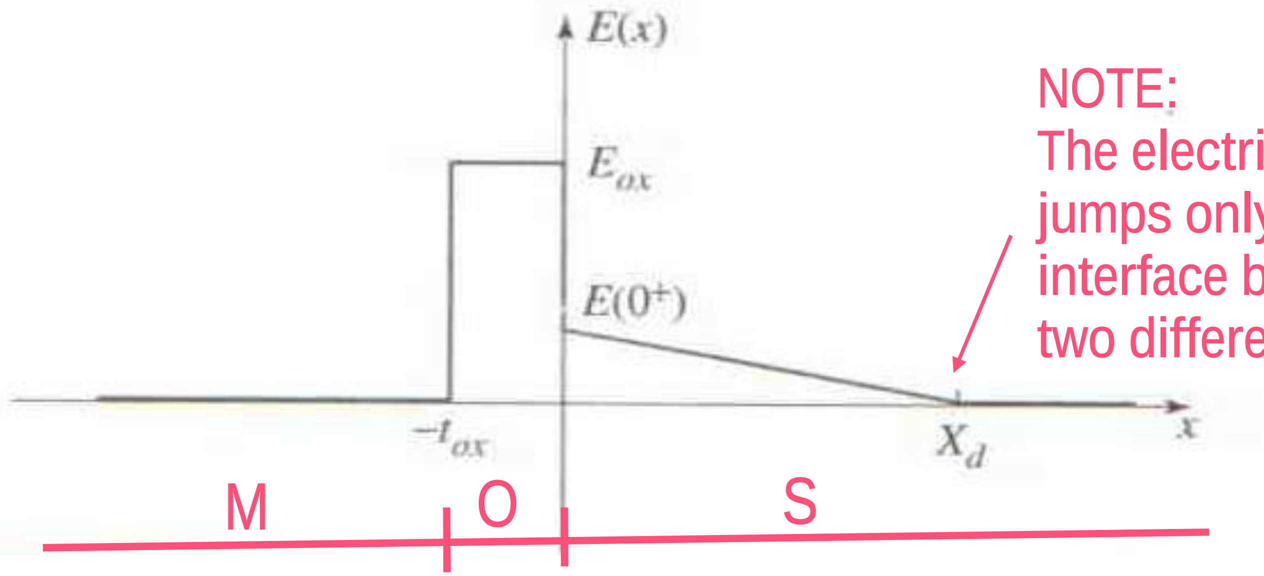
M-O-S Electric Field



- In the metal the electric field is 0
- In the oxide ($-t_{ox} < x < 0$) the charge density is zero ($\rho(x)=0$), therefore the electric field is constant

$$for -t_{ox} < x < 0: \quad \frac{dE(x)}{dx} = \frac{\overset{0}{\cancel{\rho(x)}}}{\epsilon_{ox}} = 0 \quad \Rightarrow \quad E(x) = E_{ox}$$

M-O-S Electric Field

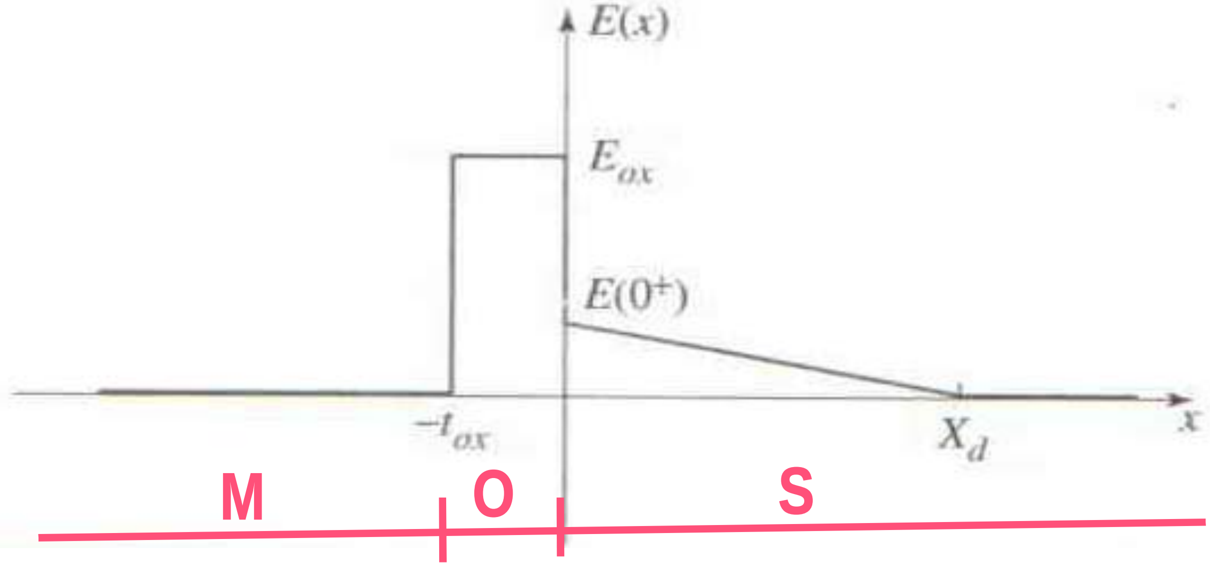


NOTE:
The electric field jumps only at the interface between two different materials

- Outside the charged region of the silicon ($x > X_d$) the electric field is 0
- In the charged region of the silicon ($0 < x < X_d$) the charge density is constant (ρ_0) therefore the electric field is a linear function of x

$$dE(x) = \frac{\rho_0}{\epsilon_s} dx \;\rightarrow\; \epsilon_s \cancelto{0}{E(x_d)} - \epsilon_s E(0^+) = \rho_0 \int_{0^+}^{X_d} dx \;\Longrightarrow\; E(0^+) = \frac{-\rho_0 X_d}{\epsilon_s}$$

M-O-S Electric Field



- The boundary condition at the oxide/silicon interface is:

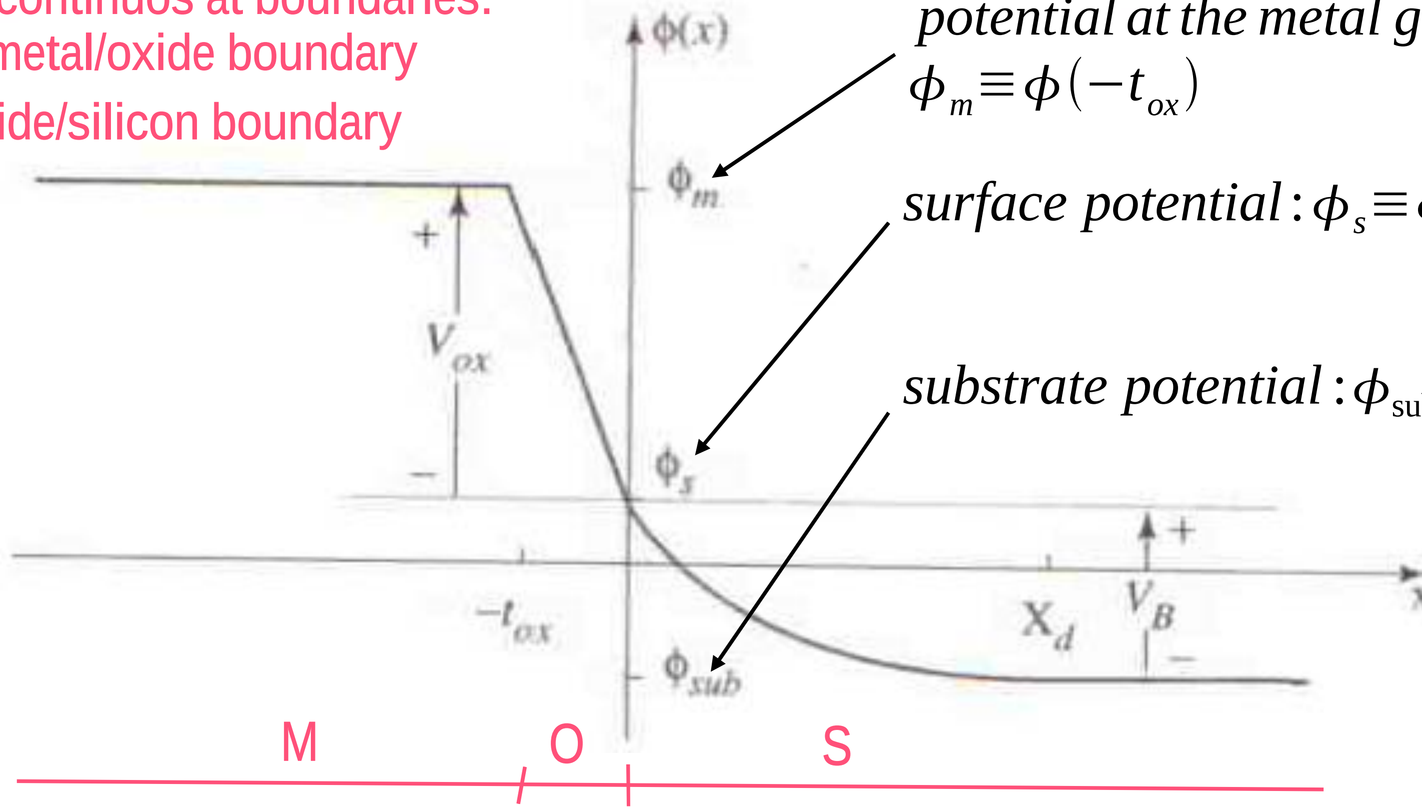
$$\epsilon_{ox} E_{ox} = \epsilon_s E(0^+) \rightarrow E_{ox} = \underbrace{\frac{\epsilon_s}{\epsilon_{ox}}}_{\approx 3} E(0^+) \quad \Rightarrow \quad E_{ox} = \frac{-\rho_0 X_d}{\epsilon_{ox}}$$

Potential plot through M-O-S

Potential is continuos at boundaries:

$x = -t_{ox}$ metal/oxide boundary

$x = 0$ oxide/silicon boundary



potential at the metal gate : $\phi_m \equiv \phi(-t_{ox})$

surface potential : $\phi_s \equiv \phi(0)$

substrate potential : $\phi_{\text{sub}} \equiv \phi(X_d)$

Poisson's Equation :

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon}$$

drop across the oxide

$$V = \phi_m - \phi_{\text{sub}} = V_{ox} + V_B$$

drop across the charged region of the silicon

Surface potential

for $-t_{ox}<x<0$:

$$\phi_m-\phi_s=\int_0^{-t_{ox}}(-E_{ox})\,dx=E_{ox}\cdot t_{ox}\equiv V_{ox}$$

$$\downarrow$$

$=Q_G=-Q_B$

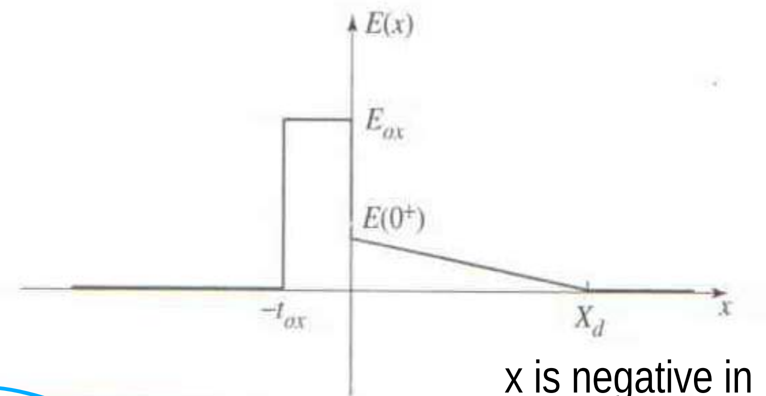
$=1/C_{ox}$

$$\phi_s=\phi_m-E_{ox}\cdot t_{ox}=\phi_m-\frac{(-\rho_0)X_d\,t_{ox}}{\epsilon_{ox}}$$

$$\downarrow$$

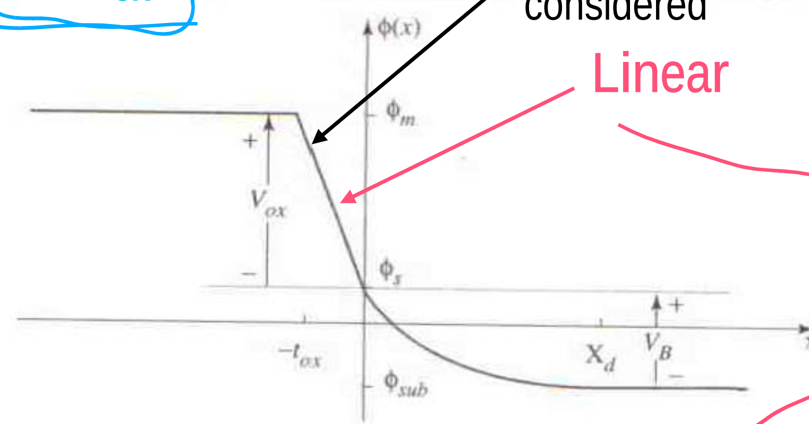
$$\phi_s=\phi_m+\frac{Q_B}{C_{ox}}\quad\text{and}\quad V_{ox}=-\frac{Q_B}{C_{ox}}=\frac{Q_G}{C_{ox}}$$

The drop across the oxide is proportional to the charge stored on each side of the oxide



x is negative in the region considered

Linear



$$\phi_m-\phi(x)=\int_x^{-t_{ox}}(-E_{ox})\,dx=$$

$$=E_{ox}\cdot(t_{ox}+x)=\frac{V_{ox}}{t_{ox}}\cdot(t_{ox}+x)$$

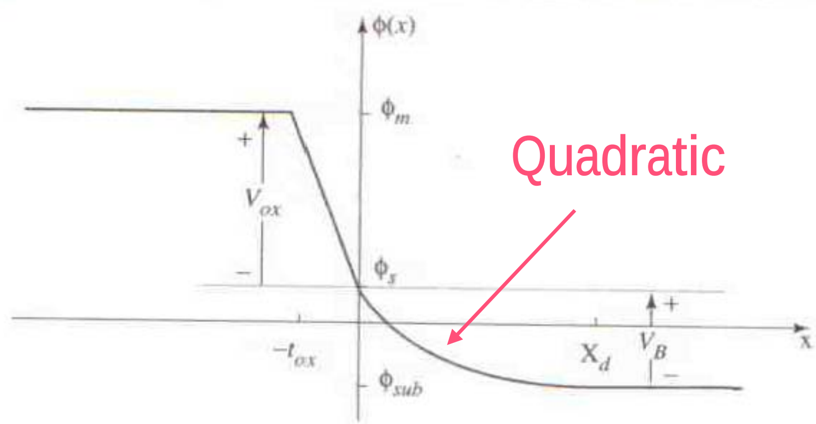
Potential drop across the substrate

$for \quad 0 < x < X_d:$ charged region of silicon

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho_0}{\epsilon_s} \quad (>0)$$

The potential is "concave up"
in the charged region

$$V_B \equiv \phi(0) - \phi(X_d) = \frac{1}{2}\left(\frac{-\rho_0}{\epsilon_s}\right) X_d^2 = \left(-\frac{\rho_0 X_d}{2\epsilon_s}\right) X_d = \frac{-Q_B}{2\epsilon_s} X_d$$



PN Junction in Thermal Equilibrium

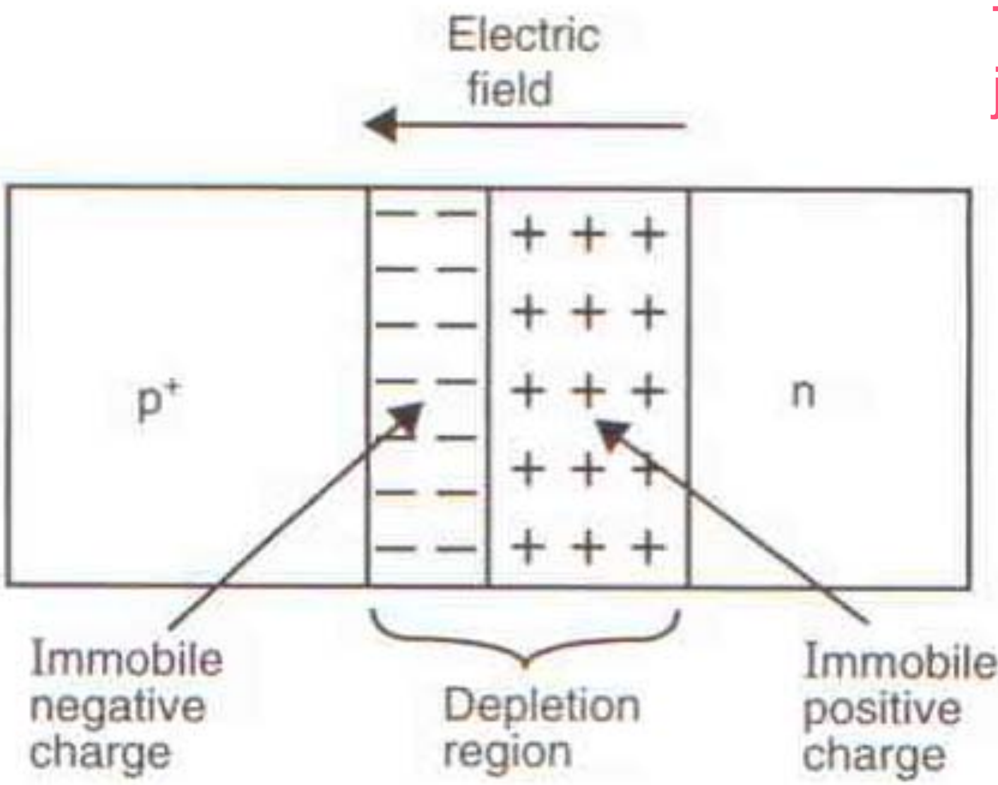
- If no external stimulus is applied (zero applied voltage, no external light source, etc) the device will eventually reach a steady state status of thermal equilibrium
- In this state (“open circuit” and steady state condition) the current density must be zero:

$$J_{tot,0} = J_{p,0} + J_{n,0} = 0$$

- Eventually, the populations of electrons and holes are each in equilibrium and therefore must have zero current densities

$$\begin{cases} J_{p,0} = 0 \\ J_{n,0} = 0 \end{cases}$$

Diffusion Mechanism



The charge on the two sides of the junction must be equal (charge neutrality)

Under "open circuit" and steady state conditions the built in electric field opposes the diffusion of free carriers until there is no net charge movement

Fig. 1.2 A simplified model of a diode. Note that a depletion region exists at the junction due to diffusion and extends farther into the more lightly doped side.

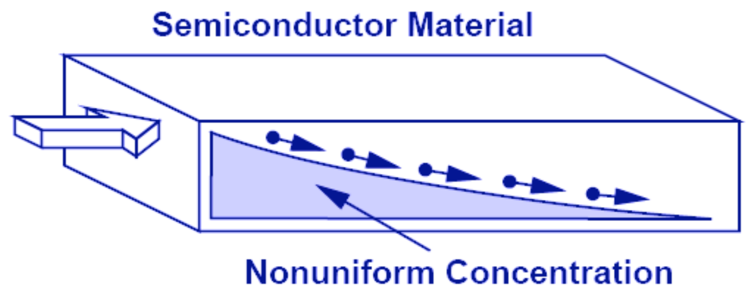
$N_A > N_D$

We assumed the n-side is the more lightly doped

Diffusion Current

- It's a manifestation of thermal random motion of particles (statistical phenomenon)
- In a material where the concentration of particles is uniform the random motion balances out and no net movement result (drunk sail-man walk $\longrightarrow$ Brownian walks)
- If there is a difference (gradient) in concentration between two parts of a material, statistically there will be more particles crossing from the side of higher concentration to the side of lower concentration than in the reverse direction.
- Then we expect a net flux of particles

Diffusion Equations



charge in the cross section

$$I_n \propto A q_e \frac{dn}{dx}$$

The more non uniform the concentration the larger the current

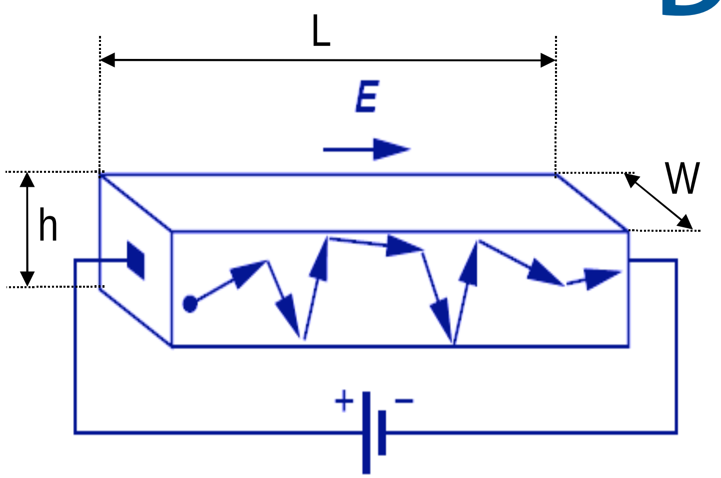
Proportionality Constants

$$I_n = -D_n A q_e \frac{dn}{dx} = D_n A q \frac{dn}{dx} \qquad I_p = -D_p A q_h \frac{dn}{dx} = -D_p A q \frac{dn}{dx}$$

Assuming the charge concentration decreases with increasing x
It means that dn/dx and dp/dx are negative, so to conform with conventions we must put a – sign in front of the equations.

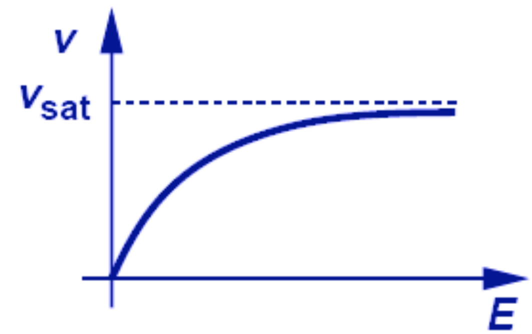
$$\boxed{\begin{aligned} J_n &= D_n q \frac{dn}{dx} \\ J_p &= -D_p q \frac{dp}{dx} \end{aligned}}$$

Drift Current



$$\vec{v}_n = -\mu_n \vec{E}$$
$$\vec{v}_p = +\mu_p \vec{E}$$

Mobility (proportionality constant)



Eventually v saturates
- too many collisions
- effective electrons' mass increases

cross section area

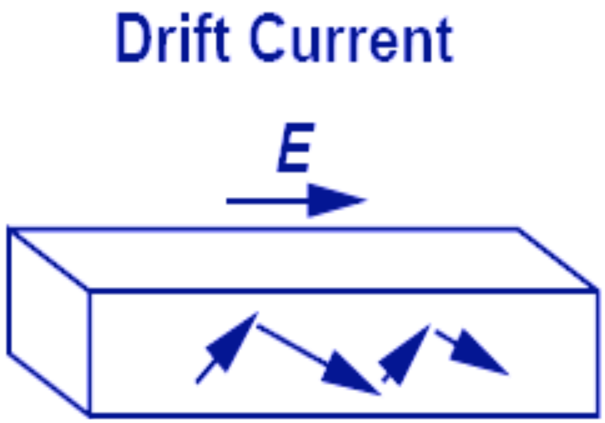
$$I_n = \underbrace{v_n W h}_{\text{volume travelled per unit of time}} n q_e \qquad I_p = v_p W h p q_h$$

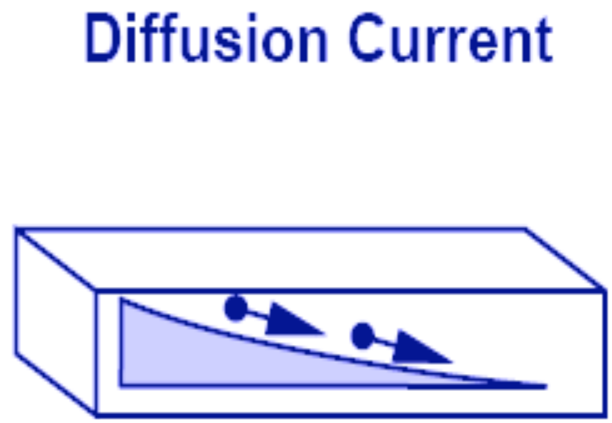
charge per unit of volume

volume travelled per unit of time

$$J_n = -\mu_n E n q_e = \mu_n E n q$$
$$J_p = \mu_p E p q_h = \mu_p E p q$$

Drift and Diffusion currents





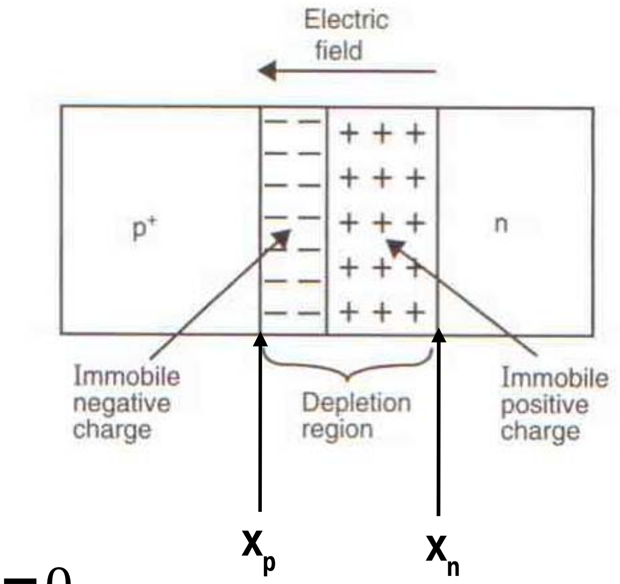
$$J_{n,drift} = -\mu_n E\, n\, q_e = \mu_n E\, n\, q$$
$$J_{p,drift} = \mu_p E\, p\, q_h = \mu_p E\, p\, q$$

$$J_{n,diff} = -D_n q_e \frac{dn}{dx} = D_n q \frac{dn}{dx}$$

$$J_{p,diff} = -D_p q_h \frac{dp}{dx} = -D_p q \frac{dp}{dx}$$

$$q \equiv q_h = -q_e = 1.6 \times 10^{-19}\, C$$

Built in Voltage



- At equilibrium:
  (drift and diffusion balance out)

$$J_n = J_{n,drift} + J_{n,diff} = \mu_n E\, n\, q + D_n q \frac{dn}{dx} = 0$$

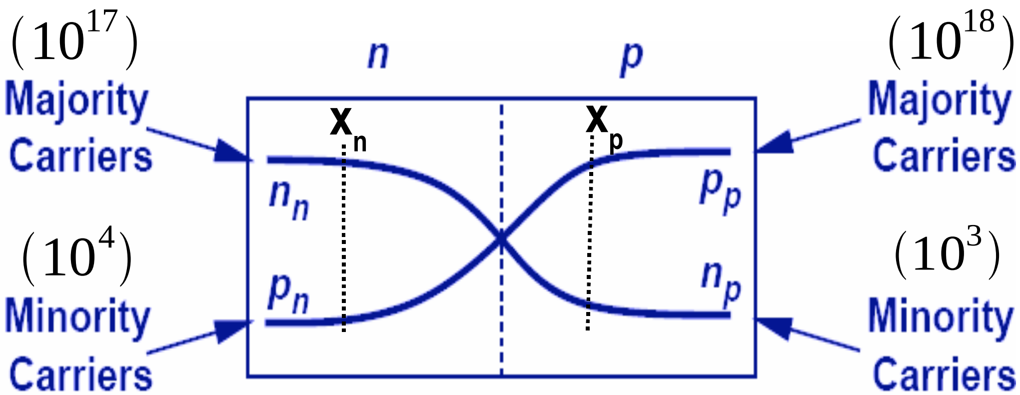
$$J_p = J_{p,drift} + J_{p,diff} = \mu_p E\, p\, q - D_p q \frac{dp}{dx} = 0$$

- Let's consider the second equation:

$$\mu_p E\, p \cancel{q} = D_p \cancel{q} \frac{dp}{dx} \;\rightarrow\; -\mu_p \frac{dV}{dx} p = D_p \frac{dp}{dx} \;\rightarrow\; -\mu_p\, p\, dV = D_p\, dp \;\rightarrow$$

$$-\mu_p \int_{V(x_n)}^{V(x_p)} dV = D_p \int_{p(x_n)}^{p(x_p)} \frac{dp}{p} \;\rightarrow\; V(x_p) - V(x_n) = -\frac{D_p}{\mu_p} \ln\left(\frac{p(x_p)}{p(x_n)}\right) \;\rightarrow$$

Built in Voltage



n_n : Concentration of electrons on n side $\approx N_D \quad (e.g.\ 10^{17})$

p_n : Concentration of holes on n side $\approx n_i^2 / N_D$

p_p : Concentration of holes on p side $\approx N_A \quad (e.g.\ 10^{18})$

n_p : Concentration of electrons on p side $\approx n_i^2 / N_A$

Since both μ and D are manifestations of thermal random motion (i.e. statistical thermodynamics phenomena) they are not independent

$$Einstein's\ Relation:$$

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{KT}{q} \equiv V_T$$

$$Mass-Action\ Law:$$

$$n \cdot p = n_i^2$$

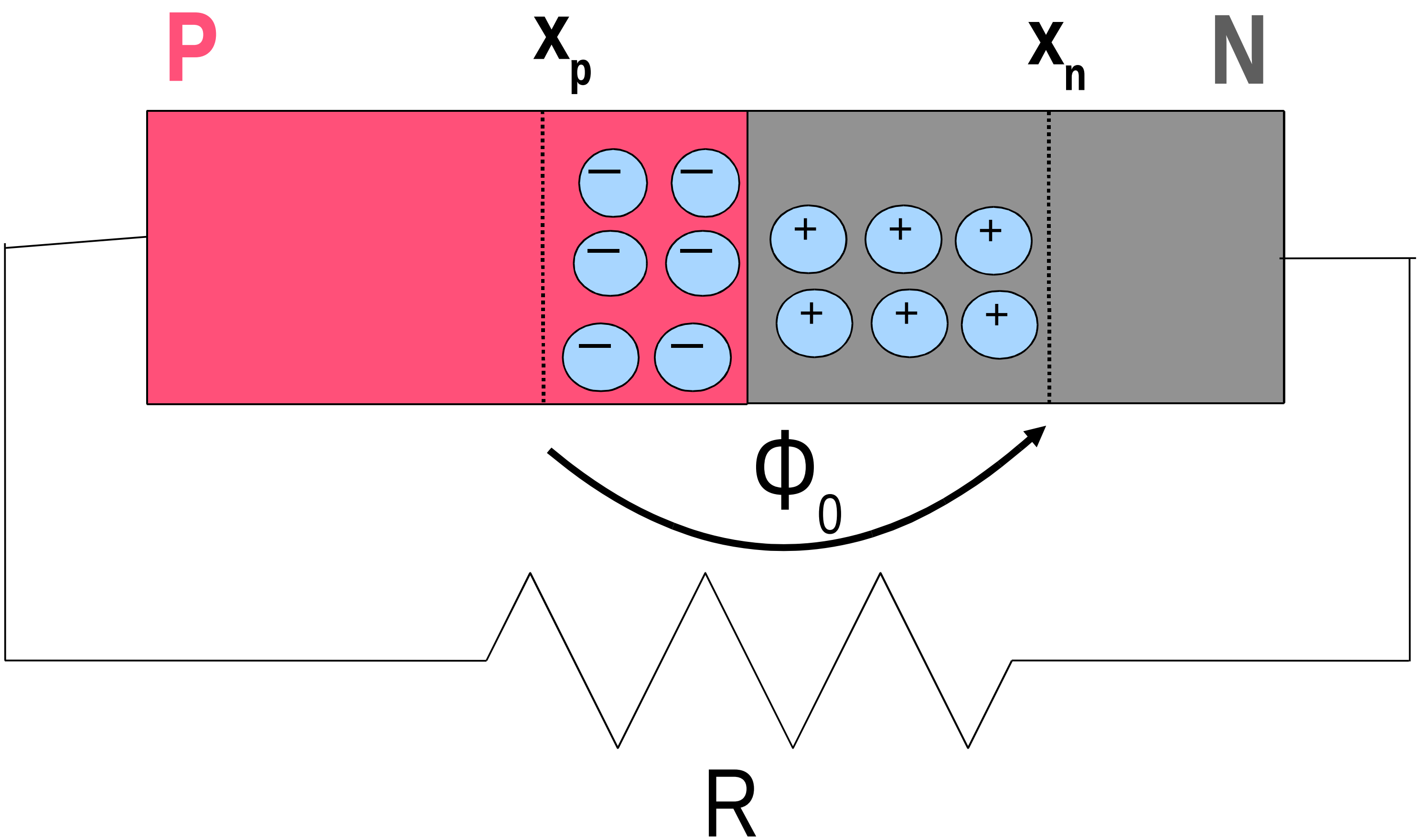
If n ↑ then p ↓
A larger number of electrons causes the recombination rate of electrons with holes to increase

$$\phi_0 \equiv V(x_n) - V(x_p) = \frac{D_p}{\mu_p} \ln\left(\frac{p(x_p)}{p(x_n)}\right) \Leftrightarrow \boxed{\phi_0 = \frac{KT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

Applying KVL to the PN junction in equilibrium

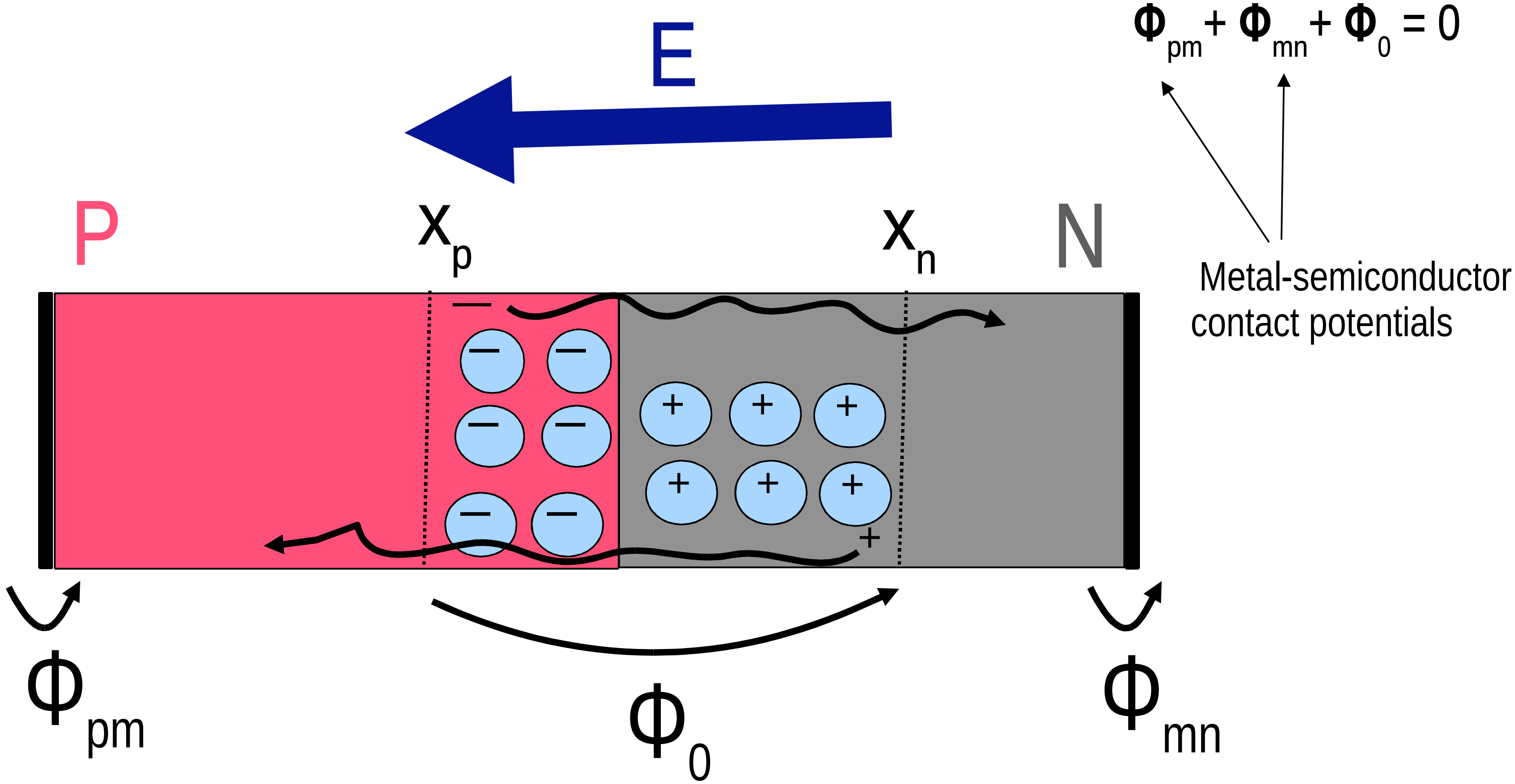
$$\cancel{I = \frac{\phi_0}{R}}$$

We cannot have current. Something is wrong !



Built in Voltage

KVL at equilibrium:
$\Phi_{pm} + \Phi_{mn} + \Phi_0 = 0$



If a free electron in the P region or a hole in the N region somehow reach the edge of the depletion region get swept by the electric field (⟶ drift)

Neutrality of charge

- There are 4 charged particles in silicon, two mobiles (holes and electrons) and two fixed (ionized donors and ionized acceptors)
- The total positive change density and the total negative charge must be equal

$$N_D + p = N_A + n$$

electrons concentration

positive ions concentration

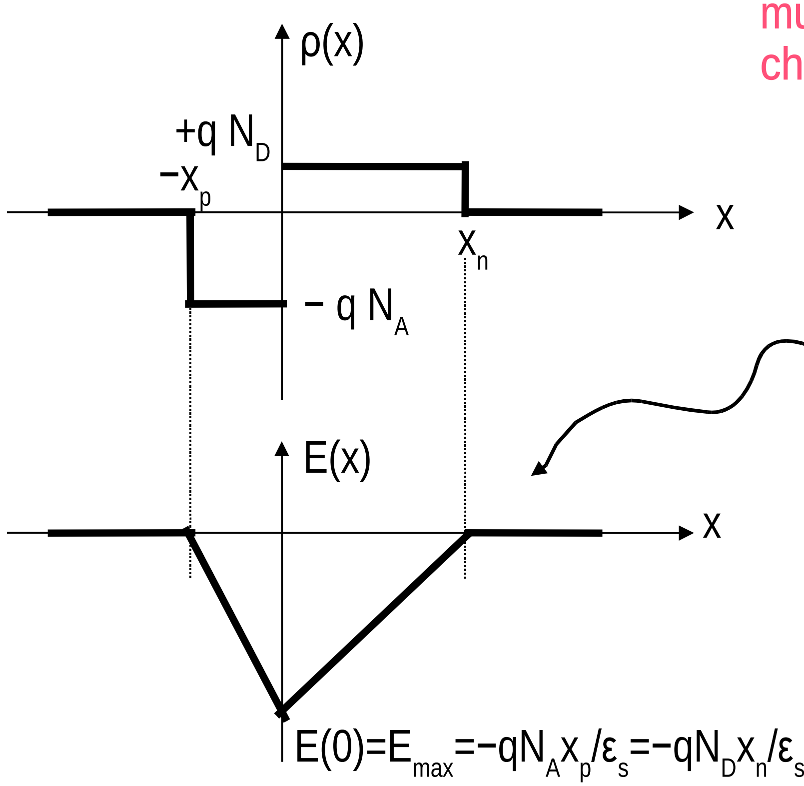
negative ions concentration

holes concentration

Depletion region in equilibrium

- The doping concentrations N_A on the p side and N_D on the n side are assumed constants

$$\rho(x) = q(p - n + N_D - N_A)$$
$$\downarrow$$
the depletion region is free of electrons and holes
$$\downarrow$$
$$N_D - N_A \gg n - p$$
$$\downarrow$$
$$\rho(x) \approx q(N_D - N_A)$$
$$\downarrow$$
$*$*on the P side* $N_D = 0$:
$$\rho(x) = -q N_A$$
$*$*on the N side* $N_A = 0$:
$$\rho(x) = q N_D$$

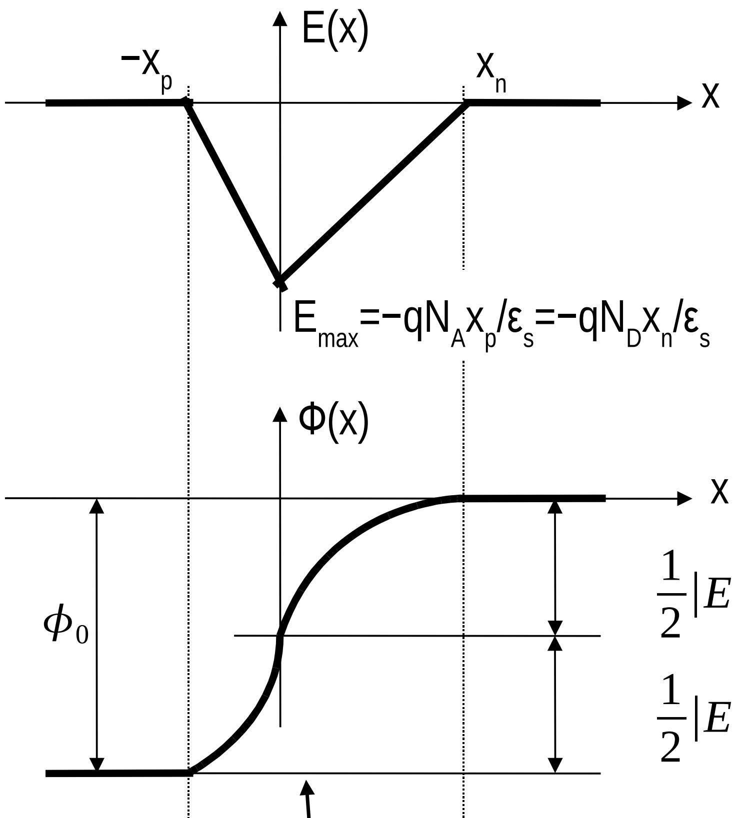


Positive and negative excess charge in the depletion region must balance out (neutrality of charge):

$$q N_D x_n = q N_A x_p$$

Gauss' Law:
$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon_s}$$

Depletion region in equilibrium



$$N_D x_n = N_A x_p \longrightarrow \boxed{\begin{cases} x_n = \dfrac{N_A}{N_D} x_p \\ x_p = \dfrac{N_D}{N_A} x_n \end{cases}}$$

$$\frac{1}{2}|E_{max}|x_n = \frac{1}{2}\frac{qN_D x_n^2}{\epsilon_s} \Leftrightarrow Area\ Triangle\ \widehat{0\,E_{max}\,x_n}$$

$$\frac{1}{2}|E_{max}|x_p = \frac{1}{2}\frac{qN_A x_p^2}{\epsilon_s} \Leftrightarrow Area\ Triangle\ \widehat{0\,E_{max}\,x_p}$$

Potential Equation :

$$d\phi = -E(x)\,dx$$

$$\boxed{\phi_0 = \frac{1}{2}\frac{qN_D}{\epsilon_s}x_n^2 + \frac{1}{2}\frac{qN_A}{\epsilon_s}x_p^2}$$

Depletion region in equilibrium

$$\begin{cases} x_n = \dfrac{N_A}{N_D} x_p \\ x_p = \dfrac{N_D}{N_A} x_n \end{cases} \longrightarrow X_{dep} = x_n + x_p = \left(1 + \frac{N_A}{N_D}\right) x_p = \left(1 + \frac{N_D}{N_A}\right) x_n$$

$$\downarrow$$

$$x_p = X_{dep} \left(\frac{N_D}{N_A + N_D} \right) \quad \text{and} \quad x_n = X_{dep} \left(\frac{N_A}{N_A + N_D} \right)$$

$$\downarrow$$

$$\phi_0 = \frac{1}{2} \frac{q N_D}{\epsilon_s} x_n^2 + \frac{1}{2} \frac{q N_A}{\epsilon_s} x_p^2 = \frac{1}{2} \frac{q N_D}{\epsilon_s} X_{dep}^2 \left(\frac{N_A}{N_A + N_D} \right)^2 + \frac{1}{2} \frac{q N_A}{\epsilon_s} X_{dep}^2 \left(\frac{N_D}{N_A + N_D} \right)^2 =$$

$$= \frac{1}{2} \frac{q}{\epsilon_s} \left(\frac{X_{dep}}{N_A + N_D} \right)^2 (N_D \cdot N_A^2 + N_A \cdot N_D^2) = \frac{q X_{dep}^2}{2 \epsilon_s} \frac{N_A N_D}{N_A + N_D}$$

$$\downarrow$$

Width and max field of the depletion region in equilibrium

$$\boxed{X_{dep} = \sqrt{\frac{2\epsilon_s}{q}\frac{N_A + N_D}{N_A N_D}\phi_0}}$$

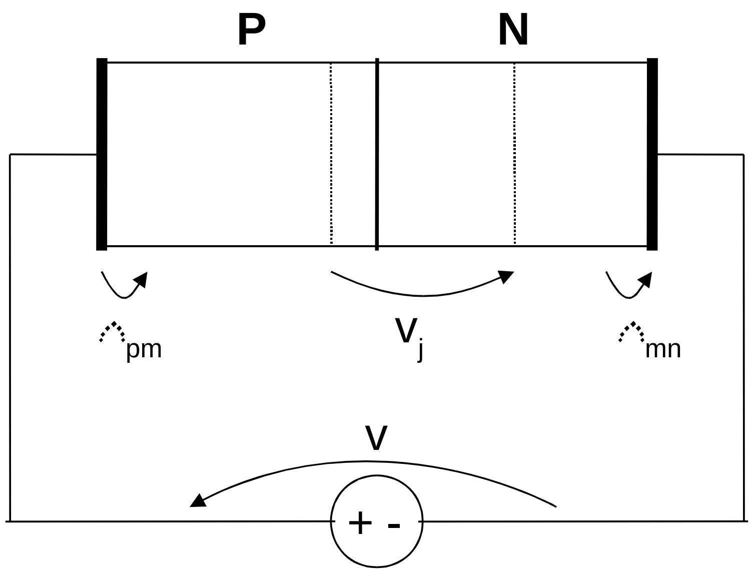
$$with: \quad \phi_0 = \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$|E_{max}| = \frac{q N_A}{\epsilon_s} x_p = \frac{q N_A}{\epsilon_s} X_{dep}\frac{N_D}{N_A + N_D} = \frac{q}{\epsilon_s}\frac{N_A N_D}{N_A + N_D} X_{dep}$$

$$\downarrow$$

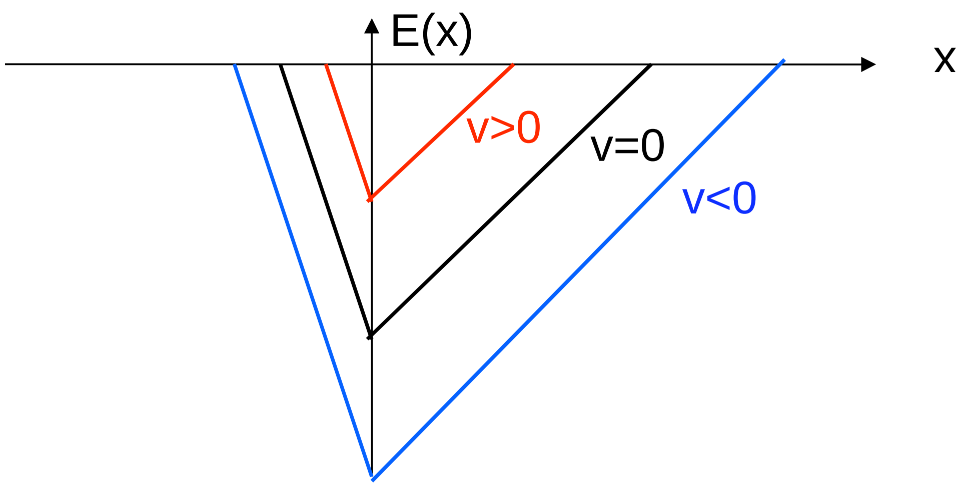
$$\boxed{|E_{max}| = \sqrt{\frac{2q}{\epsilon_s}\frac{N_A N_D}{N_A + N_D}\phi_0}}$$

Biased PN Junction



$$\underbrace{v+\phi_{pm}+\phi_{mn}}_{-\phi_0}+v_j=0$$

$$\boxed{v_j=\phi_0-v}$$

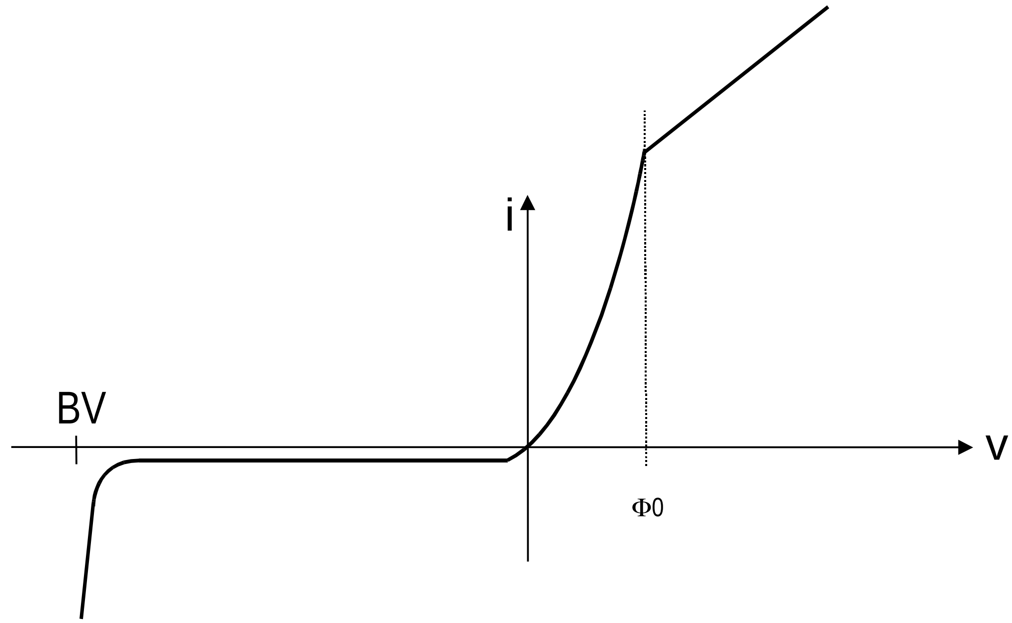


Biased PN Junction

$$\phi_0 = \frac{KT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$X_{dep} = \sqrt{\frac{2\epsilon_s}{q}\frac{N_A + N_D}{N_A N_D}(\phi_0 - v)}$$

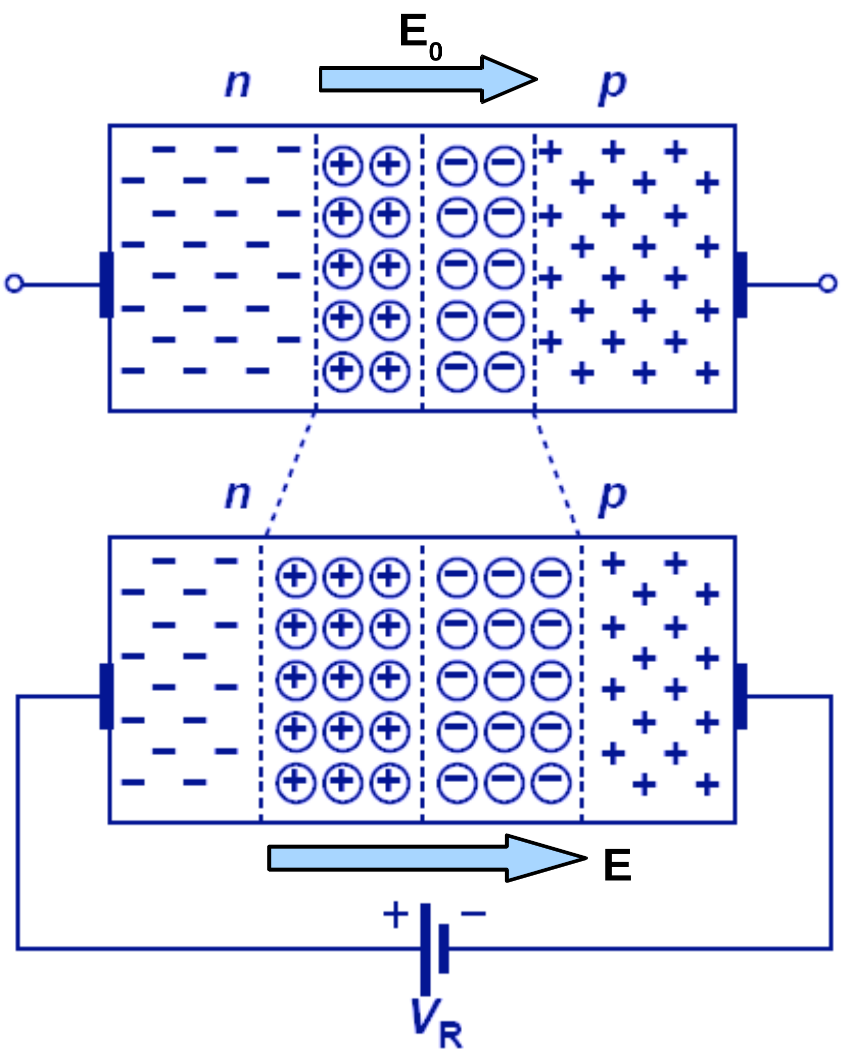
$$|E_{max}| = \sqrt{\frac{2q}{\epsilon_s}\frac{N_A N_D}{N_A + N_D}(\phi_0 - v)}$$



- if we keep decreasing the voltage eventually we'll break the material.
  For silicon the breakdown point is reached for an electric field of approx. 10^7 V/m.
  NOTE: the depletion region can't get bigger than the length of the bar !

- if we keep increasing the voltage the depletion region will disappear ($v=\Phi_0$).
  As v becomes comparable with Φ_0 the PN junction behave like a sort of resistor (the current is determined by the ohmic contacts and the resistance of the semiconductor)

Reverse Biased PN Junction



- Under reverse bias the depletion region becomes wider

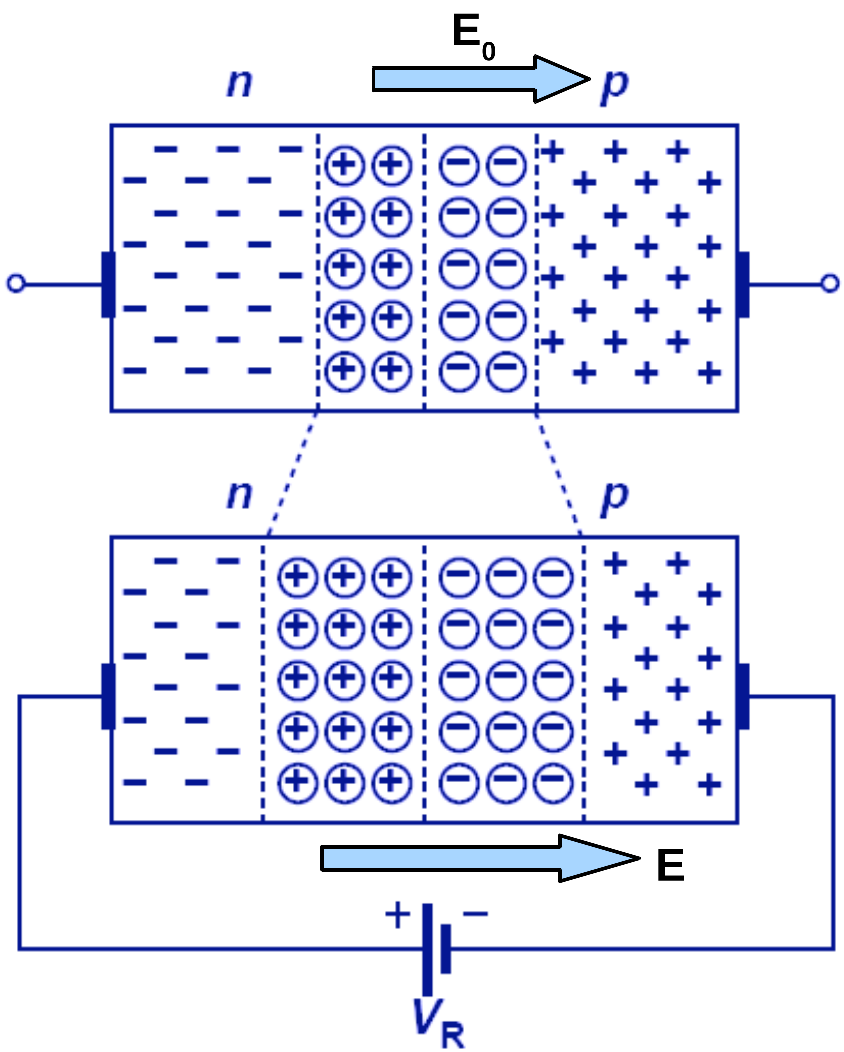
↓

- Then, it gets harder for the majority carriers to cross (diffuse through) the junction and easier for the minority carrier to be swept (drifted) across the junction

↓

- Since there are only a FEW minority carriers, the current carried under reverse bias is negligible

Reverse Biased PN Junction

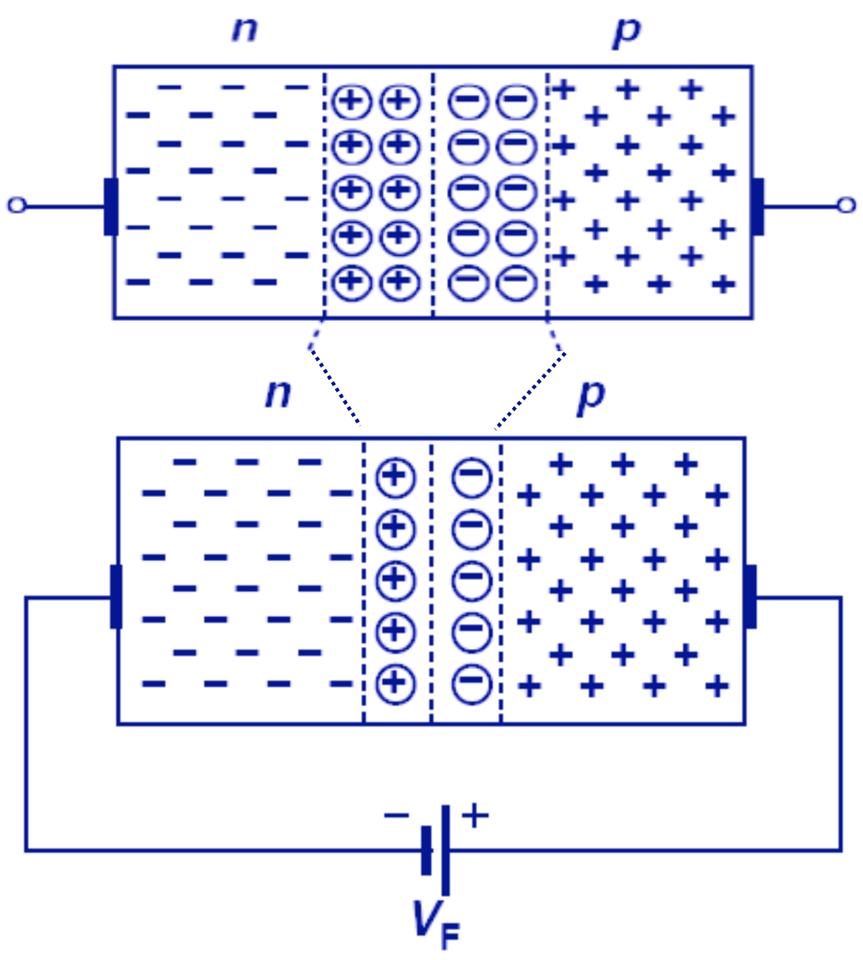


NOTE:

As soon as a minority carrier, let's say an electron on the P side, is swept across the junction, on the N side it becomes a majority carrier. Every time a minority carrier on the P side is swept toward the N side it leaves one less minority carrier on the edge of the depletion region in the P region. The same is true for holes swept from the N side to the P side.

Under reverse bias the minority carrier concentration at the edges of the depletion regions is depleted below their equilibrium value. Since the number of minority carriers is small anyway this won't be a major difference

Forward Biased PN Junction

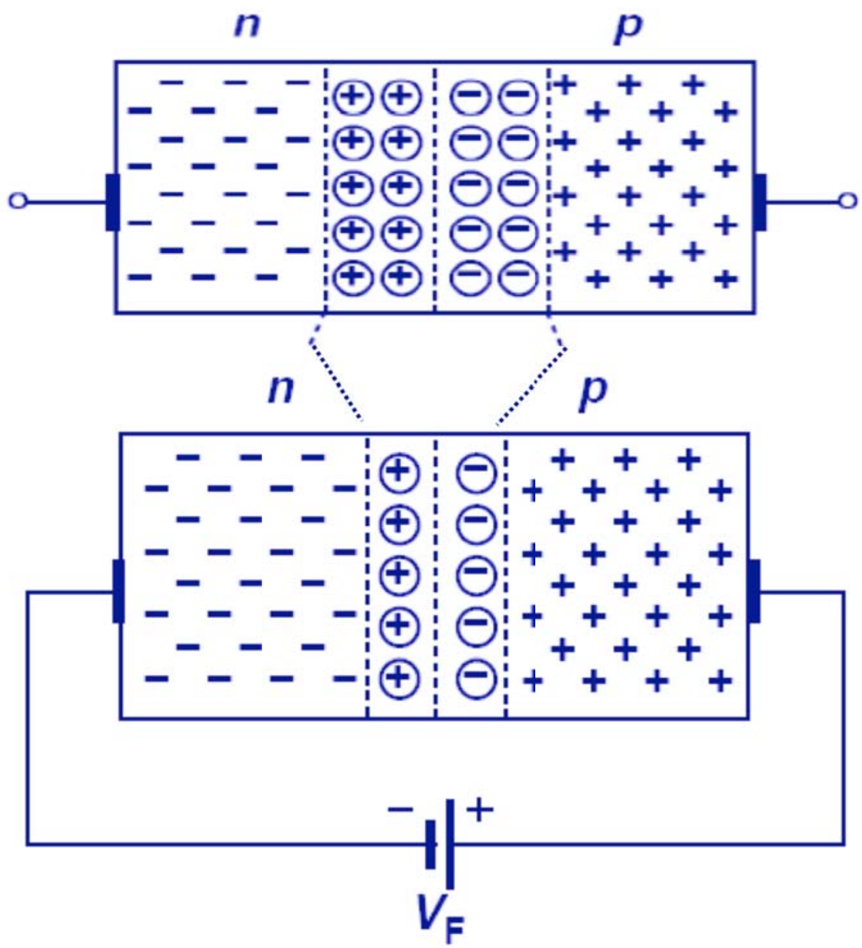


- Under forward bias the depletion region shrinks

- Then, it gets easier for the majority carriers to cross (diffuse through) the junction and harder for the minority carrier to be swept (drifted) across the junction.

- Since there are a LOT of majority carriers we expect the current to be considerable

Forward Biased PN Junction



NOTE:
As soon as a majority carrier (let's say a hole on the P side) crosses the junction it becomes a minority carrier. Thus at the edge of the N side of the depletion region we have an excess of minority carriers compared with the concentration of minority carriers on the rest of the N region far from the junction. This gradient causes a considerable diffusion current. The same is true for electrons crossing from the N side to the P side.

As VF is increased the excess minority concentration is increased
If VF = 0 (equilibrium) there is no excess minority concentration

I/V characteristic of PN Junction

$$I_{diode} = \underbrace{\left(\frac{qAD_n}{L_n} \frac{n_i^2}{N_A} + \frac{qAD_p}{L_p} \frac{n_i^2}{N_D} \right)}_{I_s} \left(e^{\frac{V_{diode}}{V_T}} - 1\right) = I_s \left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

with:

$A =$ cross section area of the diode

$\frac{n_i^2}{N_D} =$ holes' concentration in the N region (minority carriers)

$\frac{n_i^2}{N_A} =$ electrons' concentration in the P region (minority carriers)

$D_p =$ diffusion constant for the holes in the N region

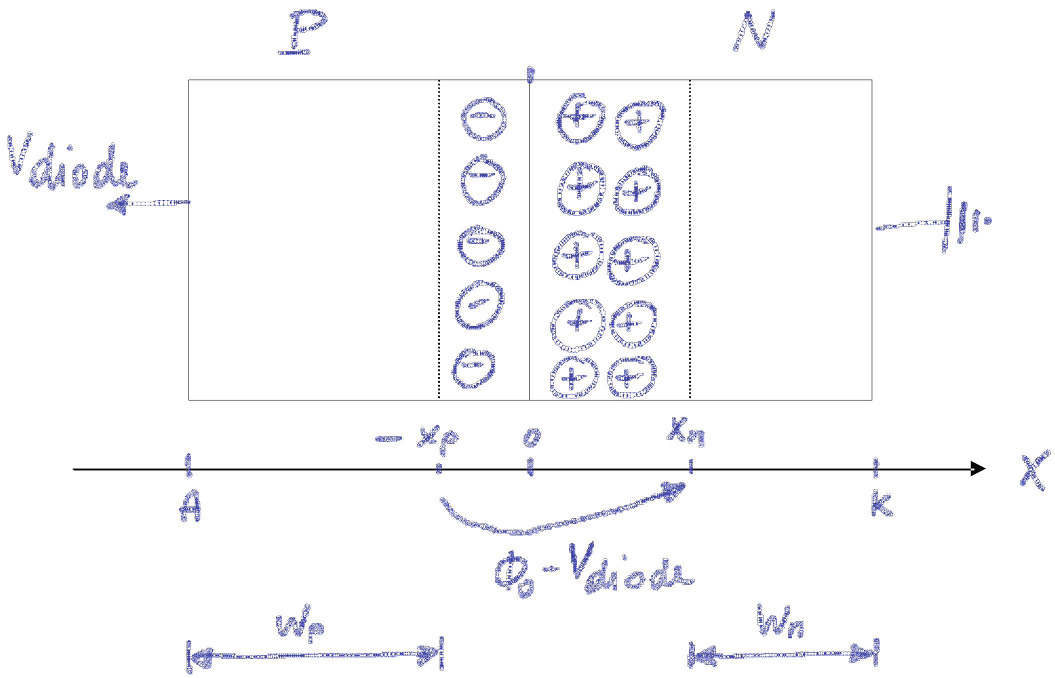
$L_p =$ diffusion lenght for the holes in the N region $= \sqrt{D_p \tau_p}$

$\tau_p =$ average time it takes for a hole into the N region to recombine with a majority electron

...

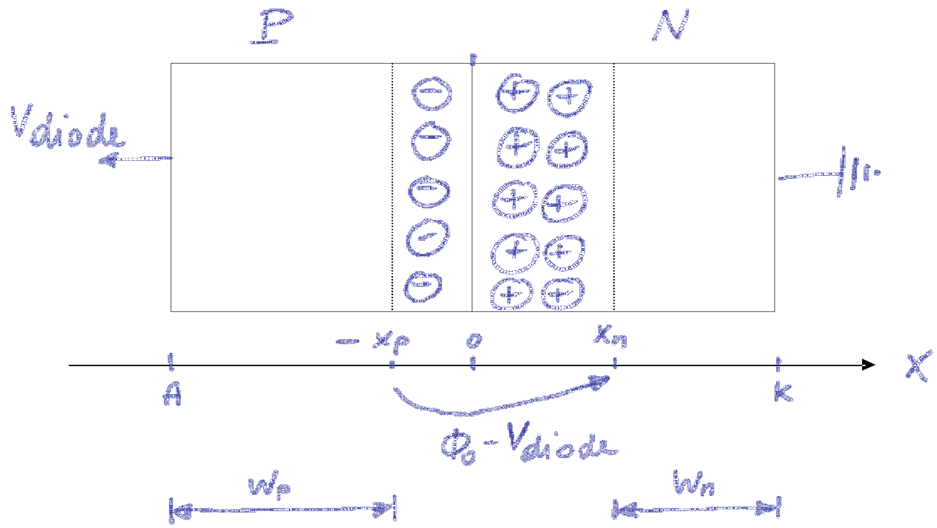
I/V characteristic of PN Junction

Since the only region where we have "net charge" is between $-x_p$ and x_n such region (a.k.a. space charge region) is the only one where there is electric field.

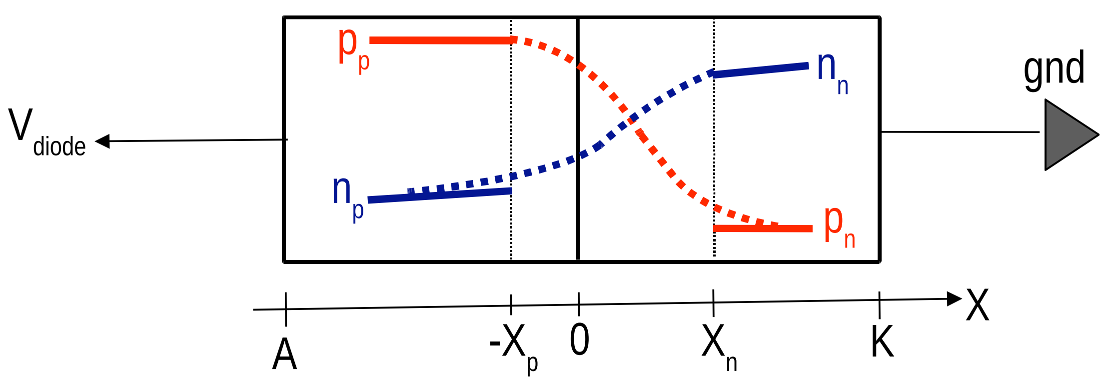


The regions from A to $-x_p$ and from x_n to K are quasi-neutral (it is like they were perfect conductors and in perfect conductors there is no electric field inside)

I/V characteristic of PN Junction

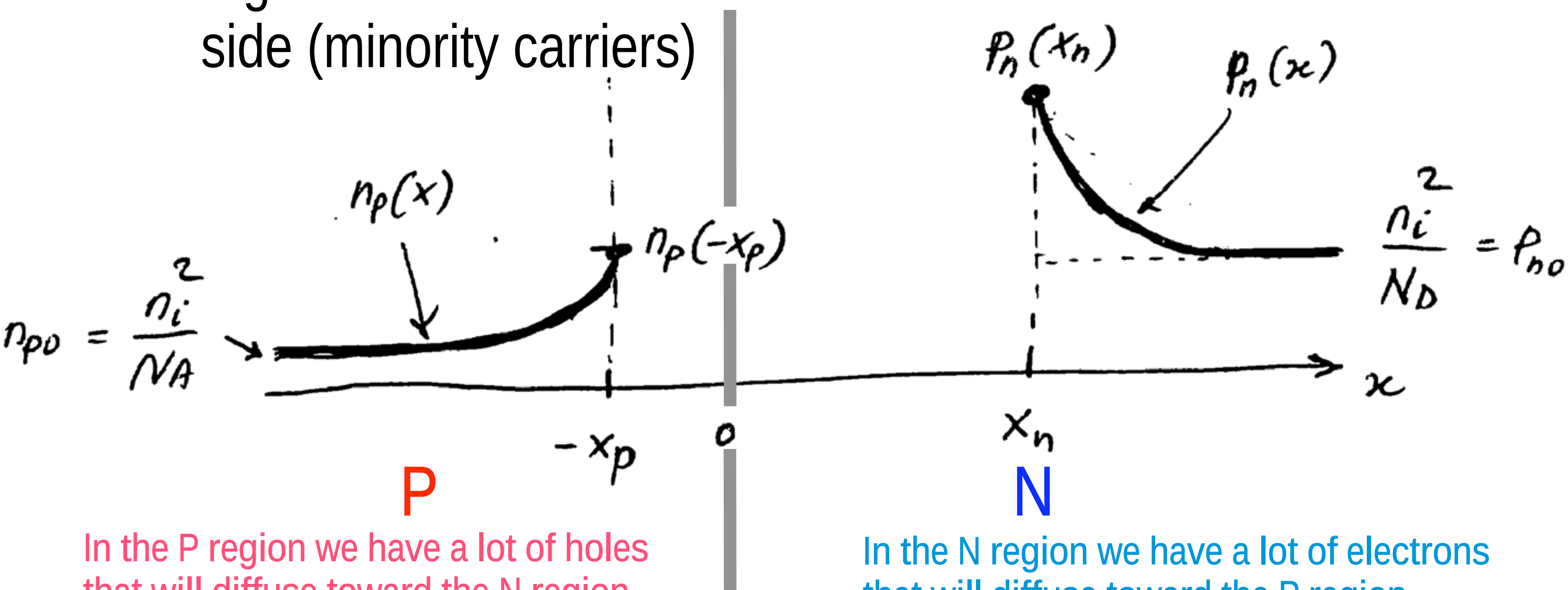


The situation is similar to the one at equilibrium but now the "built in voltage" is $\Phi_0 - V_{diode}$ instead of Φ_0



I/V characteristic of PN Junction

- Under forward bias close to the depletion edges we have:
  - a greater hole concentration than normal on the N side (minority carriers)
  - a greater electrons concentration than normal on the P side (minority carriers)



In the P region we have a lot of holes that will diffuse toward the N region

In the N region we have a lot of electrons that will diffuse toward the P region

I/V characteristic of PN Junction

Extending the result derived at equilibrium we can write the voltage across the space charge region (between $-x_p$ and x_n) as:

$$V_j \equiv V(x_n) - V(-x_p) = \phi_0 - V_{diode} = V_T \ln\left(\frac{p(-x_p)}{p(x_n)}\right) = V_T \ln\left(\frac{n(x_n)}{n(-x_p)}\right)$$

$$\begin{cases} \dfrac{\phi_0 - V_{diode}}{V_T} = \ln \dfrac{p(-x_p)}{p(x_n)} \\[2ex] \dfrac{\phi_0 - V_{diode}}{V_T} = \ln \dfrac{n(x_n)}{n(-x_p)} \end{cases} \qquad \begin{cases} p(x_n) = \dfrac{p(-x_p)}{e^{\frac{\phi_0 - V_{diode}}{V_T}}} = \dfrac{p(-x_p)}{e^{\frac{\phi_0}{V_T}}} e^{\frac{V_{diode}}{V_T}} \\[2ex] n(-x_p) = \dfrac{n(x_n)}{e^{\frac{\phi_0 - V_{diode}}{V_T}}} = \dfrac{n(x_n)}{e^{\frac{\phi_0}{V_T}}} e^{\frac{V_{diode}}{V_T}} \end{cases}$$

I/V characteristic of PN Junction

- And noting that:
  - at the boundary of the quasi neutral P region at -Xp the hole density (majority carriers) is approximately equal at equilibrium as well as under bias,
  - and the same is true for the electron density (majority carriers) at the boundary of the quasi neutral N region (at Xn)

$$\frac{p(-x_p)}{e^{\frac{\phi_0}{V_T}}} \approx \frac{p_{p0}}{e^{\frac{\phi_0}{V_T}}} = p_{n0} = \frac{n_i^2}{N_D}$$

$$\frac{n(x_n)}{e^{\frac{\phi_0}{V_T}}} \approx \frac{n_{n0}}{e^{\frac{\phi_0}{V_T}}} = n_{p0} = \frac{n_i^2}{N_A}$$

the concentration of the majority carriers in the quasi neutral regions is approximately the same as the concentration at equilibrium

I/V characteristic of PN Junction

- Thus:

$$\begin{cases} p(x_n) = \dfrac{p(-x_p)}{e^{\frac{\phi_0 - V_{diode}}{V_T}}} = \dfrac{p(-x_p)}{e^{\frac{\phi_0}{V_T}}} e^{\frac{V_{diode}}{V_T}} \\ n(-x_p) = \dfrac{n(x_n)}{e^{\frac{\phi_0 - V_{diode}}{V_T}}} = \dfrac{n(x_n)}{e^{\frac{\phi_0}{V_T}}} e^{\frac{V_{diode}}{V_T}} \end{cases} \longrightarrow \boxed{\begin{cases} p(x_n) \approx \dfrac{n_i^2}{N_D} e^{\frac{V_{diode}}{V_T}} \\ n(-x_p) \approx \dfrac{n_i^2}{N_A} e^{\frac{V_{diode}}{V_T}} \end{cases}}$$

$$\frac{p(-x_p)}{e^{\frac{\phi_0}{V_T}}} \approx \frac{p_{p0}}{e^{\frac{\phi_0}{V_T}}} = p_{n0} = \frac{n_i^2}{N_D}$$

$$\frac{n(x_n)}{e^{\frac{\phi_0}{V_T}}} \approx \frac{n_{n0}}{e^{\frac{\phi_0}{V_T}}} = n_{p0} = \frac{n_i^2}{N_A}$$

I/V characteristic of PN Junction

$$I_{diode} = I_n + I_p = I_{n,drift} + I_{n,diff} + I_{p,drift} + I_{p,diff}$$

- If we consider the quasi neutral regions, since in the quasi neutral regions there is no field there will be no drift

$$I_n \approx I_{n,diff} \quad (neutral\ region\ W_N)$$
$$I_p \approx I_{p,diff} \quad (neutral\ region\ W_P)$$

- Then, the most suitable traverse sections for the evaluation of the total current I_{diode} are those at the boundary of the depletion layer ($x=-x_p$ or $x=x_n$)

$$I_{diode} = I_n(-x_p) + I_p(-x_p) \approx I_{n.diff}(-x_p) + I_{p,diff}(-x_p)$$

I/V characteristic of PN Junction

- If we make the simplifying assumption that the flow of the carriers in the depletion region is approximately constant (in other words we assume the recombination in the depletion region is negligible)

$$I_{p,diff}(-x_p) \approx I_{p,diff}(x_n)$$

$$\Downarrow$$

$$I_{diode} \approx I_{n.diff}(-x_p) + I_{p,diff}(-x_p) \approx I_{n.diff}(-x_p) + I_{p,diff}(x_n)$$

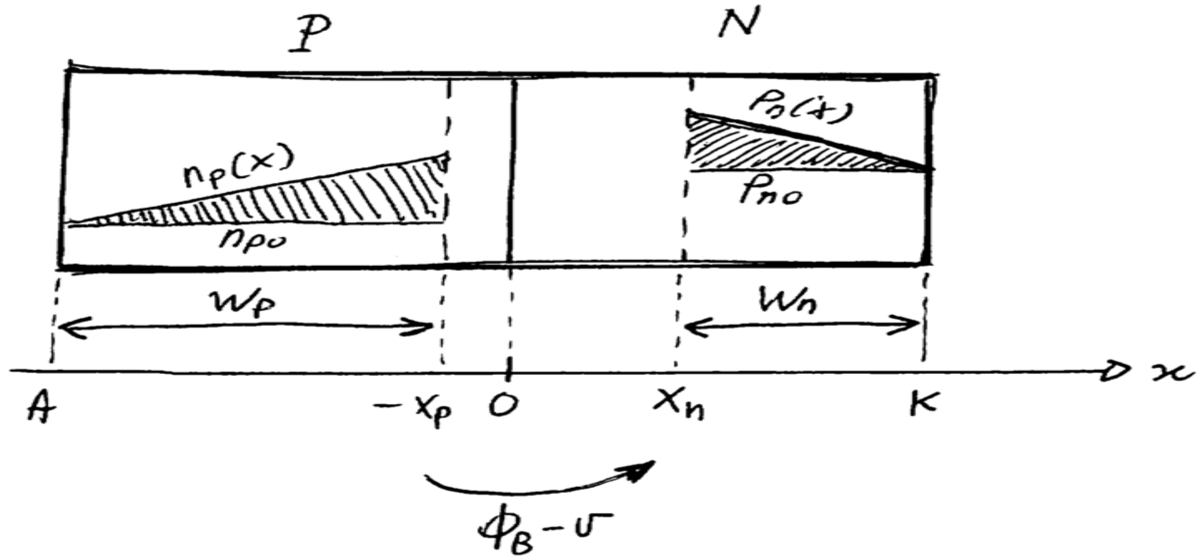
- The currents due to diffusing carriers moving away from the junction are given by the well know diffusion equations:

$$I_{n,diff}(x) = q A D_n \frac{dn_p(x)}{dx}$$

$$I_{p,diff}(x) = -q A D_p \frac{dp_n(x)}{dx}$$

I/V characteristic of PN Junction

- If we assume that the carriers distribution is linear (SHORT DIODE)



$$\frac{dn_p(x)}{dx}\bigg|_{-x_p} = \frac{n_p(-x_p) - n_p(A)}{W_p} = \frac{n_{p0}e^{\frac{V_{diode}}{V_T}} - n_{p0}}{W_p} = \frac{n_{p0}(e^{\frac{V_{diode}}{V_T}} - 1)}{W_p}$$

$$\frac{dp_n(x)}{dx}\bigg|_{x_n} = -\frac{p_n(x_n) - p_n(K)}{W_n} = -\frac{p_{n0}e^{\frac{V_{diode}}{V_T}} - p_{n0}}{W_n} = -\frac{p_{n0}(e^{\frac{V_{diode}}{V_T}} - 1)}{W_n}$$

PN Junction: I/V characteristic

- Recalling that:

$$n_{p0} = \frac{n_i^2}{N_A} \quad \text{(electrons are minority carriers in P)}$$

$$p_{n0} = \frac{n_i^2}{N_D} \quad \text{(holes are minority carriers in N)}$$

$$\frac{dn_p(-x_p)}{dx} = \frac{n_{p0}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)}{W_p} = \frac{n_i^2}{N_A}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

$$\frac{dp_n(x_n)}{dx} = -\frac{p_{n0}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)}{W_n} = \frac{-n_i^2}{N_D}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

$$I_{n,diff}(-x_p) = qAD_n\frac{dn_p(-x_p)}{dx} = qAn_i^2\frac{D_n}{N_A W_p}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

$$I_{p,diff}(x_n) = -qAD_p\frac{dp_n(x_n)}{dx} = qAn_i^2\frac{D_p}{N_D W_n}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

PN Junction: I/V characteristic

- And finally:

$$I_{n,diff}(-x_p) = qAn_i^2 \frac{D_n}{N_A W_p}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

$$I_{p,diff}(x_n) = qAn_i^2 \frac{D_p}{N_D W_n}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

$$I_{diode} \approx I_{n.diff}(-x_p) + I_{p,diff}(-x_p) = \underbrace{qAn_i^2\left(\frac{D_n}{N_A W_p} + N_D W_n\right)}_{I_s}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

- In the case of a LONG DIODE the minority carriers will recombine before reaching the diode terminals

$$I_{diode} \approx I_{n.diff}(-x_p) + I_{p,diff}(-x_p) = qAn_i^2\left(\frac{D_n}{N_A \boldsymbol{L_n}} + \frac{D_p}{N_D \boldsymbol{L_p}}\right)\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

PN Junction: I/V characteristic

SHORT DIODE:

$$I_{diode} \approx I_{n.diff}(-x_p) + I_{p,diff}(-x_p) = \underbrace{qAn_i^2\left(\frac{D_n}{N_A W_p} + \frac{D_p}{N_D W_n}\right)}_{I_s}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

LONG DIODE:

$$I_{diode} \approx I_{n.diff}(-x_p) + I_{p,diff}(-x_p) = \underbrace{qAn_i^2\left(\frac{D_n}{N_A \boldsymbol{L_n}} + \frac{D_p}{N_D \boldsymbol{L_p}}\right)}_{I_s}\left(e^{\frac{V_{diode}}{V_T}} - 1\right)$$

Where L_n is a constant known as the diffusion length of electrons in the P side and L_p is a constant known as the diffusion length for holes in the N side. The constants L_n and L_p are dependent on the doping concentrations N_A and N_D respectively.

Diode Capacitances

- Depletion Capacitance (= Junction Capacitance) $\longleftarrow$ C_j
- Diffusion Capacitance $\longleftarrow$ C_d

- Reverse Biased Diode
  - Depletion Capacitance
- Forward Biased Diode
  - Diffusion Capacitance + Depletion Capacitance

Depletion Charge

- The depletion region stores an immobile charge of equal amount on each side of the junction ($\longrightarrow$ it forms a capacitance !!)

$$q_J \;=\; q_P = -q\,N_A\,x_p\,A \;=\; -q_N = -q\,N_D\,x_n\,A$$

N side has “+” ions

P side has “−” ions

$$x_p = X_{dep}\left(\frac{N_D}{N_A + N_D}\right) \quad \text{and} \quad x_n = X_{dep}\left(\frac{N_A}{N_A + N_D}\right)$$

$$q_J \;= -q\frac{N_A N_D}{(N_A + N_D)}\,A\,X_{dep}$$

The decision to take q_J negative is totally arbitrary. (But it turns out to be a good one if we prefer to work with positive capacitances)

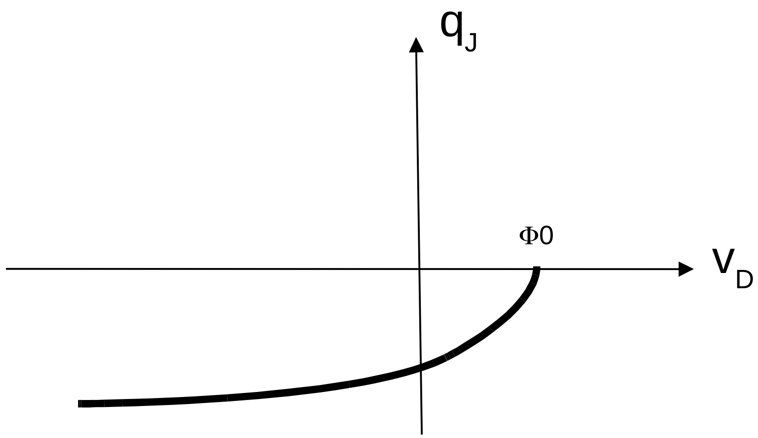
$$X_{dep} = \sqrt{\frac{2\epsilon_s}{q}\,\frac{N_A + N_D}{N_A \cdot N_D}\left(\phi_0 - v_D\right)}$$

$$\boxed{q_J(v_D) = -A\sqrt{2q\,\epsilon_s\,\frac{N_A \cdot N_D}{N_A + N_D}\left(\phi_0 - v_D\right)}}$$

The charge of the depletion region is a function of the voltage v_D applied to the diode

Depletion Capacitance

- Since the depletion charge does not change linearly with the applied voltage the resulting capacitor is non linear !!

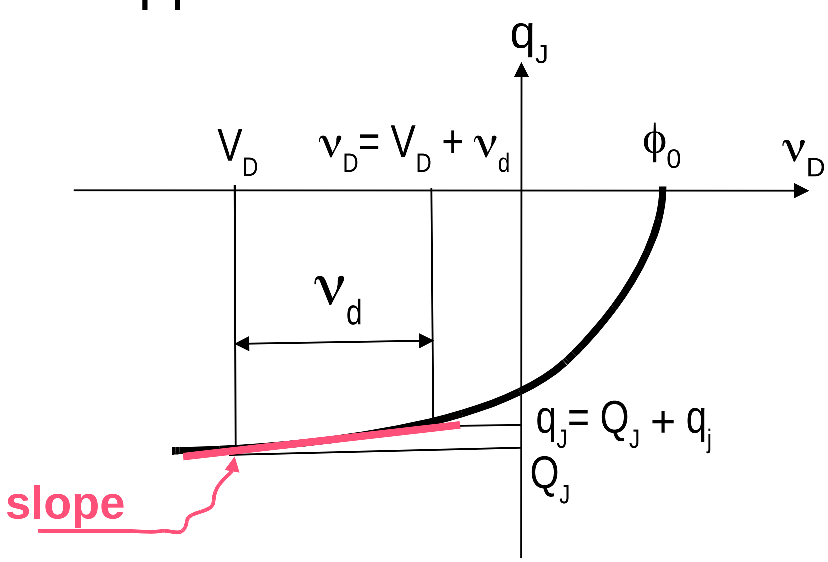


$$q_J(v_D) = -A\sqrt{2q\epsilon_s \frac{N_A \cdot N_D}{N_A + N_D}(\phi_0 - v_D)}$$

- An important physical consideration:
  we are dealing with a capacitor that no matter where I put the + of the applied voltage it always accumulate the positive charge on the N side of the junction, and the negative charge on the P side of the junction.

Small Signal Depletion Capacitance

- For small changes of the applied voltage about a specified DC voltage V_D we can derive an equivalent linear capacitor approximation



since q_J vs. ν_D relationship is non linear the capacitor is non linear $q_J \propto \sqrt{(-v_D)}$

$$q_J(v_D) - q_J(V_D) = \left.\frac{dq_J}{dv_D}\right|_{V_D} \times (v_D - V_D)$$

slope

$$q_j = \left.\frac{dq_J}{dv_D}\right|_{V_D} \times v_d$$

Common conventions:

total "quantities" (applied)

$$v_D = V_D + v_d$$
$$V_d = V_D + v_d$$

small "quantities"

DC "quantities"

Small Signal Depletion Capacitance

$$C_j \equiv C_j(V_D) = \frac{dq_J}{dv_D}\Big|_{V_D} = \frac{d}{dv_D}\left[-A\left(2q\epsilon_s\frac{N_A\cdot N_D}{N_A+N_D}(\phi_0-v_D)\right)^{1/2}\right]_{V_D} =$$

$$= -A\left(2q\epsilon_s\frac{N_A\cdot N_D}{N_A+N_D}\right)^{1/2}\left(\frac{d}{dv_D}(\phi_0-v_D)^{1/2}\right)_{V_D} =$$

$$= -A\left(2q\epsilon_s\frac{N_A\cdot N_D}{N_A+N_D}\right)^{1/2}\left(-\frac{1}{2}(\phi_0-v_D)^{-1/2}\right)_{V_D} =$$

$$= A\left(\frac{q\epsilon_s}{2}\frac{N_A\cdot N_D}{N_A+N_D}\right)^{1/2}\left((\phi_0-V_D)^{-1/2}\right) = A\left(\frac{2}{q\epsilon_s}\frac{N_A+N_D}{N_A\cdot N_D}(\phi_0-V_D)\right)^{-1/2} =$$

$$= \frac{A}{\sqrt{\frac{2}{q\epsilon_s}\frac{N_A+N_D}{N_A\cdot N_D}(\phi_0-V_D)}} = \frac{A}{\sqrt{\frac{2}{q\epsilon_s}\frac{N_A+N_D}{N_A\cdot N_D}\phi_0\left(1-\frac{V_D}{\phi_0}\right)}}$$

$= C_{j0}$

Small Signal Depletion Capacitance

Zero Bias Capacitance = junction capacitance in thermal equilibrium (V_D=0)

$$C_{j0} = C_j(V_D = 0) \equiv \frac{A}{\sqrt{\frac{2}{q\epsilon_s}\frac{N_A+N_D}{N_A \cdot N_D}\phi_0}} = \frac{A\epsilon_s}{X_{dep,0}}$$

$$X_{dep,0} = \sqrt{\frac{2\epsilon_s}{q}\frac{N_A+N_D}{N_A \cdot N_D}\phi_0} = \epsilon_s\sqrt{\frac{2}{q\epsilon_s}\frac{N_A+N_D}{N_A \cdot N_D}\phi_0}$$

$$C_j \equiv C_j(V_D) = \frac{q_j}{v_d} = \frac{dq_J}{dv_D}\Big|_{V_D} = \frac{C_{j0}}{\sqrt{\left(1-\frac{V_D}{\phi_0}\right)}} = \frac{A\epsilon_s}{X_{dep,0}\sqrt{\left(1-\frac{V_D}{\phi_0}\right)}} = \frac{A\epsilon_s}{X_{dep}}$$

Small Signal Depletion Capacitance: Physical Interpretation

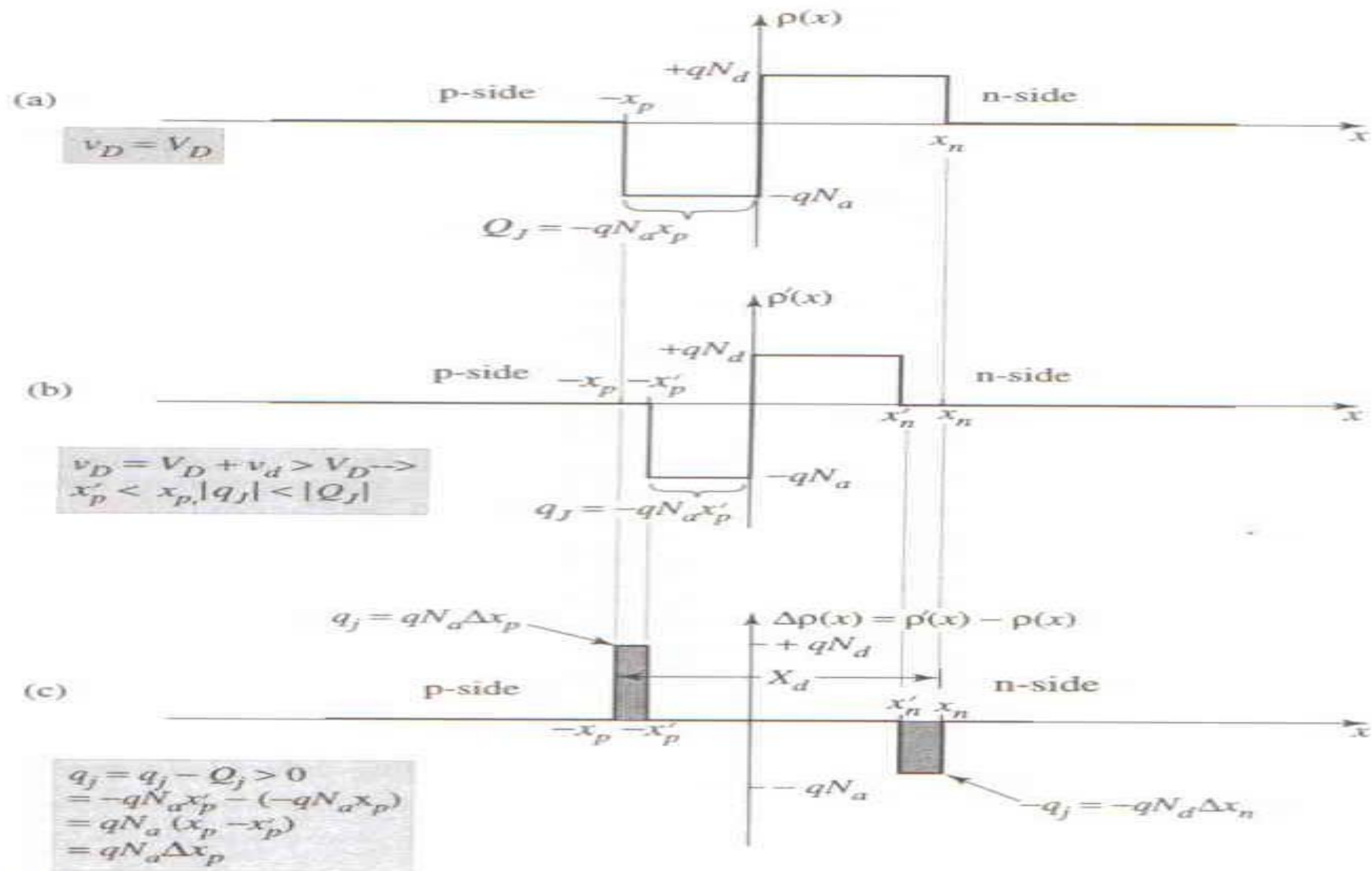
$$C_j = \frac{q_j}{v_d} = \frac{A\epsilon_s}{X_{dep}}$$

Capacitance of a parallel plate capacitor with its plates separated by the depletion width $X_{dep}(V_D)$ at the particular DC voltage V_D.

The charges separated by X_{dep} are the small signal charge layers $\pm q_j$.

For vd $\rightarrow$ 0, the small signal charges become sheets that are separated by a gap width of exactly Xdep

Physical Interpretation



➤ **Figure 3.19** (a) Charge density $\rho(x)$ in depletion region for a reverse bias of $V_D < 0$, (b) charge density $\rho'(x)$ in depletion region for a perturbed reverse bias $v_D = V_D + v_d$ with $v_d > 0$, and (c) difference $\Delta\rho(x)$ between (b) and (a) showing incremental depletion charge $\pm q_j$ separated by approximately the depletion width X_d. Note that the magnitude of v_d is exaggerated in order to clarify its effect on the depletion region width.

Small Signal Depletion Capacitance

- In the practice the depletion capacitance is usually provided per cross-section area:

$$\mathbb{C}_{j0} = \mathbb{C}_j(V_D = 0) \equiv \frac{\epsilon_s}{X_{dep,0}} = \frac{1}{\sqrt{\frac{2}{q\epsilon_s}\frac{N_A + N_D}{N_A \cdot N_D}\phi_0}}$$

$$\mathbb{C}_j = \mathbb{C}_j(V_D) \equiv \frac{dq_J}{dv_D}\bigg|_{V_D} = \frac{\epsilon_s}{X_{dep}} = \frac{\epsilon_s}{X_{dep,0}\sqrt{\left(1 - \frac{V_D}{\phi_0}\right)}} = \frac{\mathbb{C}_{j0}}{\sqrt{\left(1 - \frac{V_D}{\phi_0}\right)}}$$

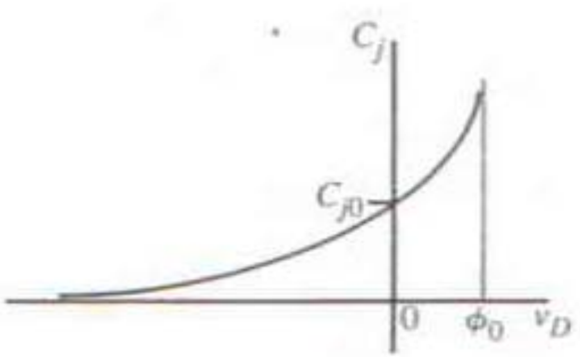


Figure 2.2-3 Depletion capacitance as a function of externally applied junction voltage.

NOTE: when the diode is forward biased with $v_D = \phi_0$ the equation for C_j "blows up" (i.e., is equal to infinity). As v_D approaches ϕ_0 the assumption that the depletion region is free of charged carriers is no longer true.

Graded Junctions

- All the equations derived for the depletion capacitance are based on the assumption that the doping concentration change abruptly at the junction. Although this is a good approximation for many integrated circuits is not always true.
- More in general:

$$C_j = \frac{C_{j0}}{\left(1 - \frac{V_D}{\phi_0}\right)^{M_j}}$$

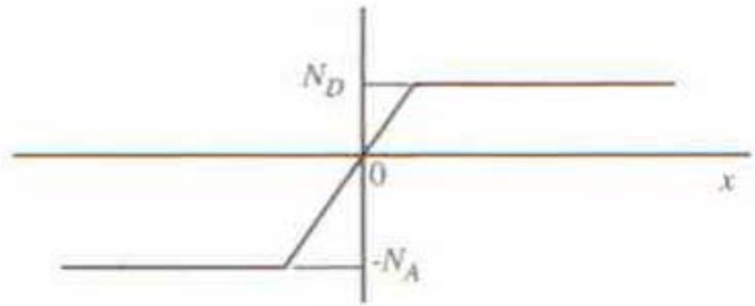


Figure 2.2-2 Impurity concentration profile for diffused pn junction.

- Mj is a constant called **grading coefficient** and its value ranges from 1/3 to ½ depending on the way the concentration changes from the P to the N side of the junction

Large Signal Depletion Capacitance

- The equations for the depletion capacitance given before are valid only for small changes in the applied voltage
- It is extremely difficult and time consuming to accurately take this non linear capacitance into account when calculating the time to charge or discharge a junction over a large voltage change
- A commonly used approximation is to calculate the charge stored in the junction for the two extreme values of applied voltage, and then through the use of $\Delta Q = C\Delta V$, calculate the average capacitance accordingly

$$C_{j-av} = \frac{|\, Q(V_2) - Q(V_1) \,|}{|\, V_2 - V_1 \,|}$$

The approximation is pessimistic

Large Signal Depletion Capacitance

$$\mathbb{Q}(V)=\sqrt{2q\epsilon_s\frac{N_A\cdot N_D}{N_A+N_D}(\phi_0-V)}=\sqrt{2q\epsilon_s\frac{N_A\cdot N_D}{N_A+N_D}\phi_0\left(1-\frac{V}{\phi_0}\right)}=2\phi_0\mathbb{C}_{j0}\sqrt{\left(1-\frac{V}{\phi_0}\right)}$$

$$\mathbb{C}_{j0}=\sqrt{\frac{q\epsilon_s}{2\phi_0}\frac{N_A\cdot N_D}{N_A+N_D}} \quad \rightarrow \quad \sqrt{q\epsilon_s\frac{N_A\cdot N_D}{N_A+N_D}}=\mathbb{C}_{j0}\sqrt{2\phi_0}$$

$$\mathbb{C}_{j-av}=\frac{|\mathbb{Q}(V_2)-\mathbb{Q}(V_1)|}{|V_2-V_1|} = 2\phi_0\mathbb{C}_{j0}\left(\frac{\left|\sqrt{1-\frac{V_2}{\phi_0}}-\sqrt{1-\frac{V_1}{\phi_0}}\right|}{|V_2-V_1|}\right)$$

Example

Find a rough approximation for the junction capacitance to be used to estimate the charging time of a reverse biased junction from 0V to 5V (or vice versa). Assume $\phi_0 \approx 0.9$ V

$$C_{j-av} = 2\phi_0 C_{j0}\left(\frac{\left|\sqrt{1-\frac{V_2}{\phi_0}} - \sqrt{1-\frac{V_1}{\phi_0}}\right|}{|V_2 - V_1|}\right) = 2\cdot 0.9 C_{j0}\left(\frac{\left|\sqrt{1-\frac{-5}{0.9}} - 1\right|}{|-5-0|}\right) =$$

$$\approx \frac{C_{j0}}{2}$$