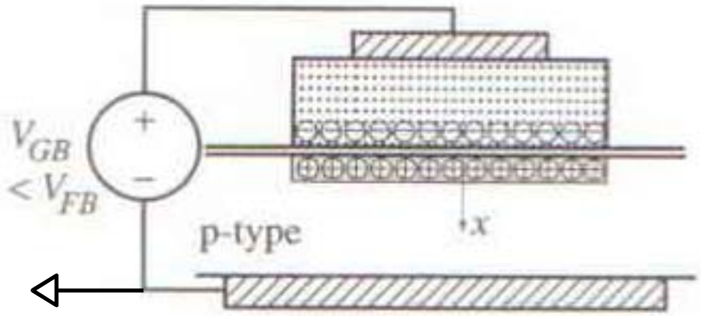
Accumulation

We apply a gate to bulk voltage that is smaller than the flat-band voltage:

$$\boxed{V_{GB} < V_{FB}}$$

NOTICE:
for an MOS structure with
n+ poly-silicon gate and
p-type substrate this is a
negative value



The gate potential is more negative than the potential of he bulk

Since the bulk is of p-type, there is plenty of positively charged holes in it that will be attracted toward the substrate surface, so the MOS capacitor start to store positive charge at the substrate surface. (And as a by-product of the neutrality principle the gate charge become negative)

Accumulation

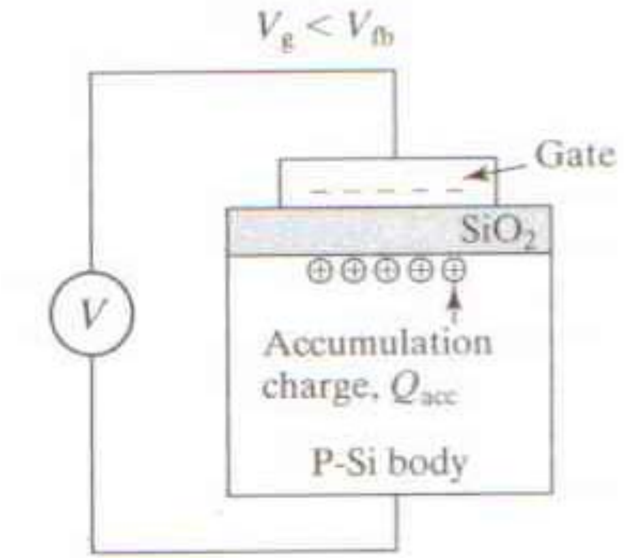


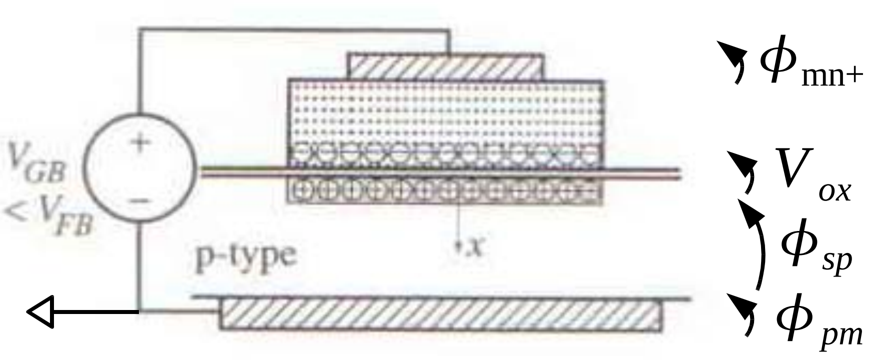
FIGURE 5–5 This MOS capacitor is biased into surface accumulation ($p_s > p_0 = N_a$). Types of charge present. ⊕ represents holes and – represents negative charge.

The piling-up of holes at the surface increases the density of holes at the surface p_s above the normal bulk level of N_A.

$$p_s = n_i e^{-\phi_s/V_T} \rightarrow \phi_s = -V_T \ln\left(\frac{p_s}{n_i}\right) < \underbrace{-V_T \ln\left(\frac{N_A}{n_i}\right)}_{=\phi_p}$$

However, the logarithmic function is weak so it is a reasonable approximation that $\phi_s \approx \phi_p$ in accumulation (for $p_s \approx 10\, N_A$ at room temperature the Φ_s would increases of only 60 mV)

Accumulation



drop across the semiconductor

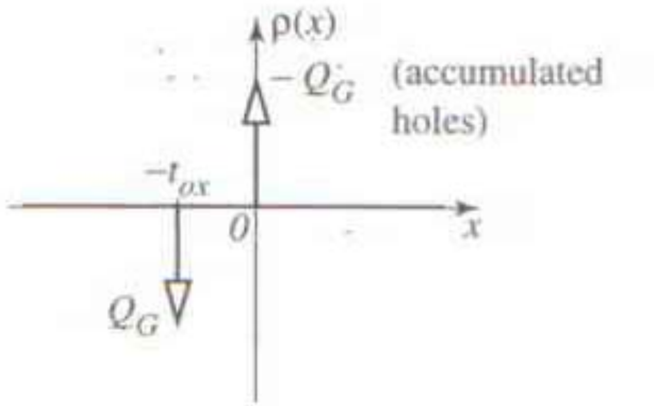
$$V_{GB} = \phi_{mn+} + V_{ox} + \phi_{sp} + \phi_{pm}$$

$$\downarrow$$

$$V_{GB} = \underbrace{\phi_{mn+} + \phi_{pm}}_{= -\phi_{\text{BUILT-IN}} \equiv V_{FB}} + V_{ox} + \overset{\approx 0}{(\phi_s - \phi_p)}$$

$$\downarrow$$

$$\boxed{V_{GB} - V_{FB} \approx V_{ox}}$$



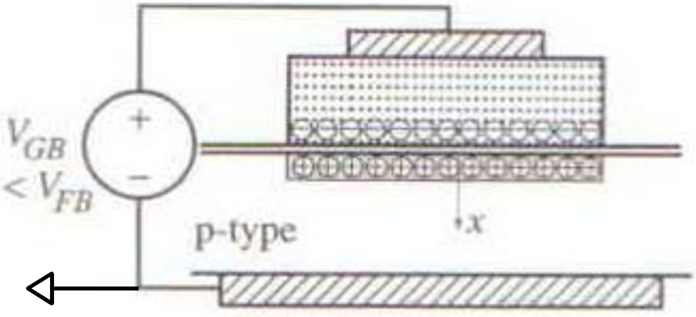
In accumulation the charge density is identical to that of a parallel plate capacitor, thus:

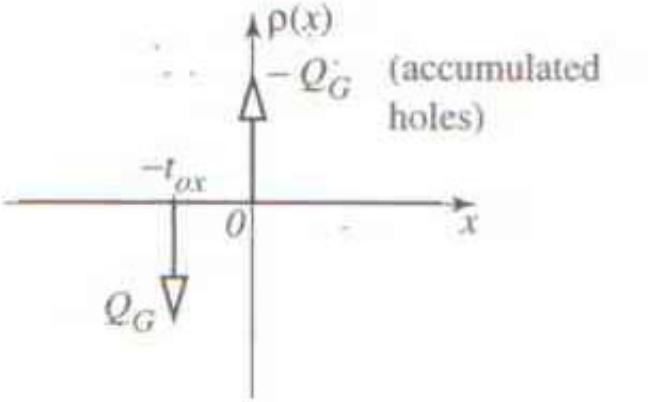
$$\text{for } V_{GB} \leqslant V_{FB}: \quad \mathbb{Q}_{acc} = -\mathbb{Q}_G = -\frac{\epsilon_{ox}}{t_{ox}} V_{ox} = -\mathbb{C}_{ox} V_{ox}$$

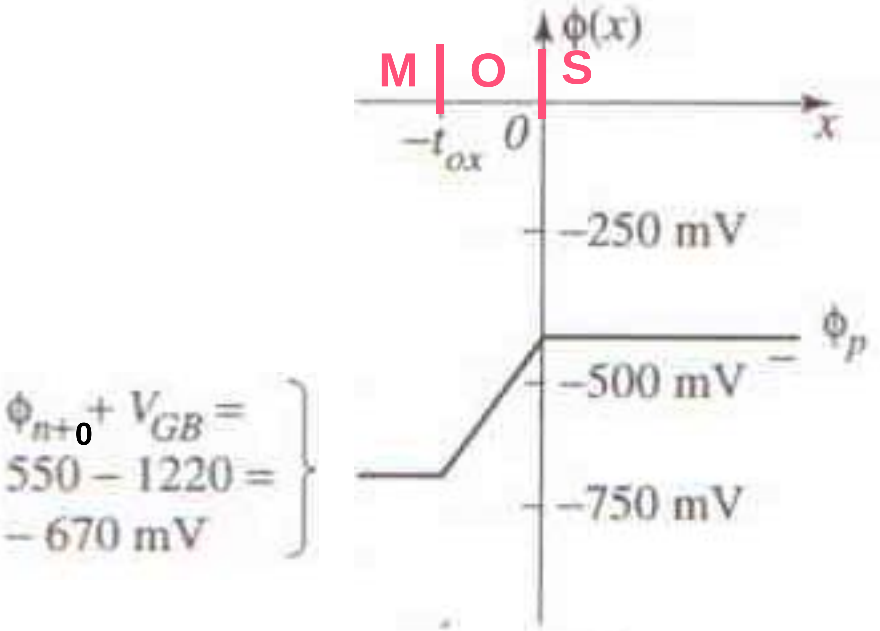
$$\downarrow$$

for $V_{GB} \leqslant V_{FB}$:	$\mathbb{Q}_G = \mathbb{C}_{ox} V_{ox} = \mathbb{C}_{ox}(V_{GB} - V_{FB})$

Accumulation







Depletion

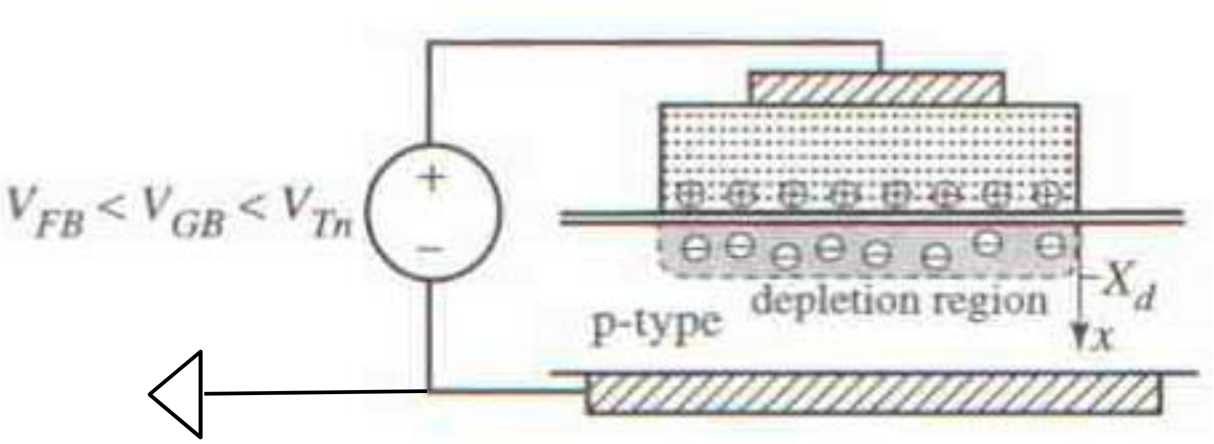
We apply a gate to bulk voltage bigger than the flat-band voltage but smaller than a certain threshold:

$$\boxed{V_{FB} < V_{GB} < V_{TH}}$$

With respect to the flat-band condition the potential of the poly-silicon will be shifted V_{GB} above the potential of the substrate

↓

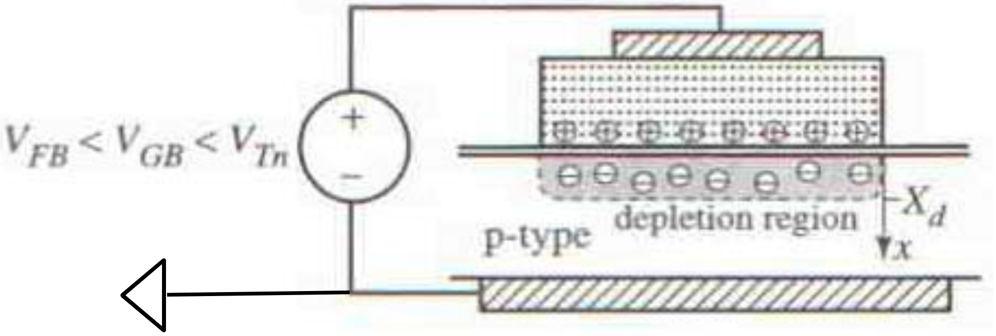
The holes in the semiconductor will be repelled down in the substrate and leave negatively charged fixed acceptor ions behind (i.e. a depletion region is created near the surface).
The negatively charged depletion region in the substrate is mirrored by a positive charge sheet on the gate

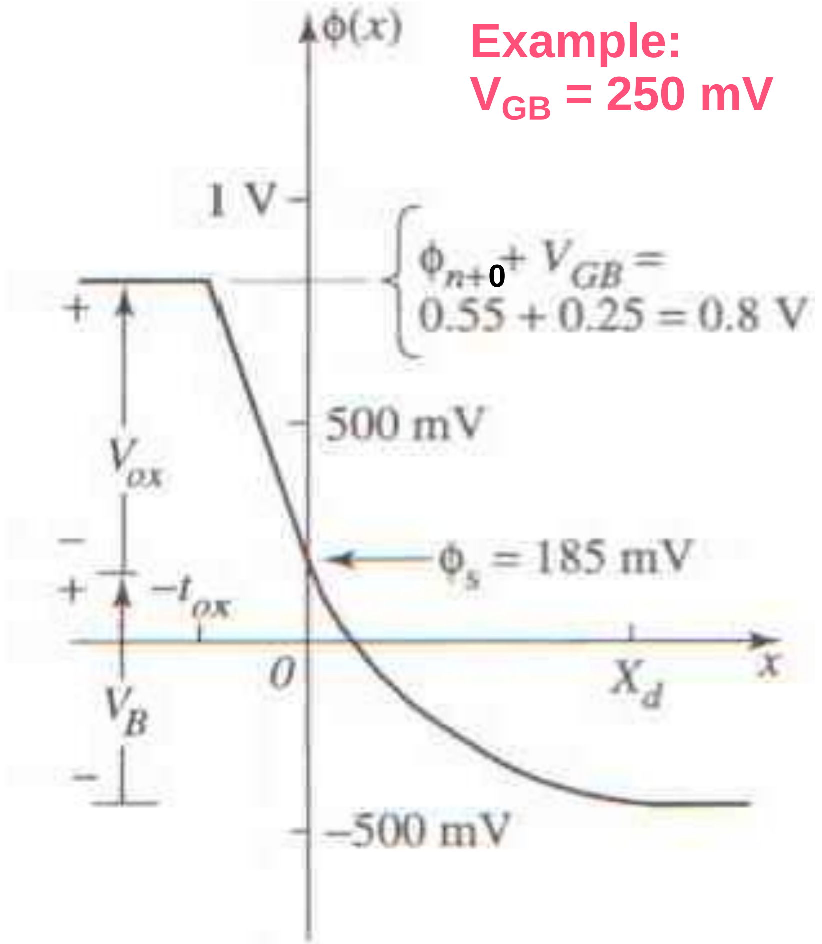


NOTE:
we are already familiar with this case: this the same case we have analyzed for the MOS capacitor in thermal equilibrium

Depletion

Example:
V_{GB} = 250 mV





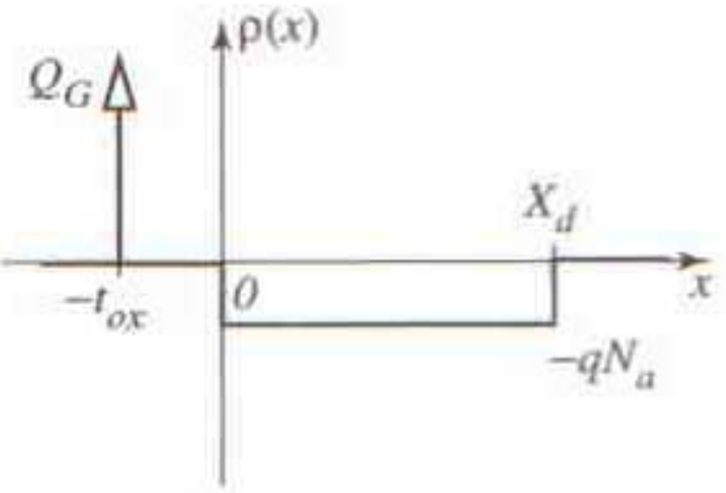
$$= -\phi_{\text{BUILT-IN}} \equiv V_{FB} \quad (= -\phi_{n+,0} + \phi_{p,0})$$

$$V_{GB} = \overbrace{\phi_{mn} + \phi_{pm}} + \underbrace{V_{MOS}}$$

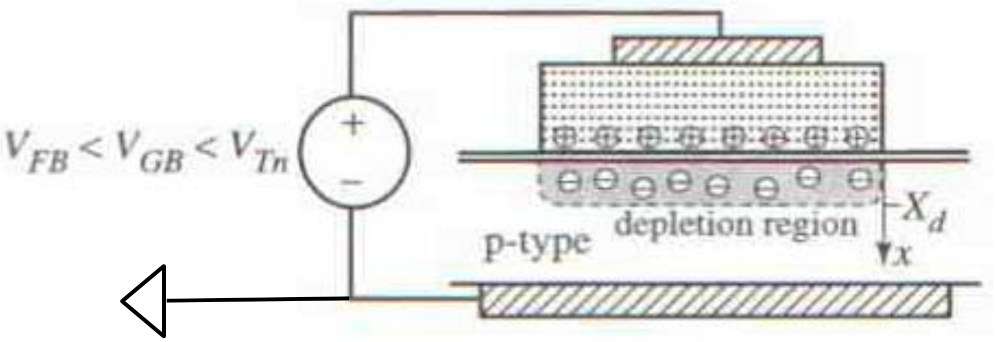
$$= V_{ox} + V_B \quad (= \phi_{n+} - \phi_p)$$

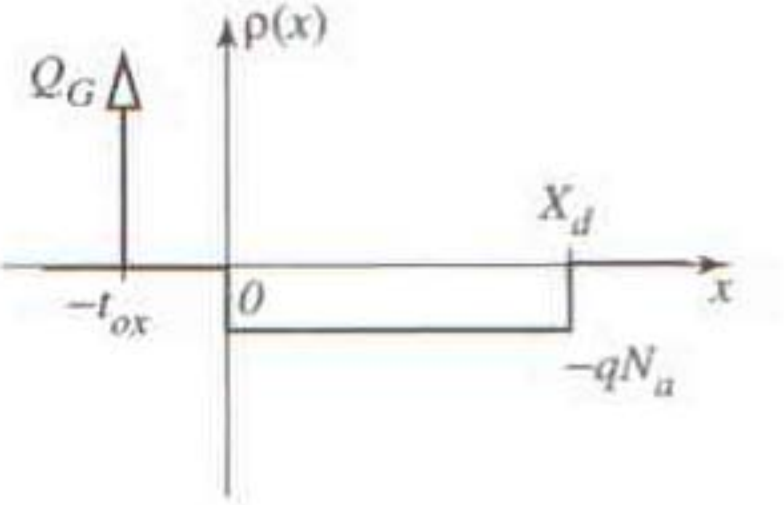
$$\downarrow$$

$$\boxed{V_{GB} - V_{FB} = V_{ox} + V_B}$$



Depletion





Carefully adapting
the results found in TE

$$V_{GB} - V_{FB} = V_{ox} + V_B$$

$$Depletion\ Width: \quad X_d = \frac{\epsilon_s}{\mathbb{C}_{ox}} \left(\sqrt{1 + \frac{2\mathbb{C}_{ox}^2}{qN_A \epsilon_s}(V_{GB} - V_{FB})} - 1 \right) \quad for\ V_{FB} < V_{GB} < V_{TH}$$

$$Charge\ of\ the\ MOS\ capacitor: \quad Q_G = -Q_B = q N_A X_d \quad for\ V_{FB} < V_{GB} < V_{TH}$$

$$Surface\ Potential: \quad \phi_s = \phi_{\mathrm{n+}} - \frac{q N_A X_d}{\mathbb{C}_{ox}} = V_{GB} + \phi_{\mathrm{n+,0}} - \frac{q N_A X_d}{\mathbb{C}_{ox}} \quad for\ V_{FB} < V_{GB} < V_{TH}$$

Depletion

NOTE:
In reality in the substrate besides the holes being repelled down we have also free electrons (minority carriers) being attracted up, toward the surface.

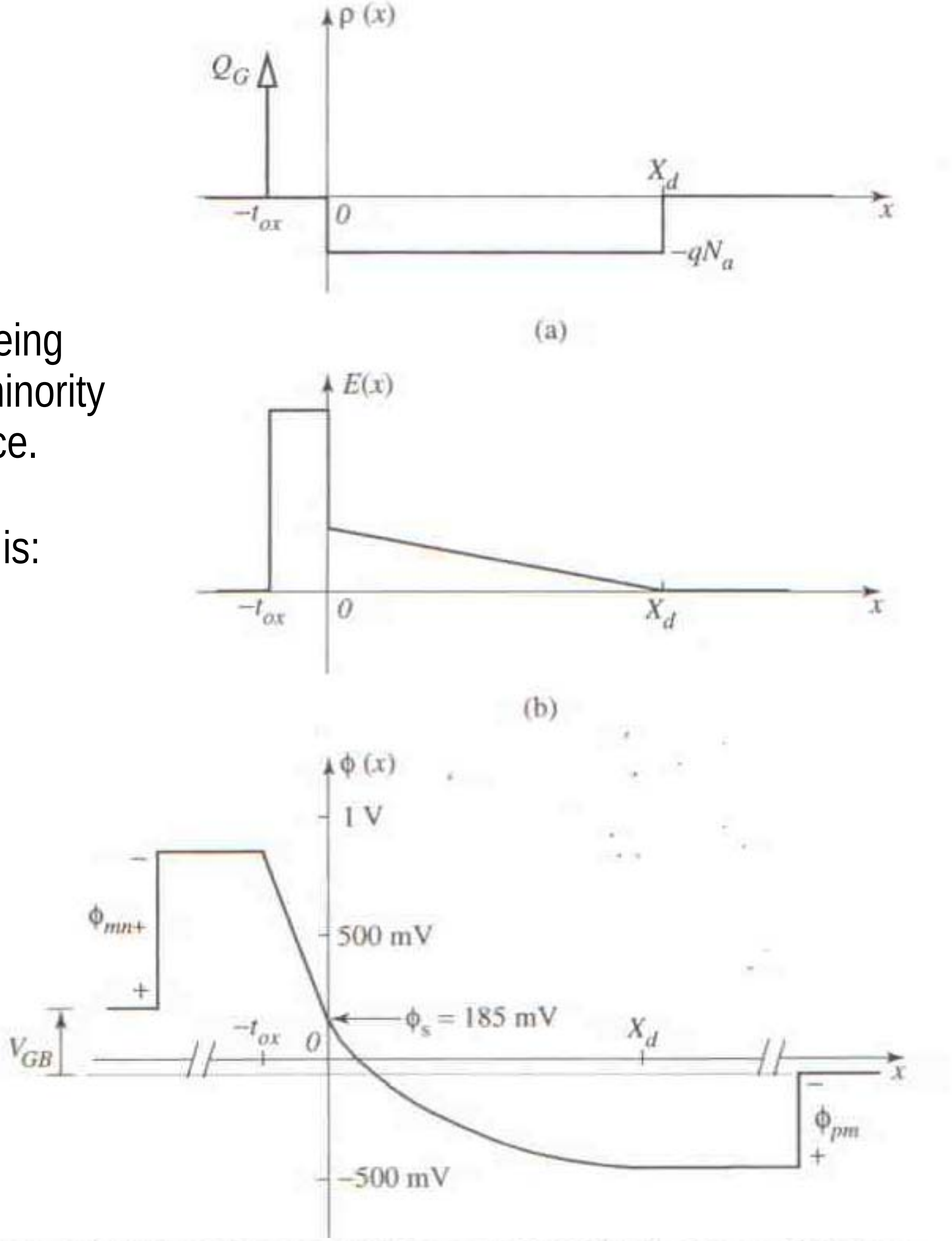
The concentration of electrons at the surface is:

$$\boxed{n_s = n_i e^{\phi_s/V_T}}$$

Example:
with Φs=185 mV the surface electron concentration at room temperature is:

$$n_s = n_i e^{\phi_s/V_T} = 10^{10} e^{185/26} \approx 1.2 \times 10^{13} cm^{-3}$$

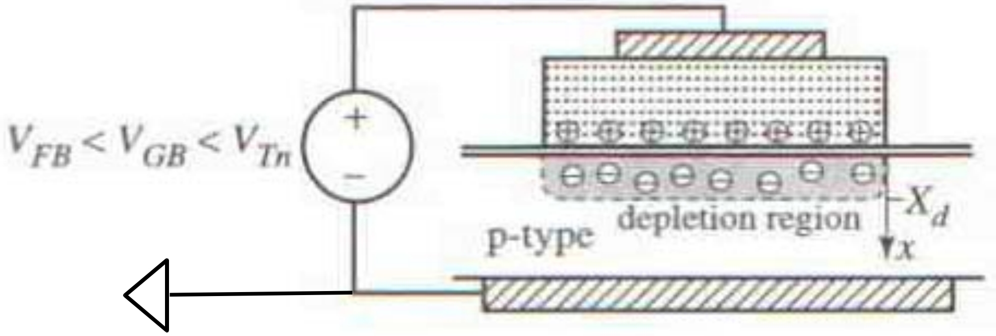
which is negligible compared to the ionized acceptor concentration N_A ($=10^{17}$ cm^{-3})

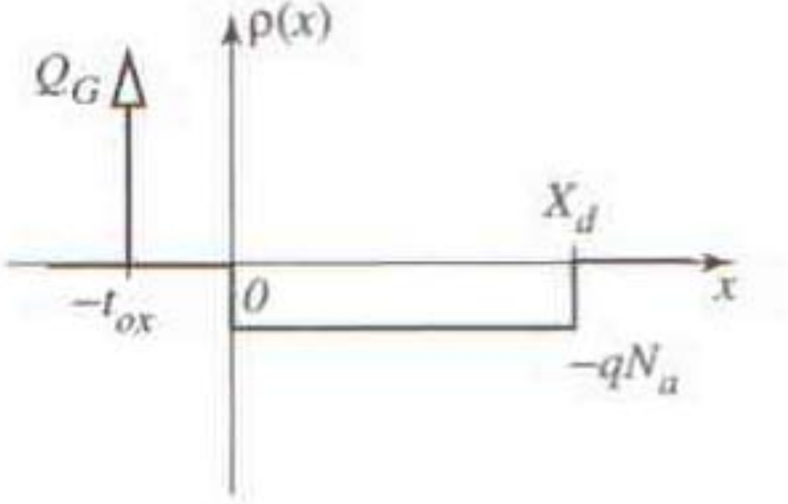


➤ **Figure 3.32** MOS capacitor on a p-type substrate biased in depletion with an applied bias $V_{GB} = 250$ mV for $V_{FB} = -970$ mV: (a) charge density, (b) electric field, and (c) potential.

$V_{FB} < V_{GB} < V_{TH}$

Depletion

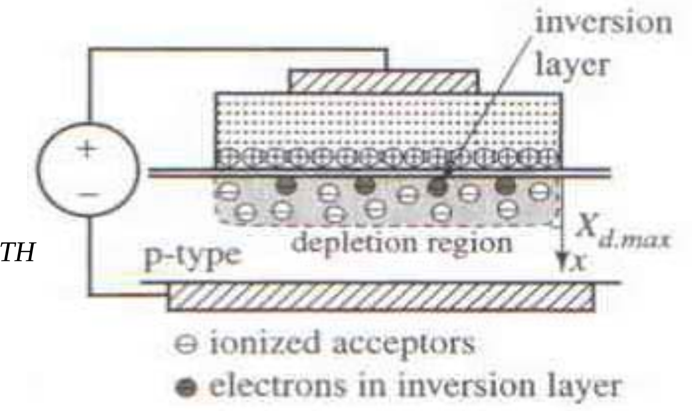




- In order to keep the analysis simple we assume that in depletion condition the surface electron concentration is always negligible
- As V_{GB} get closer and closer to the critical value V_{TH} this is not necessarily very accurate

Depletion

$V_{GB} > V_{FB}$
&
close to V_{TH}



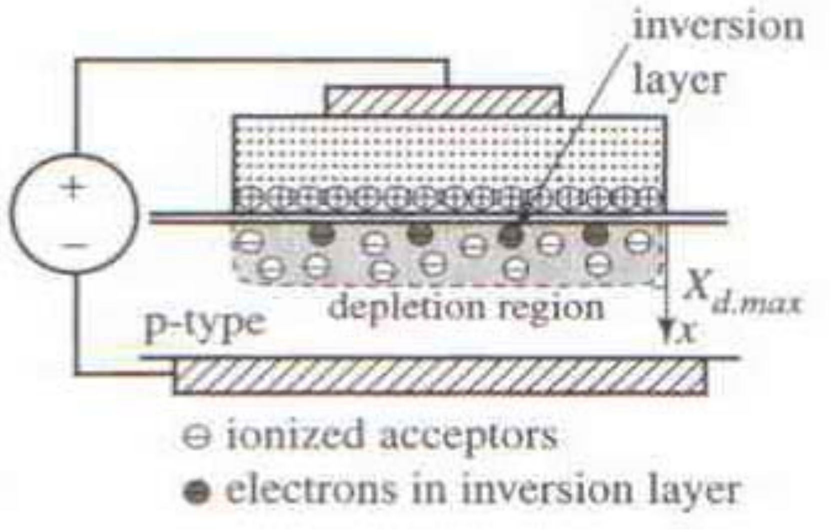
- The electron concentration increases exponentially as the surface potential increases, to the point where it eventually becomes the dominant component of the negative charge in the silicon substrate

$$n_s = n_i e^{\phi_s / V_T}$$

- As a result the charge density in the silicon substrate must be modified to include the electron contribution

$$\rho(x) = -q(N_A + n(x)) = -q\left(N_A + n_i e^{\phi(x)/V_T}\right) \quad for\ 0 \le x \le X_d$$

Threshold



As we keep increasing the gate to bulk voltage above the flat-band level the surface potential continue to rise and eventually, there will be a significant electron concentration at the surface.

$$\text{for}\; V_{FB} < V_{GB} < V_{TH} : \;\; \phi_s = V_{GB} + \phi_{n+,0} - \frac{q N_A X_d(V_{GB})}{C_{ox}}$$

At some point the surface will become so rich of electrons that it is no longer in depletion but at the threshold of inversion. In other words, it looks like if the surface inverted from p-type to n-type (a “material” with a lot of free mobile electrons).

The threshold is defined as the condition when the surface electron concentration n_s is equal to the bulk doping concentration N_A

Threshold

$$\boxed{V_{GB} = V_{TH}}$$

- At the threshold the surface is as much n-type as the bulk is p-type:

$$n_s^{(threshold)} = n_i \, e^{\phi_s^{(threshold)}/V_T} \equiv N_A$$

- And since at T.E. the bulk concentration is:

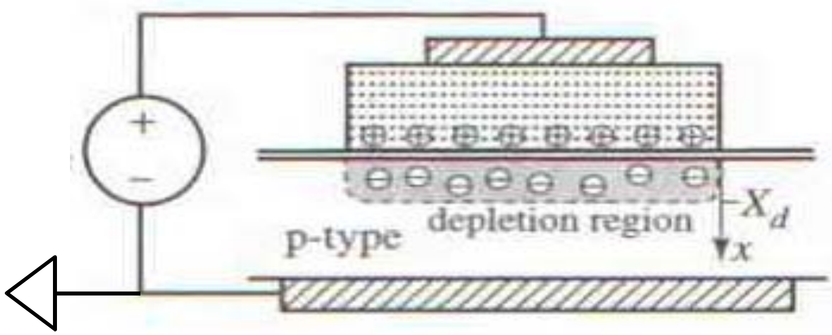
$$\frac{N_A}{n_i} = -e^{\phi_{\mathrm{p},0}/V_T}$$

↓

- It follows that at the threshold the surface potential becomes equal and opposite to the fermi potential of the bulk:

$$\boxed{\phi_s^{(threshold)} = -\phi_{\mathrm{p},0} \equiv -\phi_{\mathrm{F\text{-}bulk}}}$$

Threshold Voltage



- Let's write KVL at the onset of inversion (threshold):

$$\underbrace{V_{GB}^{\text{(threshold)}}}_{\equiv V_{TH}} = V_{FB} + V_{ox}^{\text{(threshold)}} + V_{B}^{\text{(threshold)}}$$

$$\updownarrow$$

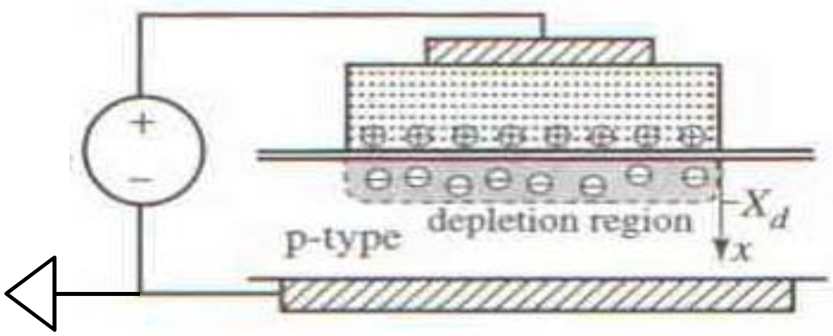
$$V_{TH} - V_{FB} = V_{ox}^{\text{(threshold)}} + V_{B}^{\text{(threshold)}}$$

All we have to do is to find V_{ox} and V_B

- The potential drop across the depletion region is a known quantity:

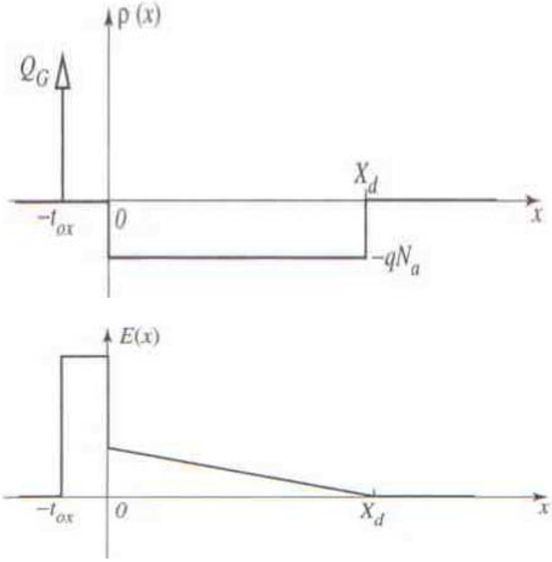
$$V_{B}^{\text{(threshold)}} = \phi_{s}^{\text{(threshold)}} - \phi_{p0} = -2\phi_{p0} = -2\phi_{\text{F-bulk}} = 2\phi_{s}^{\text{(threshold)}}$$

Threshold Voltage



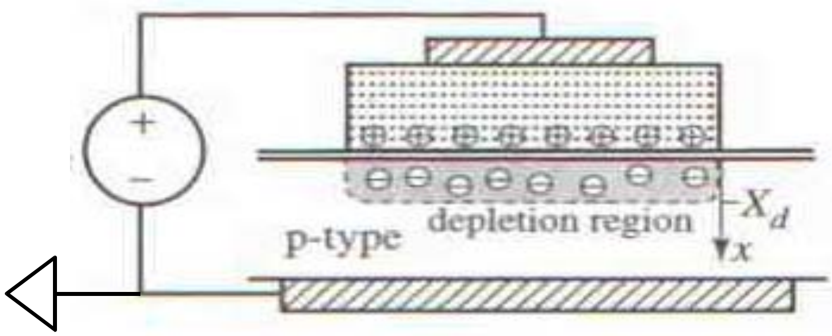
- At the onset of inversion the depletion width has increased to its maximum value $X_{d,max}$
- The charge density in the depletion region is $\rho(x) = -qN_A$

Gauss at the Boundary: $\int_{0+}^{X_{\text{d,max}}} dE(x) = \int_{0+}^{X_{\text{d,max}}} \frac{\rho(x)}{\epsilon_s} dx = \frac{-qN_A X_{\text{d,max}}}{\epsilon_s}$



$$E(\underset{0}{\cancel{X_{\text{d,max}}}}) - E(0+) = \frac{-qN_A X_{\text{d,max}}}{\epsilon_s} \longrightarrow E(0+) = \frac{qN_A X_{\text{d,max}}}{\epsilon_s}$$

Threshold Voltage



Gauss:

$$\int_{0+}^{x} dE(x) = \int_{0+}^{x} \frac{\rho(x)}{\epsilon_s}\,dx = \frac{-q\,N_A\,x}{\epsilon_s}$$

$$\Big\downarrow$$

$$E(x) - E(0+) = \frac{-q\,N_A\,x}{\epsilon_s}$$

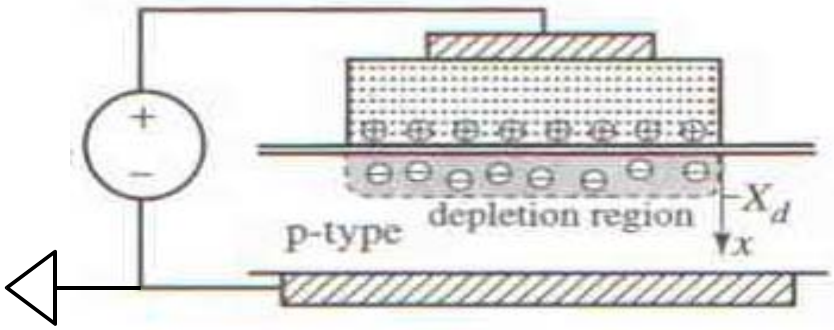
$$\Big\downarrow$$

$$E(x) = \underbrace{E(0+)}_{=\frac{q\,N_A\,X_{\mathrm{d,max}}}{\epsilon_s}} - \frac{q\,N_A\,x}{\epsilon_s} = \frac{qN_A}{\epsilon_s}\left(X_{\mathrm{d,max}} - x\right)$$

Potential:

$$\int_{0+}^{X_{\mathrm{d,max}}} E(x)\,dx = -\underbrace{\int_{0+}^{X_{\mathrm{d,max}}} d\,\phi(x)}_{=V_B^{(\text{threshold})}} \longrightarrow$$

Threshold Voltage



$$V_B^{(\text{threshold})} = \int_{0+}^{X_{\text{d,max}}} E(x)\,dx = \int_{0+}^{X_{\text{d,max}}} \frac{qN_A}{\epsilon_s}(X_{\text{d,max}} - x)\,dx = \frac{qN_A}{\epsilon_s}\frac{X_{\text{d,max}}^2}{2}$$

$$V_B^{(\text{threshold})} = \frac{qN_A}{\epsilon_s}\frac{X_{\text{d,max}}^2}{2} = -2\phi_{\text{F-bulk}} = 2\phi_s^{(\text{threshold})}$$

$$X_{\text{d,max}} = \frac{\sqrt{4\epsilon_s\phi_s^{(\text{threshold})}}}{qN_A}$$

Threshold Voltage

- The charge in the depletion region at the onset of inversion is:

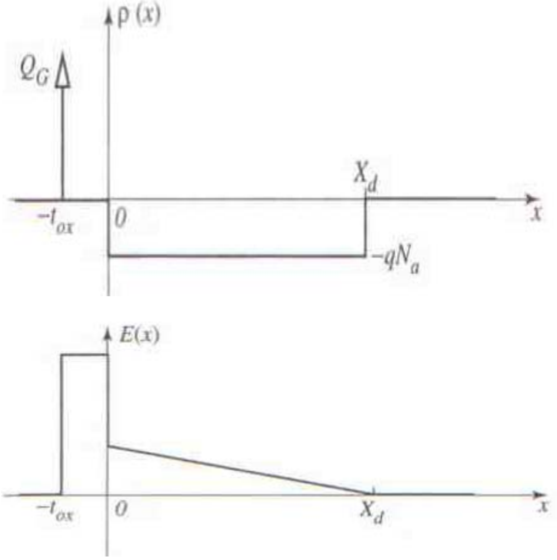
$$\boxed{Q_B^{(\text{threshold})} = Q_{B,\max} = -qN_A X_{d,\max} = -\sqrt{4qN_A\epsilon_s\phi_s^{(\text{threshold})}}}$$

- Let's know find the voltage drop across the oxide:

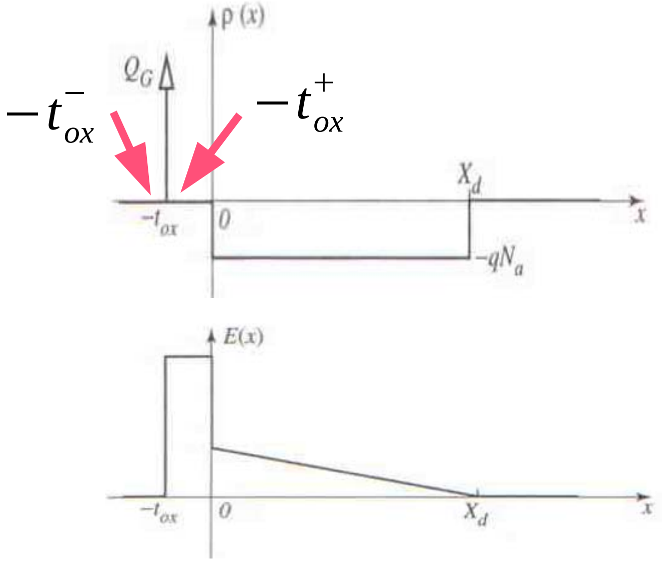
$$\underbrace{\int_{0-}^{-t_{ox}} d\phi(x)}_{=V_{ox}^{(\text{threshold})}} = -\int_{0-}^{-t_{ox}} E(x)\,dx = -\int_{0-}^{-t_{ox}} E_{ox}^{(\text{threshold})}\,dx = E_{ox}^{(\text{threshold})}\cdot t_{ox}$$

$$\downarrow$$

$$V_{ox}^{(\text{threshold})} = E_{ox}^{(\text{threshold})}\cdot t_{ox}$$



Threshold Voltage



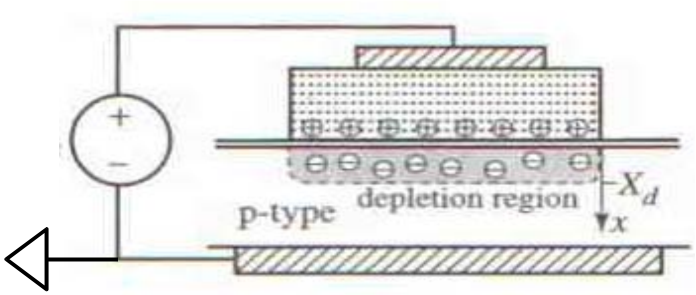
- Gauss: $$\int_{0-}^{-t_{ox}^{-}} dE(x) = \int_{0-}^{-t_{ox}^{-}} \frac{\rho(x)}{\epsilon_{ox}}\, dx$$

$$E(\cancelto{0}{-t_{ox}^{-}})\underbrace{-E(0\text{-})}_{=-E_{ox}^{\text{(threshold)}}} = \int_{0\text{-}}^{-t_{ox}^{-}} \frac{\mathbb{Q}_G^{\text{(threshold)}}\,\delta(x)}{\epsilon_{ox}}\, dx = \int_{-t_{ox}^{+}}^{-t_{ox}^{-}} \frac{\mathbb{Q}_G^{\text{(threshold)}}\,\delta(x)}{\epsilon_{ox}}\, dx = \frac{-\mathbb{Q}_G^{\text{(threshold)}}}{\epsilon_{ox}}$$

$$E_{ox}^{\text{(threshold)}} = \frac{\mathbb{Q}_G^{\text{(threshold)}}}{\epsilon_{ox}} = \frac{-\mathbb{Q}_B^{\text{(threshold)}}}{\epsilon_{ox}}$$

$$\boxed{V_{ox}^{\text{(threshold)}} = E_{ox}^{\text{(threshold)}} \cdot t_{ox} = \frac{t_{ox}}{\epsilon_{ox}} \sqrt{4\mathrm{q}\,\epsilon_s N_A \phi_s^{\text{(threshold)}}} = \frac{\sqrt{4\mathrm{q}\,\epsilon_s N_A \phi_s^{\text{(threshold)}}}}{\mathbb{C}_{ox}}}$$

Threshold Voltage



- Finally putting everything together:

$$V_{TH} = V_{FB} + V'_{ox} + V'_{B}$$

NOTE:
I got tired of dragging the superscript threshold on all the equations so I simply put a prime at its place

$$V_{TH} = V_{FB} + 2\phi'_s + \frac{\sqrt{2qN_A\epsilon_s(2\phi'_s)}}{\mathbb{C}_{ox}}$$

$= - Q_B'$

For an n-type material this is a positive value:

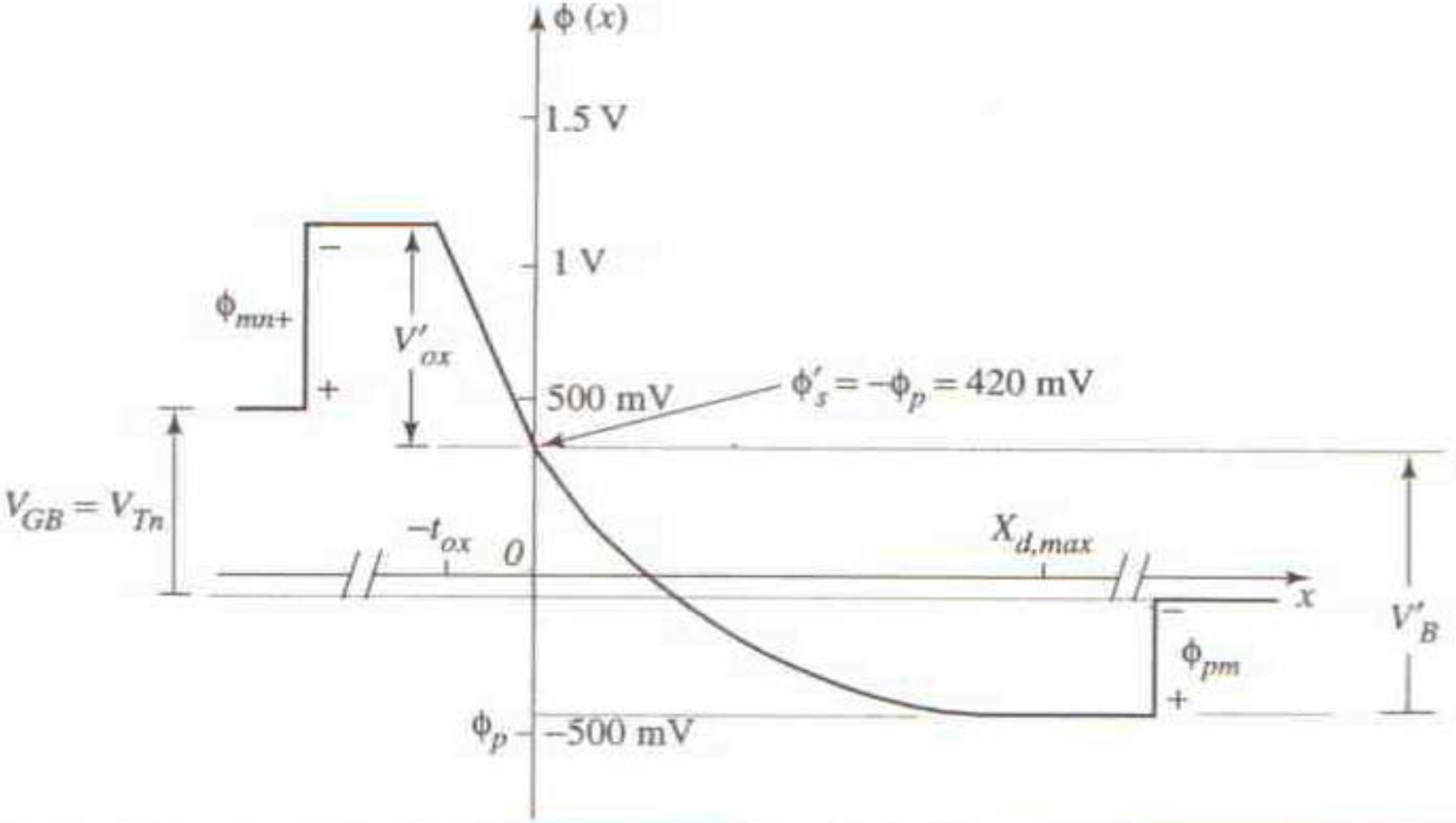
$$\phi_{\text{F-bulk}} = \frac{KT}{q} \cdot \ln\left(\frac{N_D}{n_i}\right)$$

For a p-type material this is a negative value

- Or equivalently:

$$V_{TH} = V_{FB} - 2\phi_{\text{F-bulk}} + \frac{\sqrt{2qN_A\epsilon_s(-2\phi_{\text{F-bulk}})}}{\mathbb{C}_{ox}} \quad \text{with:} \quad \phi_{\text{F-bulk}} = -\frac{KT}{q} \cdot \ln\left(\frac{N_A}{n_i}\right)$$

Threshold

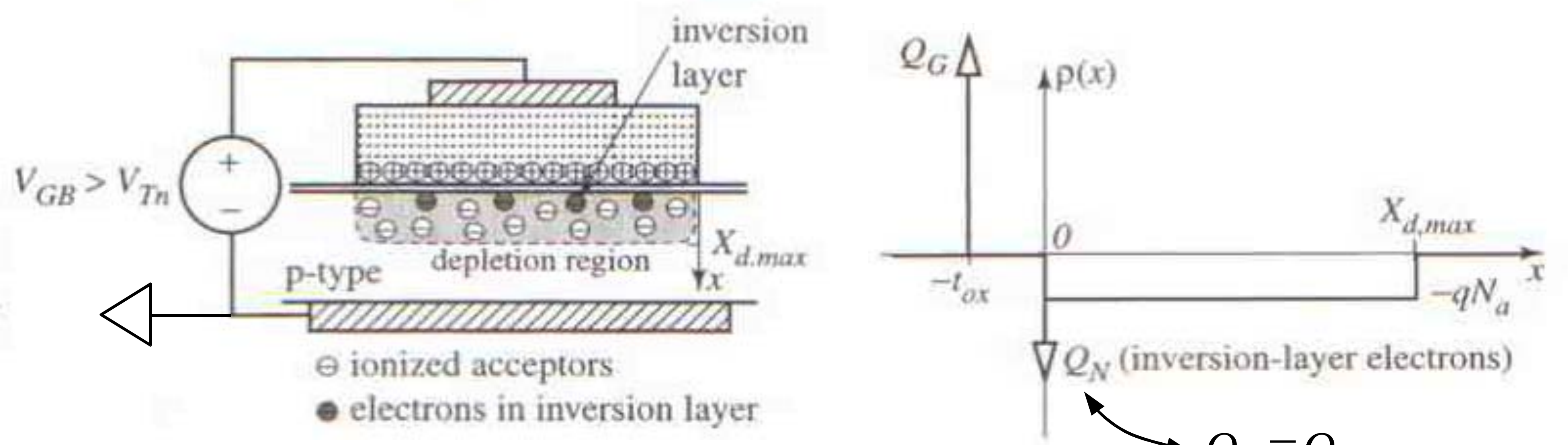


➤ **Figure 3.33** Potential for gate biased at the threshold voltage $V_{GB} = V_{Tn}$, the onset of inversion. The surface potential is equal and opposite to the bulk potential.

$\boxed{V_{GB} > V_{TH}}$

Inversion

- We apply a gate to bulk voltage bigger than the threshold voltage



⊖ ionized acceptors

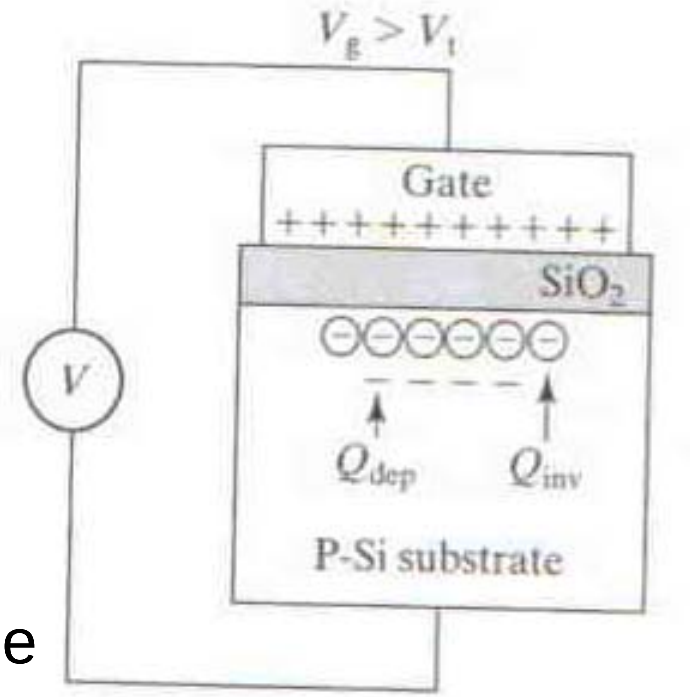
● electrons in inversion layer

$Q_N \equiv Q_{inv}$

$$Q_G = -\underbrace{(Q_{inv} + Q_{dep})}_{= Q_B}$$

electrons at the oxide-substrate surface constitute a sheet of charge

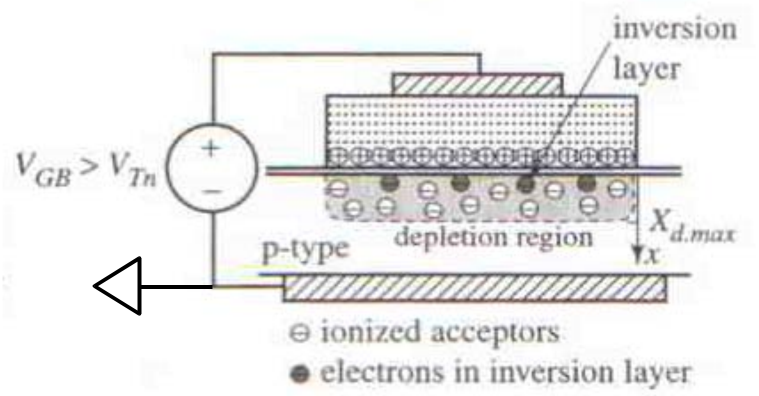
Inversion



- After inversion occurs, further increases in V_{GB} only slightly increase the surface potential Φ_s. Any further increase in the surface potential would induce a much larger increases in the surface electron density and we cannot expect the surface electron density to increase indefinitely
- For sake of simplification we assume that for $V_{GB} > V_{TH}$ the surface potential stop increasing and it remain pinned to Φ'_s
- If Φ_s does not increases neither will the depletion region width, that approximately will reach its maximum value:

$$X_{d,\max} = \sqrt{\frac{2\epsilon_s(2\phi_s')}{qN_A}}$$

Inversion



$$V_{GB}=V_{FB}+V_B+V_{ox}$$

- Since we assumed the surface potential remains pinned at Φ_s' and the bulk is at a fixed reference potential, the drop across the substrate is the same as in threshold condition: $V_B=2\phi_s'$
- So any increase of V_{GB} above V_{TH} is all picked up by V_{ox}

$$V_{ox}=\frac{Q_G}{C_{ox}}=\frac{-Q_B}{C_{ox}}=-\frac{Q_{\text{dep,max}}}{C_{ox}}-\frac{Q_{inv}}{C_{ox}}=\frac{\sqrt{2qN_A\epsilon_s(2\phi_s')}}{C_{ox}}-\frac{Q_{inv}}{C_{ox}}$$

$$\downarrow$$

$$\underbrace{V_{GB}=V_{FB}+2\phi_s'+\frac{\sqrt{2qN_A\epsilon_s(2\phi_s')}}{C_{ox}}}_{=V_{TH}}-\frac{Q_{inv}}{C_{ox}} \longrightarrow V_{GB}=V_{TH}-\frac{Q_{inv}}{C_{ox}}$$

$$\downarrow$$

Inversion

$$V_{GB} = V_{TH} - \frac{\mathbb{Q}_{inv}}{\mathbb{C}_{ox}}$$

↓

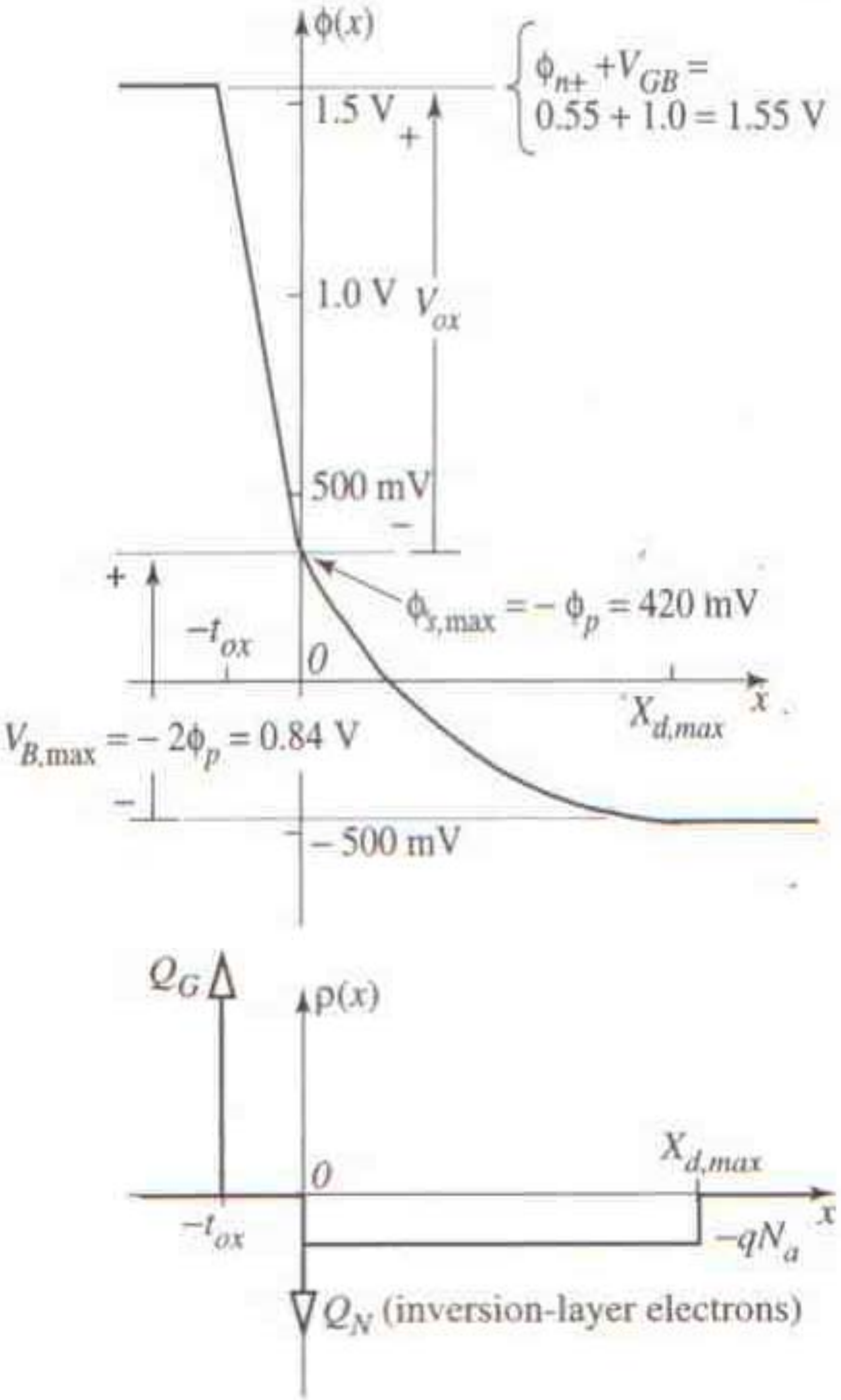
$$\boxed{\mathbb{Q}_{inv} = \mathbb{C}_{ox}(V_{GB} - V_{TH})}$$

Beside the voltage offset of V_{TH} it behaves just as a linear capacitor

↓

$$\boxed{\begin{aligned}\mathbb{Q}_G(V_{GB}) &= -\mathbb{Q}_{\mathrm{dep,max}} - \mathbb{Q}_{inv} = \\ &= -\mathbb{Q}_{\mathrm{dep,max}} - \mathbb{C}_{ox}(V_{GB} - V_{TH})\end{aligned}}$$

Example: V_{GB} = 1.0V



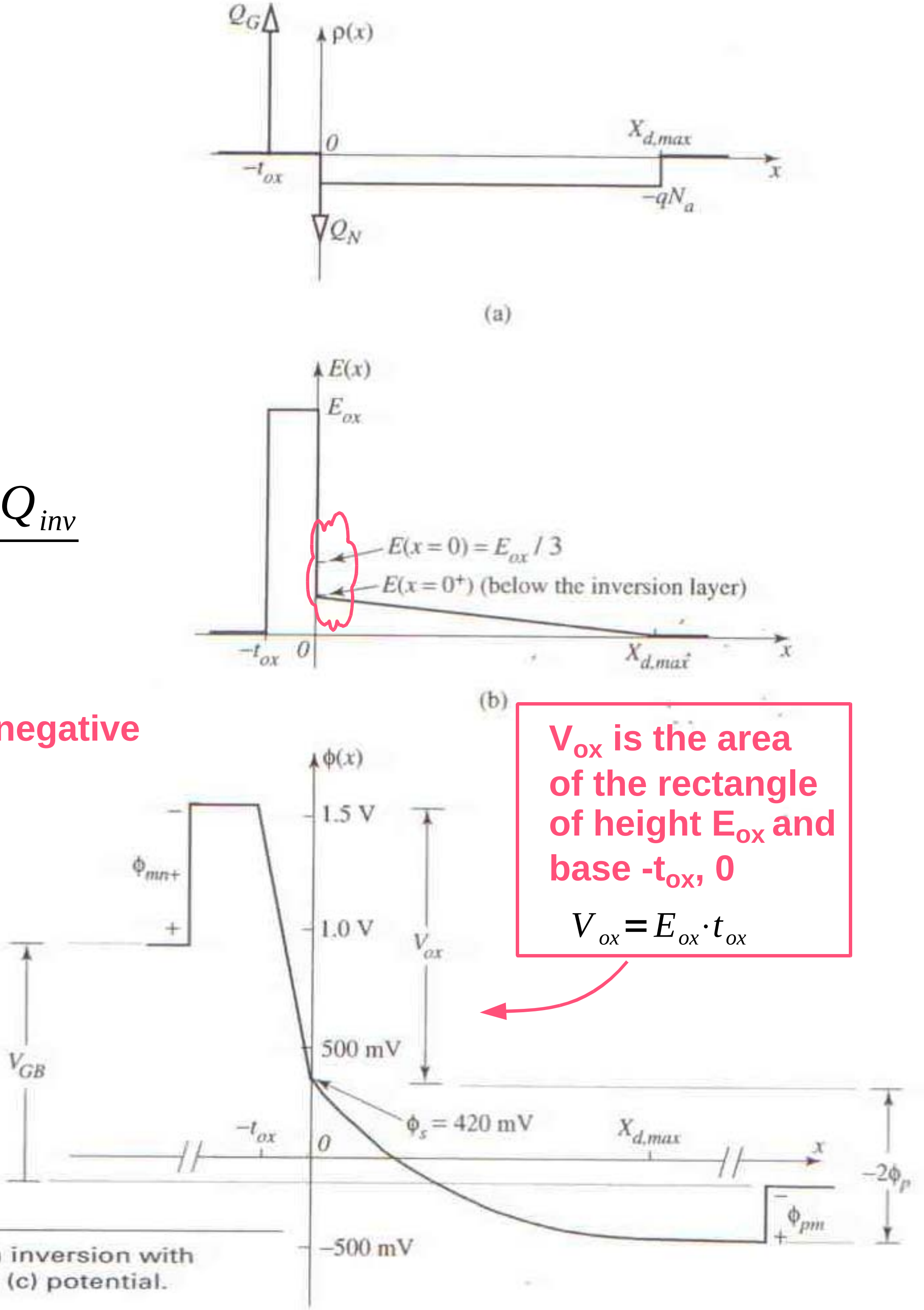
Inversion

$$E_{ox}=\frac{Q_G}{\epsilon_{ox}}=\frac{-Q_{\text{dep,max}}-Q_{inv}}{\epsilon_{ox}}$$

$$E(0)=E_{ox}\underbrace{\left(\frac{\epsilon_{ox}}{\epsilon_s}\right)}_{\frac{\epsilon_{ox}}{\epsilon_s}\approx\frac{1}{3}}=\frac{Q_G}{\epsilon_s}=\frac{-Q_{\text{dep,max}}-Q_{inv}}{\epsilon_s}$$

$$E(0+)=-\frac{Q_{\text{dep,max}}}{\epsilon_s}=E(0)+\frac{Q_{inv}}{\epsilon_s}$$

Q_{inv} is negative



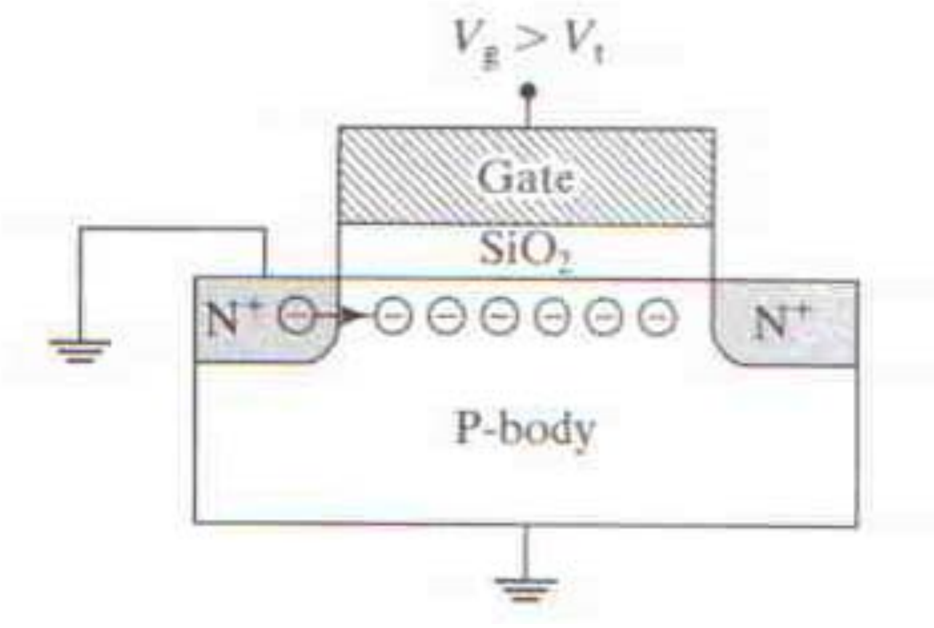
V_{ox} is the area of the rectangle of height E_{ox} and base $-t_{ox}$, 0

$$V_{ox}=E_{ox}\cdot t_{ox}$$

➤ **Figure 3.34** MOS capacitor on a p-type substrate biased in inversion with $V_{GB}=1$ V, $V_{Tn}=600$ mV: (a) charge density, (b) electric field, and (c) potential.

Inversion

- So far, we assumed that in inversion the electrons appears at the surface instantaneously.
- However, since there are few electrons in the p-type body, it can take minutes for thermal generation to generate the necessary electrons to form the inversion layer.
- The MOS transistor structure solve this problem. The inversion electrons are supplied by the n+ junctions.



Gate Charge of MOS capacitor

Accumulation:

$$Q_G = C_{ox}(V_{GB} - V_{FB}) \quad \text{for } V_{GB} \le V_{FB}$$

Depletion:

$$Q_G = -Q_B(V_{GB}) = \frac{q\varepsilon_s N_a}{C_{ox}}\left(\sqrt{1 + \frac{2C_{ox}^2(V_{GB} - V_{FB})}{q\varepsilon_s N_a}} - 1\right)(V_{FB} \le V_{GB} \le V_{Tn})$$

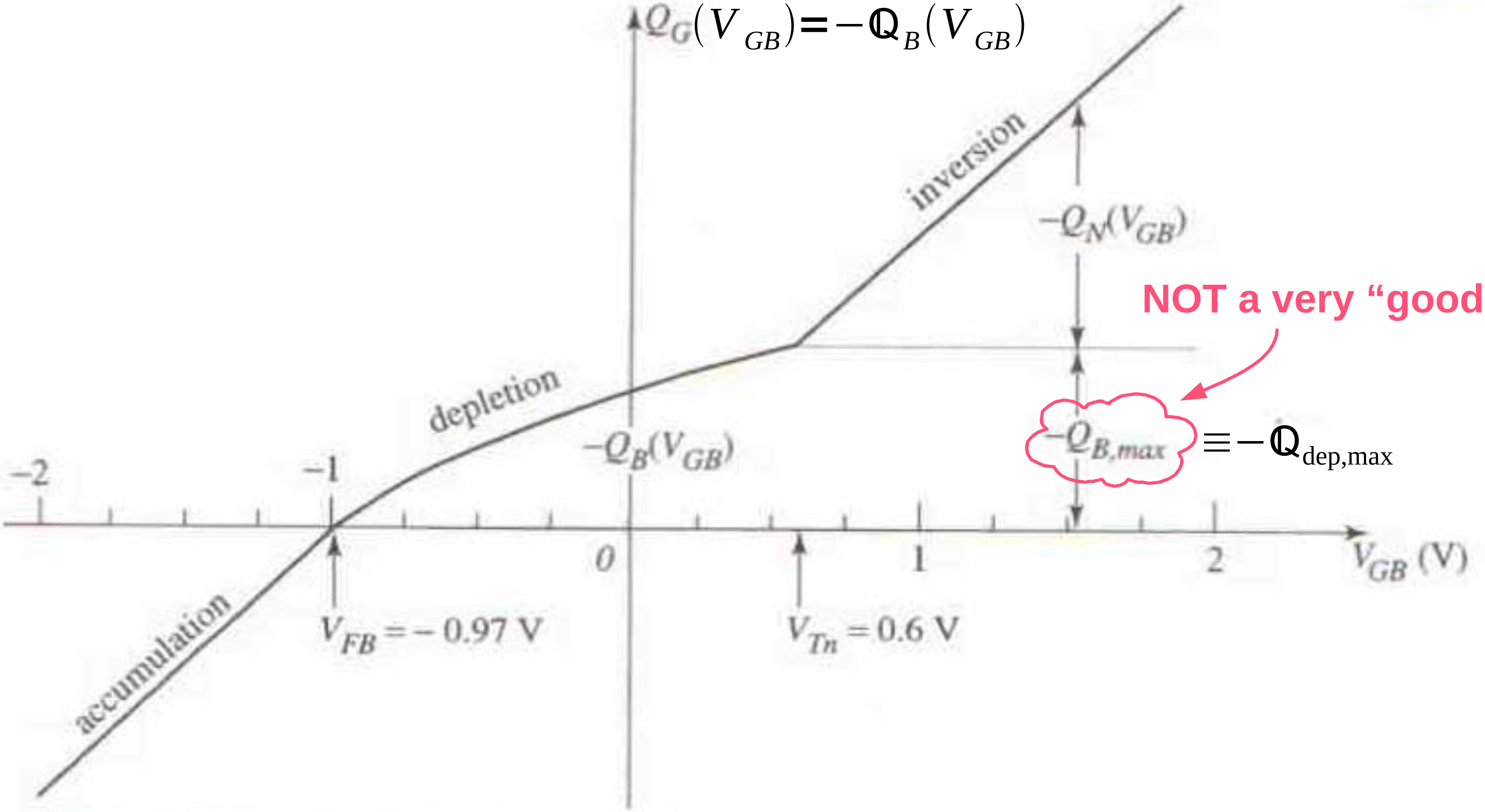
Inversion:

$$Q_G = C_{ox}(V_{GB} - V_{Tn}) + \frac{q\varepsilon_s N_a}{C_{ox}}\left(\sqrt{1 + \frac{2C_{ox}^2(V_{Tn} - V_{FB})}{q\varepsilon_s N_a}} - 1\right) \quad \text{for } V_{GB} \ge V_{Tn}$$

The depletion charge reach its maximum value for $V_{GB} = V_{TH}$

$$Q_{dep,max} = \left.\frac{q\varepsilon_s N_a}{C_{ox}}\left(\sqrt{1 + \frac{2C_{ox}^2(V_{GB} - V_{FB})}{q\varepsilon_s N_a}} - 1\right)\right|_{V_{GB} = V_{TH}}$$

Gate Charge of MOS capacitor



$Q_G(V_{GB}) = -Q_B(V_{GB})$

NOT a very "good" name

$\equiv -Q_{\text{dep,max}}$

➤ Figure 3.25 Gate charge as a function of gate-bulk voltage for an MOS capacitor with a 150 Å-thick gate oxide and a substrate doping $N_a = 10^{17}$ cm^{-3}.

Components of charge in the substrate

- Since in accumulation and inversion the substrate charge due to a change in the gate-to-bulk voltage is stored at the oxide/silicon surface (sheet of charge), the charge varies linearly
- In depletion the substrate charge is smeared over the substrate, so it doesn't vary linearly to changes in the gate-to-bulk voltage

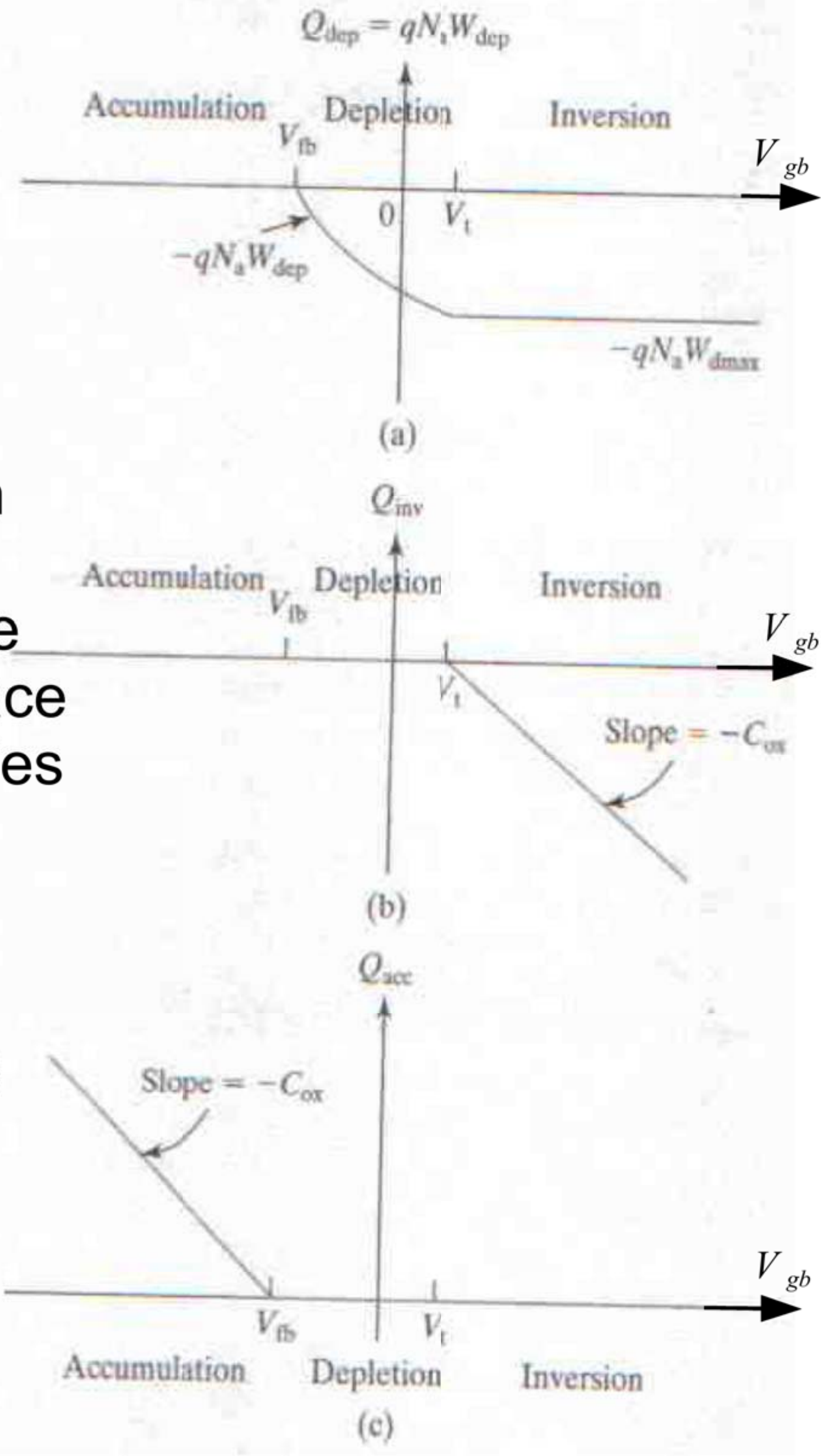


FIGURE 5–13 Components of charge (C/cm^2) in the MOS capacitor substrate: (a) depletion-layer charge; (b) inversion-layer charge; and (c) accumulation-layer charge.

Substrate Charge of MOS Cap.

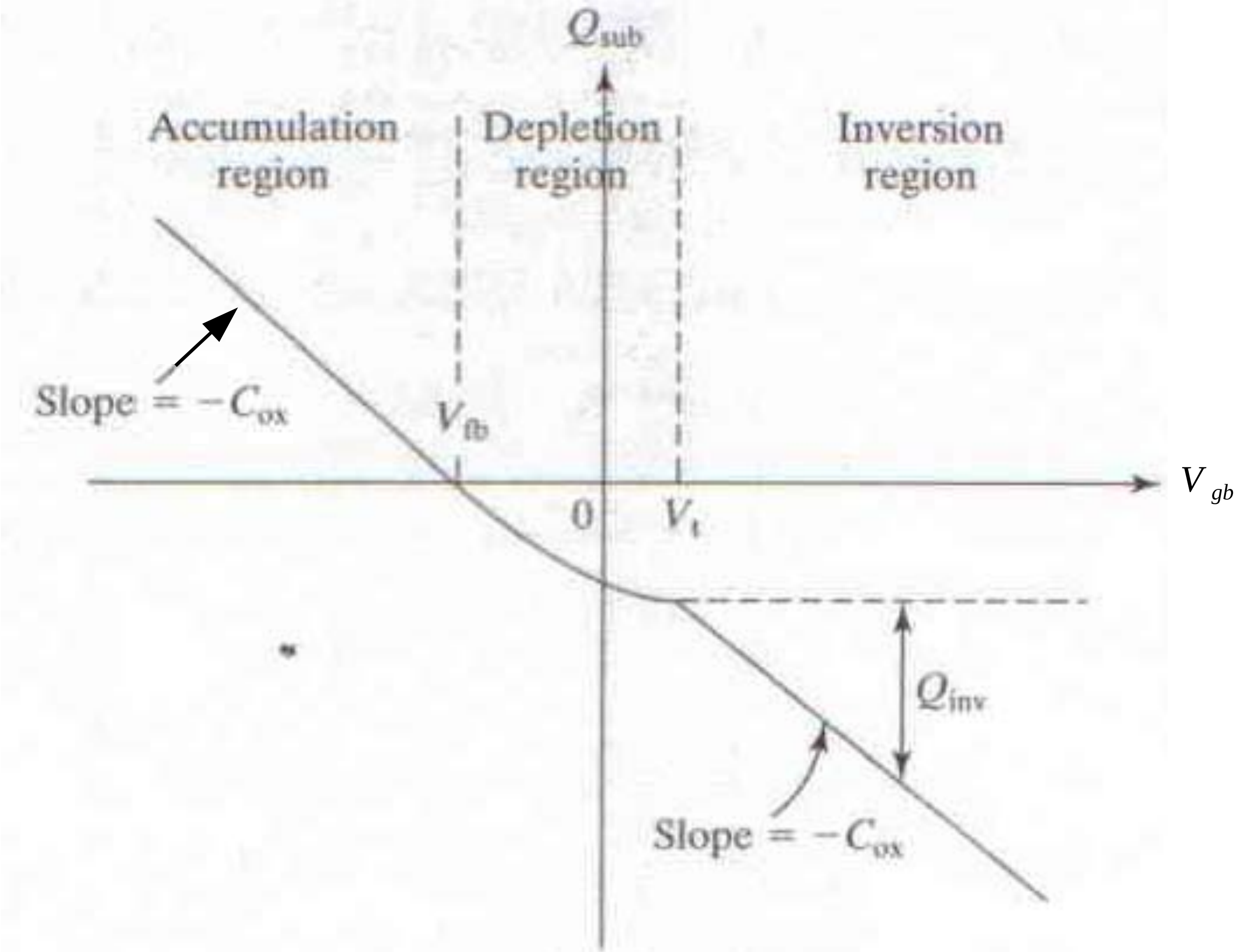


FIGURE 5–14 The total substrate charge, Q_{sub} (C/cm^2), is the sum of Q_{acc}, Q_{dep}, and Q_{inv}.