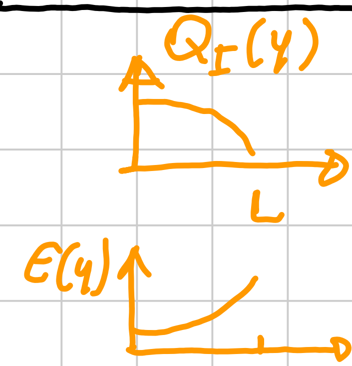
C_{gs} of MOS transistor in saturation



$Q_I(y)\,E(y) = \text{const}$

$$Q_I(y) = C_{ox}\left(V_{gs} - V(y) - V_t\right)$$

inversion layer charge per unit area

$$I_d = Q_I \cdot \frac{LW}{t} = Q_I \cdot vel \cdot W = Q_I \mu E W$$

lateral field

$$E = \frac{dV}{dy}$$

$$I_d = C_{ox}\left(V_{gs} - V(y) - V_t\right) W \mu E$$

$$C_{gs} = \frac{\partial Q_c}{\partial V_{gs}}$$

Q_c = total charge in the channel

$$I_d = Q_I \mu \frac{dV}{dy} W \rightarrow dy = \frac{Q_I \mu W \, dV}{I_d}$$

$$Q_c = W \cdot \int_0^{L'} Q_I(y)\,dy = W \int_{V(0)}^{V(L')} \frac{Q_I^2 \mu W \, dV}{I_d} =$$

no matter where L' is the voltage at pinch off is $V_{gs} - V_t$

$$= \frac{W^2 \mu}{I_d} \int_0^{V_{gs}-V_t} Q_I^2 \, dV =$$

$$= \frac{\mu W^2}{I_d} \int_0^{V_{gs}-V_t} C_{ox}^2 (V_{gs} - V - V_t)^2 dV =$$

$$= \frac{\mu C_{ox}^2 W^2}{I_d} \int_0^{V_{gs}-V_t} (V_{gs} - V_t - V)^2 dV =$$

$$= \frac{\mu C_{ox}^2 W^2}{I_d} \int_0^{V_{gs}-V_t} \left[(V_{gs}-V_t)^2 + V^2 - 2V(V_{gs}-V_t) \right] dV$$

$$= \frac{\mu C_{ox}^2 W^2}{I_d} \left[(V_{gs}-V_t)^2 V + \frac{V^3}{3} - 2\frac{V^2}{2}(V_{gs}-V_t) \right]_0^{V_{gs}-V_t}$$

$$\downarrow$$

$$Q_c = \frac{\mu C_{ox}^2 W^2}{I_d} \left[(V_{gs}-V_t)^3 + \frac{(V_{gs}-V_t)^3}{3} - (V_{gs}-V_t)^3 \right]$$

$$= \frac{\mu C_{ox}^2 W^2}{I_d} \cdot \frac{(V_{gs}-V_t)^3}{3}$$

At saturation edge:

$$I_d = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{gs} - V_t)^2$$

$$Q_c = \frac{2}{3} C_{ox} WL (V_{gs} - V_t)$$

$$C_{gs} = \frac{\partial Q_c}{\partial V_{gs}} = \frac{2}{3} C_{ox} WL$$

if we want to be picky we can use the expression of I_d with $1+\lambda V_{ds}$

$$C_{gs} = \frac{2}{3} \frac{C_{ox} WL}{(1+\lambda V_{ds})}$$

$$= \frac{2}{3} C_{ox} W L'$$