MOS Transistor Theory

Slides adapted from:

N. Weste, D. Harris, *CMOS VLSI Design*, © Addison-Wesley, 3/e, 2004

Outline

- The Big Picture
- MOS Structure
- Ideal I-V Characteristics
- MOS Capacitance Models
- Non ideal I-V Effects
- Pass transistor circuits
- Tristate Inverter
- Switch level RC Delay Models

The Big Picture

- So far, we have treated transistors as ideal switches
- An ON transistor passes a finite amount of current
 - Depends on terminal voltages
 - Derive current-voltage (I-V) relationships
- Transistor gate, source, drain all have capacitance
 - $I = C (\Delta V / \Delta t) \rightarrow \Delta t = (C / I) \Delta V$
 - Capacitance and current determine speed

MOS Transistor Symbol



Metal-to-metal Capacitor



► Figure Ex3.1B Close-up of metal-metal capacitor with applied voltage, showing surface charges on top and bottom metal plates and electric field lines.



MOS Structure

- Gate and body for MOS capacitor
- Operating modes
 - Accumulation ⁰
 - Depletion
 - Inversion



MOS Structure (a more in depth look)



MOS capacitor in accumulation





MOS capacitor in flatband

• $V_{GB} = V_{FB}$



$$V_{FB} \equiv -\phi_B$$





- Xd does not increase much beyond threshold
- All extra voltage beyond $V_{\rm T}$ used to increase inversion charge $Q_{\rm N}$

MOS Capacitor in inversion

• Existence of Q_N and control over Q_N by V_{GB} is the key to MOS electronics







Physical interpretation of MOS capacitance

- Accumulation: parallel plate capacitor --> C = Cox
- Depletion: increment in gate charge is mirrored at bottom of depletion region, so capacitance model is Cox in series with the depletion region capacitance Cb

gate
Si/SiO₂ surface
bulk
$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$
Note that X_d is
a function of V_{GB}

$$C = C_{ox} || C_b$$

 Inversion: bulk charge is no longer changing with V_{GB} --> an increment in gate charge is mirrored in the inversion layer under the gate.

The capacitance is therefore the same as in accumulation $-> C = C_{ox}$

Width of the depletion region

• VFB \leq VGB \leq VT

$$X_d(V_{GB}) = \sqrt{\frac{2\varepsilon_s(\phi_B - V_{GB})}{q}} \left(\frac{1}{N_a} + \frac{1}{N_d}\right) = X_{do}\sqrt{1 - \frac{V_{GB}}{\phi_B}}$$

$$X_{do} = \sqrt{\frac{2\varepsilon_{S}\phi_{B}}{q} \left(\frac{1}{N_{a}} + \frac{1}{N_{d}}\right)}$$
equilibrium

nMOS Transistor Terminal Voltages

• Mode of operation depends on V_g, V_d, V_s

$$- V_{gs} = V_g - V_s$$

$$- V_{gd} = V_g - V_d$$

$$- V_{ds} = V_d - V_s = V_{gs} - V_{gd}$$



- Source and drain are symmetric diffusion terminals
 - By convention, source is terminal at lower voltage
 - Hence $V_{ds} \ge 0$
- nMOS body is grounded. First assume source is 0 too.
- Three regions of operation
 - Cutoff
 - Linear
 - Saturation

nMOS in cutoff operation mode (1/2)

• No channel

(a)

Simplifying assumption VB=VS

• I_{ds} = 0



nMOS in cut-off mode (2/2)

- $V_{GS} < V_T$
- Q_N= inversion charge =0
- V_{DS} drops across the depletion region



nMOS in linear operation mode (1/3)

- Channel forms
- Current flows from D to S
 e⁻ from S to D
- I_{ds} increases with $V_{ds (b)}$
- Similar to linear resistor

 $\begin{array}{c} \textcircledleft \\ \textcircledleft \\ & \bigvee_{ds} < & \bigvee_{gs} - & \bigvee_{t} \\ & \bigvee_{ds} - & \bigvee_{gs} < - & \bigvee_{t} \\ & \bigvee_{d-V_{s}} - & \bigvee_{g+V_{s}} < - & \bigvee_{t} \\ & \bigvee_{dg} < - & \bigvee_{t} \\ & \leftarrow & \rightarrow \\ & \bigvee_{gd} < - & \bigvee_{t} \\ \hline \\ & \bigvee_{ds>0} \\ \Leftrightarrow & \bigvee_{gs} - & \bigvee_{gd>0} \\ \hline \\ & \bigvee_{gs>0} \\ \end{matrix}$



nMOS in linear (=triode) mode (2/3)

- $V_{GS} > V_T \text{ and } V_{GD} > V_T$ $(V_{GD} > V_T \Leftrightarrow V_{DS} < V_{GS} V_T)$
- V_{GS} \Rightarrow Q_N \Rightarrow I_{DS}
- $V_{DS} \uparrow \Longrightarrow E_{lateral} \uparrow \Longrightarrow I_{DS} \uparrow$



nMOS in linear (=triode) mode (3/3)

- V_{GS}[†] (more electrons in the inversion layer)
- V_{DS}↑ (higher lateral electric field ⇔ electrons in the channel move faster)



nMOS in Saturation operation mode (1/6)

- Channel pinches off
- I_{ds} independent of V_{ds}
- We say current saturates
- Similar to current source



nMOS in saturation mode (2/6)

•
$$V_{GS} > V_T$$
 and $V_{GD} < V_T$ $(V_{GD} < V_T \Leftrightarrow V_{DS} > V_{GS} - V_T)$



nMOS in saturation mode (3/6)

- When $V_{DS} > V_{GS} V_T (V_{GD} < V_T) \implies Q_N(y=L)=0$
- Initial thought: lack of a channel at the drain means that I_D must drop to zero. WRONG ... !!
- Drain terminal loses control over channel
 Drain current saturates and remain approximately constant at the value given by V_{DS, SAT} (=V_{GS}-V_T)

I_D becomes independent of V_{DS}: I_D=I_{D,SAT}

nMOS in saturation mode (4/6)



nMOS in saturation mode (5/6)

- Current flux is the same across any section y along the channel. Thus as y increases down the channel
 V(y) ↑ ⇒ Q_N(y) ↓ ⇒ E(y) ↑ (fewer carriers moving faster)
- <u>General</u> expression of channel current (*current= charge/time*) :

$$I_{y}(y) = I_{D}(y) = Q_{N}(y) \cdot W \cdot v_{y}(y)$$

$$v_{y}(y) = \mu \cdot E_{y}(y)$$

$$charge per area carriers velocity carriers mobility$$

$$v = \mu \cdot E$$

nMOS in saturation mode (6/6)

- Increase in $E_y (E_y^{\uparrow} \Leftrightarrow V_{DS}^{\uparrow})$ is compensated by decrease in Q_N
- When Q_N=0 at drain then
 I_D saturates





After channel charge goes to 0, there is a high lateral field that 'sweeps' the carriers to the drain*, and drops the extra voltage (this is a depletion region of the drain junction)

 * It is important to remember what a reverse biased PN junction does to minority carriers.
 Electrons (in the p-type material) get swept back into the n-region



FIG 2.4 pMOS transistor

I-V Characteristics (nMOS)

- In Linear region, I_{ds} depends on
 - How much charge is in the channel?
 - How fast is the charge moving?



Channel Charge

- MOS structure looks like parallel plate capacitor while operating in inversion:
 - Gate oxide channel
- $Q_{channel} = C^*V$

•
$$C = C_g = \varepsilon_{ox}WL/t_{ox} = c_{ox}WL$$

•
$$V = V_{gc} - V_t = (V_{gs} - V_{ds}/2) - V_t$$

$$c_{ox} = \epsilon_{ox} / t_{ox}$$

Gate Source + V_{gs} C_{g} V_{gd} Drain V_{gs} C_{g} V_{gd} U_{gd} V_{gd} V_{g

$$V_{gc} = (V_{gs} + V_{gd})/2 = V_{gs} - V_{ds}/2$$

FIG 2.5 Average gate to channel voltage

 $V_{gc} = \frac{V_{gs} + V_{gol}}{2} = \frac{V_{gs} + V_{gs} - V_{ds}}{2} = V_{gs} - \frac{V_{ds}}{2}$

Carrier velocity

- Charge is carried by e-
- Carrier velocity $\boldsymbol{\mathcal{V}}$ proportional to lateral E-field between source and drain
- $v = \mu E$ (μ called mobility)
- $E = V_{ds}/L$
- Time for carrier to cross channel:

 $-t = L / \mathcal{V}$

nMOS Linear (a.k.a. triode) I-V

Now we know

How much charge Q_{channel} is in the channel
 How much time t each carrier takes to cross



nMOS Triode I-V characteristic $I_{D} = Q_{N}(y) \cdot W \cdot v_{y}(y)$ $V_{y}(y) = \mu \cdot E_{y}(y)$ $E_{y}(y) = \frac{dV(y)}{dy}$ $Q_{N}(y) = C_{ox}[V_{GS} - V(y) - V_{T}]$

$$I_{D}dy = \mu \cdot Cox \cdot W \cdot [V_{GS} - V(y) - V_{T}] \cdot dV$$

$$I_{D} \int_{0}^{L} dy = \mu \cdot Cox \cdot W \cdot \int_{0}^{VDS} [V_{GS} - V(y) - V_{T}] \cdot dV$$

$$I_{D} = \mu \cdot Cox \cdot \frac{W}{L} \cdot \left(V_{GS} - V_{T} - \frac{V_{DS}}{2}\right) \cdot V_{DS}$$
with $\beta = \mu C_{ox} \frac{W}{L}$
₃₃

nMOS Saturation I-V

- If $V_{gd} < V_t$, channel pinches off near drain - When $V_{ds} > V_{dsat} = V_{gs} - V_t$
- Now drain voltage no longer increases current

$$I_{ds} = \beta \left(V_{gs} - V_t - \frac{V_{dsat}}{2} \right) V_{dsat}$$
$$= \frac{\beta}{2} \left(V_{gs} - V_t \right)^2$$

nMOS I-V Summary

first order (Shockley model) transistor models (ideal models)

$$I_{ds} = \begin{cases} 0 & V_{gs} < V_t & \text{cutoff} \\ \beta \left(V_{gs} - V_t - \frac{V_{ds}}{2} \right) V_{ds} & V_{ds} < V_{dsat} & \text{linear} \\ \frac{\beta}{2} \left(V_{gs} - V_t \right)^2 & V_{ds} > V_{t} \end{pmatrix} & \frac{\beta}{2} \left(V_{gs} - V_t \right)^2 & V_{ds} > V_{dsat} & \text{saturation} \\ \begin{pmatrix} and & V_{gs} > V_t \end{pmatrix} & V_{dsat} & \text{saturation} \end{pmatrix}$$

I-V characteristics of nMOS Transistor



FIG 2.7 I-V characteristics of ideal nMOS transistor

Example

- 0.6 μm process from AMI Semiconductor
- $-t_{ox} = 100 \text{ Å}$ 2.5 $V_{gs} = 5$ $-\mu = 350 \text{ cm}^2/(\text{V*s})$ 2 $-V_{t} = 0.7 V$ 1.5 $V_{gs} = 4$ l_{ds} (mA) 1 • Plot I_{ds} vs. V_{ds} $V_{gs} = 3$ 0.5 $V_{gs} = 2$ $-V_{qs} = 0, 1, 2, 3, 4, 5$ ۷^۵ = 1 0 2 3 4 0 1 - Use W/L = 4/2 λ V_{ds}

$$\beta = \mu C_{ox} \frac{W}{L} = (350) \left(\frac{3.9 \cdot 8.85 \cdot 10^{-14}}{100 \cdot 10^{-8}} \right) \left(\frac{W}{L} \right) = 120 \frac{W}{L} \mu A / V^2$$

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pMOS I-V Characteritics

- All dopings and voltages are inverted for pMOS
- Mobility μ_p is determined by holes
 - Typically 2-3x lower than that of electrons μ_n
 - 120 cm²/V*s in AMI 0.6 mm process
- Thus pMOS must be wider to provide same current

– In this class, assume $\mu_n / \mu_p = 2$

pMOS I-V Summary

first order (ideal) transistor models (a.k.a. Shockley model)

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$$I_{ds} = \begin{cases} 0 & V_{gs} > V_t & \text{cutoff} \\ \beta \left(V_{gs} - V_t - \frac{V_{ds}}{2} \right) V_{ds} & V_{ds} > V_{dsat} & \text{linear} \\ \frac{\beta}{2} \left(V_{gs} - V_t \right)^2 & V_{ds} < V_{dsat} & \text{saturation} \\ \frac{\alpha_{ds}}{\alpha_{ds}} < V_{t} \end{pmatrix} \end{cases}$$

I-V characteristics of pMOS Transistor



FIG 2.8 I-V characteristics of ideal pMOS transistor

MOS Transistor's Capacitances

- Any two conductors separated by an insulator have capacitance
- Gate to channel capacitor is very important
 - Creates channel charge necessary for operation (intrinsic capacitance)
- Source and drain have capacitance to body (parasitic capacitance)
 - Across reverse-biased diodes
 - Called diffusion capacitance because it is associated with source/drain diffusion

MOS transistor's Capacitances



Cgc modeled through Cgs and Cgd Ccb modedel through Cgb



Csb=AS × Cj-sb + PS × Cjsw-sb Cdb=AD × Cj-db + PD × Cjsw-db Side wall junction cap. per area



• When the transistor is off, the channel is not inverted

 $C_g = C_{gb} = C_{ox}WL || \epsilon_{si}WL/X_d < \epsilon_{ox}WL/t_{ox} = C_{ox}WL$

- Let's call $C_{ox}WL = C_0$
- When the transistor is on, the channel extends from the source to the drain (if the transistor is unsaturated, or to the pinch off point otherwise) $C_g = C_{gb} + C_{gs} + C_{gd}$

Gate Capacitance

Table 2.1	Approximation of intrinsic MOS gate capacitance			
Parameter		Cutoff	Linear	Saturation
C_{gb}		≈ C ₀	0	0
C_{gs}		0	$C_0/2$	2/3 C ₀
C_{gd}		0	$C_0/2$	0
$C_g = C_{gs} + C_g$	$_{gd}$ + C_{gb}	≈ C ₀	C_0	2/3 C ₀



In reality the gate overlaps source and drain. Thus, the gate capacitance should include not only the intrinsic capacitance but also parasitic overlap capacitances:

$$C_{gs}(overlap) = C_{ox} W L_{ov}$$

 $C_{gs}(overlap) = C_{ox} W L_{ov}$

Detailed Gate Capacitance

Capacitance	Cutoff	Linear	Saturation
C _{gb} (total)	≈C ₀	0	0
C _{gd} (total)	C _{ox} WL _{ov}	$C_0/2 + C_{ox}WL_{ov}$	C _{ox} WL _{ov}
C _{gs} (total)	C _{ox} WL _{ov}	$C_0/2 + C_{ox}WL_{ov}$	$2/3 C_0 + C_{ox}WL_{ov}$







Figure 3.31 Schematic representation of MOSFET oxide capacitances during (a) cut-off, (b) linear, and (c) saturation modes.

Source: M-S Kang, Y. Leblebici, *CMOS Digital ICs*, 3/e, 2003, McGraw-Hill

Overlap Capacitance

- It has two components
 - Direct overlap $\approx C_{ox}WL_{ov}$
 - Additional component due to fringing fields

$$-Cov = Cov' \cdot W$$



$$C_{ov-GS} = CGSO \cdot W$$
$$C_{ov-GD} = CGDO \cdot W$$

Junction Capacitance

- C_{sb}, C_{db}
- Undesired capacitance (parasitic)
- Due to the reverse biased p-n junctions between source and body and drain and body
- Capacitance depends on area and perimeter
 - Use small diffusion nodes
 - Comparable to C_g for contacted diffusion
 - ½ C_g for uncontacted
 - Varies with process



FIG 2.9 Diffusion region geometries

(a) Isolated contacted diffusion

- (b) Shared contacted diffusion
- (c) Merged uncontacted diffusion

Junction Capacitance



FIG 2.9 Diffusion region geometries

	AD1/AD2	PS1/PD2	AD1/AS2	PD1/PS2
(a) Isolated contacted diffusion	Wx5	2xW+10	Wx5	2xW+10
(b) Shared contacted diffusion	Wx5	2xW+10	Wx3	W+6
(c) Merged uncontacted diffusion	Wx5	2xW+10	Wx1.5	W+3

Junction Capacitance

	L ↔	,	
S	G	D	Ĵ,
L _{diff} ←→		L _{diff} ↔	

$$\begin{split} AS &= W \cdot L_{diff} \\ PS &= W + 2L_{diff} \\ W \\ PD &= W \cdot L_{diff} \\ PD &= W + 2L_{diff} \end{split}$$

 $Cjdb = \frac{AD \cdot CJ}{\left(1 + \frac{VDB}{PB}\right)^{MJ}} + \frac{PD \cdot CJSW}{\left(1 + \frac{VDB}{PBSW}\right)^{MJSW}}$ $Cjsb = \frac{AS \cdot CJ}{\left(1 + \frac{VSB}{PB}\right)^{MJ}} + \frac{PS \cdot CJSW}{\left(1 + \frac{VSB}{PBSW}\right)^{MJSW}}$

For long channel transistors the side of the perimeter abutted to the gate is shielded by the electrons in the channel



Lumped representation of the MOSFET capacitances





MOS transistor's caps Summary

extrinsic cap. intrinsic cap.				
	Subthreshold	Triode	Saturation	
C _{gs}	C _{ov}	¹ / ₂ WLC _{ox} +C _{ov}	⅔WLC _{ox} +C _{ov}	
C _{gd}	C _{ov}	¹ ∕₂WLC _{ox} +C _{ov}	C _{ov}	
C _{gb}	$\left(\frac{1}{C_{CB}} + \frac{1}{WLC_{ox}}\right)^{-1}$	0	0	
C_{sb}	C _{jsb}	C _{jsb} + ¹ / ₂ C _{CB}	C _{jsb} + ² / ₃ C _{CB}	
C _{db}	C _{jdb}	C _{jdb} + ¹ / ₂ C _{CB}	C _{jdb}	

$$C_{CB} = \frac{\varepsilon_{si}}{X_d} \cdot WL$$

Xd is the width of the depletion region at the silicon interface



Non-ideal I-V effects

- The saturation current increases less than quadratically with increasing V_{gs}
 - Velocity saturation
 - Mobility degradation
- Channel length modulation
- Body Effect
- Leakage currents
 - Sub-threshold conduction
 - Junction leakage
 - Tunneling
- Temperature dependence and supply dependence (environmental dependence)
- Geometry Dependence (process dependence)

Pass Transistors

- nMOS pass transistors pull no higher than V_{DD}-V_{tn}
 - Called a degraded "1"
 - Approach degraded value slowly (low I_{ds})
- pMOS pass transistors pull no lower than $|V_{to}|$
 - Called a degraded "0"
 - Approach degraded value slowly (low I_{ds})

Pass transistor Circuits

(a)
$$V_{DD}$$

(a) $V_{DD} \downarrow V_s = V_{DD} - V_{tn}$







FIG 2.31 Pass transistor threshold drops

Transmission gate ON resistance



Tri-state Inverter







Effective resistance of a transistor

- First-order transistor models have limited value
 - Not accurate enough for modern transistors
 - Too complicated for hand analysis
- Simplification: treat transistor as resistor
 - Replace $I_{ds}(V_{ds}, V_{gs})$ with effective resistance R
 - $I_{ds} = V_{ds}/R$
 - R averaged across switching range of digital gate
- Too inaccurate to predict current at any given time
 - But good enough to predict RC delay (propagation delay of a logic gate)

RC Values

- Capacitance
 - C = C_g = C_s = C_d = 2 fF/ μ m of gate width
 - Values similar across many processes
- Resistance
 - − R ≈ 6 KΩ^{*}µm in 0.6um process
 - Improves with shorter channel lengths
- Unit transistors
 - May refer to minimum contacted device (4/2 λ)
 - or maybe 1 μm wide device
 - Doesn't matter as long as you are consistent

RC Delay Models

- Use equivalent circuits for MOS transistors
 - ideal switch + capacitance and ON resistance
 - unit nMOS has resistance R, capacitance C
 - unit pMOS has resistance 2R, capacitance C
- Capacitance proportional to width
- Resistance inversely proportional to width

Switch level RC models





FIG 2.34 Equivalent RC circuit models

Inverter Delay Estimate

• Estimate the delay of a fanout-of-1 inverter



FIG 2.35 Inverter propagation delay

delay = 6RC

Resistance of a unit transmission gate

- The effective resistance of a transmission gate is the parallel of the resistance of the two transistor
- Approximately R in both directions

direction

double

- Transmission gates are commonly built using equalsized transistors
- Boosting the size of the pMOS only slightly improve the effective resistance while significantly increasing the capacitance



FIG 2.36 Effective resistance of a unit transmission gate

Summary

- Models are only approximations to reality, not reality itself
- Models cannot be perfectly accurate
 - Little value in using excessively complicated models, particularly for hand calculations
- To first order current is proportional to W/L
 - But, in modern transistors L_{eff} is shorter than L_{drawn}
 - Doubling the L_{drawn} reduces current more than a factor of two
 - Two series transistors in a modern process deliver more than half the current of a single transistor
- Use Transmission gates in place of pass transistors
- Transistor speed depends on the ratio of current to capacitance
 - Sources of capacitance (voltage dependents)
 - Gate capacitance
 - Diffusion capacitance