

# Chapter 5

## $g_m/I_D$ - Based Design

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Partly adapted from Stanford's analog circuit  
design sequence

Reading:  
See 'References' at the end of this chapter for optional reading.

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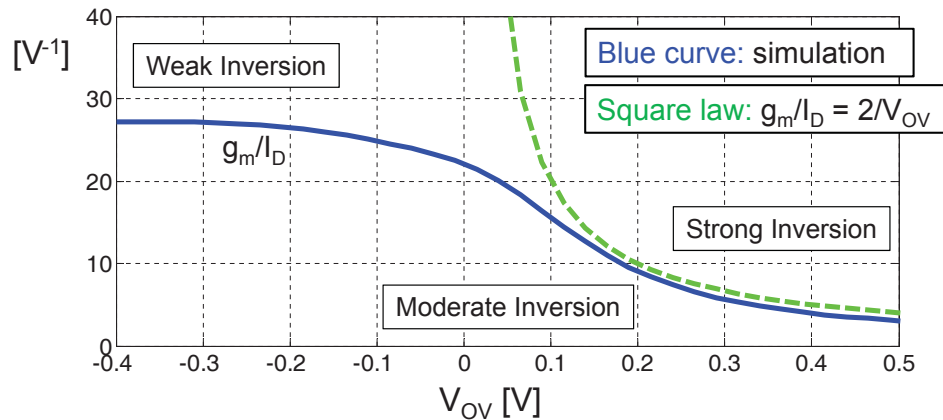
### Outline

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- Review the three main figures of merit for transistors:  $g_m/I_D$ ,  $\omega_T$ ,  $g_m/g_{ds}$ 
  - Transistor characteristics that are directly linked to circuit performance
- See how the square law fails in practice
- Discuss  $g_m/I_D$ -based design using lookup tables/charts
  - Quantitative design methodology based on characterizing transistors
  - Intuitive framework for exploring and optimizing circuit performance
    - Transistor figures of merit become useful design variables
- Design example using  $g_m/I_D$  methodology
- Learning objectives
  - Understand tradeoffs between speed, power efficiency, and gain
    - In transistors, and in circuits
  - Be able to use technology characterization tables/charts to size transistors
    - Get practice in the design project
  - Develop a systematic design methodology in the absence of simple analytical models like the square law equation

## FOM #1: $g_m/I_D$ as a Figure of Merit

- “Transconductance efficiency” figure of merit for transistors
  - Circuit performance often dictates the required  $g_m$ 
    - e.g. gain of a common source stage:  $|A_v| = g_m R_L$
  - $g_m/I_D$  quantifies how much  $g_m$  you get for the amount of bias current you invest
    - i.e.  $g_m/I_D = 10\text{S/A} \rightarrow 10\mu\text{S per } 1\mu\text{A}$



## Why Doesn't the Square Law “Work”?

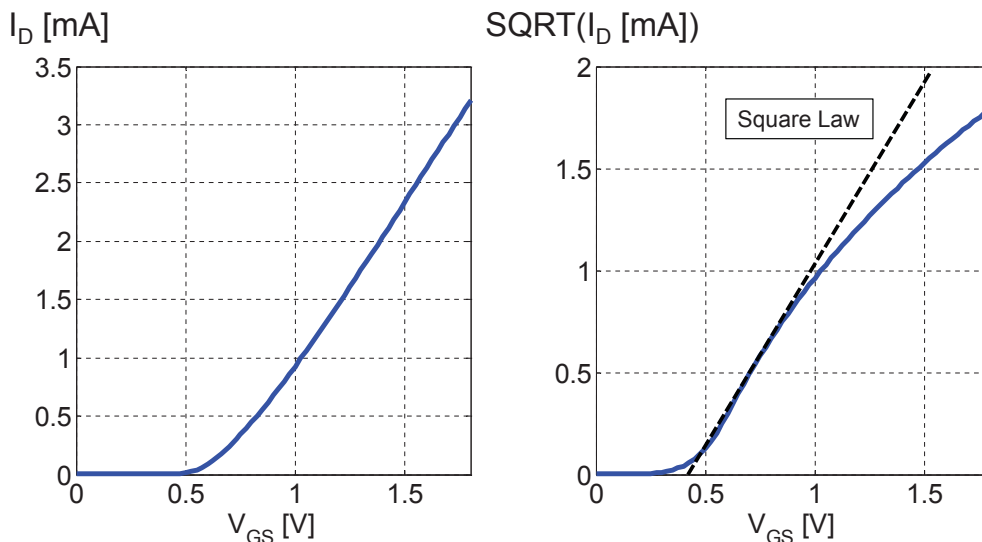
- MOSFETs are complicated!
- The IV-behavior in saturation can be roughly categorized according to the channel's inversion level: weak, moderate and strong inversion
- The current is due to diffusion in weak inversion and mostly due to drift in strong inversion; the transition is smooth and complicated
- The classic square law model is based on an ideal drift model, and applies only near the onset of strong inversion
  - And even then, the predictions are inaccurate unless “short channel effects” are taken into account
- The bottom line is that there is no modeling expression that is simple enough for hand analysis and sufficiently accurate to match real world device behavior

## What are $\mu C_{ox}$ (“KP”) and $\lambda$ (“LAMBDA”) for our Technology?

```
.MODEL nmos6720 nmos
+acm = 3          hdlf = 0.32e-6      LEVEL = 49
+VERSION = 3.1    TNOM = 27          TOX = 4.1E-9
+XJ = 1E-7        NCR = 2.3549E17    VTHO = 0.3618397
+K1 = 0.5916053  K2 = 3.225139E-3                 K3 = 1E-3
+K3B = 2.3938862  W0 = 1E-7            NLX = 1.776268E-7
+DVTOW = 0        DVT1W = 0          DVT2W = 0
+DVT0 = 1.3127368 DVT1 = 0.3876801    DVT2 = 0.0238708
+U0 = 256.74093  UA = -1.585658E-9  UB = 2.528203E-18
+UC = 5.182125E-11 VSAT = 1.003268E5 AO = 1.981392
+AGS = 0.4347252  BO = 4.989266E-7    B1 = 5E-6
+KETA = -9.888408E-3 A1 = 6.164533E-4    A2 = 0.9388917
+RDSW = 128.705483 PRWG = 0.5        PRWB = -0.2
+WR = 1          WINT = 0           LINT = 1.617316E-8
+XL = 0          XN = -1E-8         DWG = -5.383413E-9
+DWB = 9.111767E-9 VOFF = -0.0854824    NFACTOR = 2.2420572
+CIF = 0         CDSC = 2.4E-4       CDSCD = 0
+CDSUB = 0       ETAB = 2.981159E-3  ETAB = 9.289544E-6
+DSSUB = 0.0159753 PCIM = 0.7245546  PDIRLCL1 = 0.1568183
+PDIBLC2 = 2.543351E-3 PDIBLCB = -0.1    DROUT = 0.7445011
+PSCBE1 = 8E10   PSCBE2 = 1.876443E-9  PVAG = 7.200284E-3
+DELTA = 0.01    RSH = 6.6          MOBMOD = 1
+PRT = 0         UTE = -1.5         KT1 = -0.11
+KT1L = 0        KT2 = 0.022        UA1 = 4.31E-9
+UB1 = -7.61E-18 UCI = -5.6E-11         AT = 3.3E4
+WL = 0          WLN = 1           WW = 0
+WVN = 1         WWL = 0           LL = 0
+LLN = 1         LW = 0            LMN = 1
+LNL = 0         CAPMOD = 2         XPART = 1
+CGDO = 4.91E-10 CGSO = 4.91E-10    CGBO = 1E-12
+CJ = 9.652028E-4  PB = 0.8                MJ = 0.3836899
+CJSW = 2.326465E-10 PBSW = 0.8                MJSW = 0.1253131
+CJSWG = 3.3E-10  PBSWG = 0.8                MJSWG = 0.1253131
+CF = 0           PVTRO = -7.714081E-4    PRDSW = -2.5827257
+PK2 = 9.619963E-4 WKETA = -1.060423E-4  LKETA = -5.373522E-3
+PU0 = 4.5760891  PUA = 1.469028E-14         PUB = 1.783193E-23
+PVSAT = 1.19774E3 PETAO = 9.968409E-5  PKETA = -2.51194E-3
+nlv = 3          kf = 0.5e-25
```

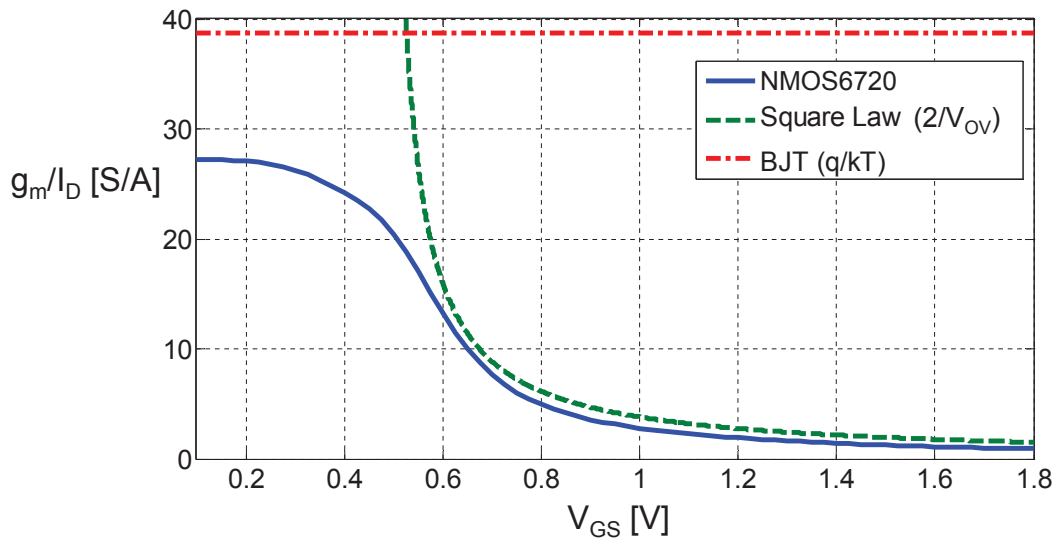
- The Spice model for an NMOS device in our course technology is shown to the left
- This is a 110-parameter BSIM3v3 model, quite complex even though 180nm is a relatively old process
  - More recent models may require even more parameters (e.g. PSP, BSIM6)
  - **KP and LAMBDA are nowhere to be found!**
- The I-V characteristics of a modern MOSFET cannot be accurately described by the square law

## Simulation (NMOS, 5/0.18 $\mu$ m, $V_{DS}=1.8V$ )



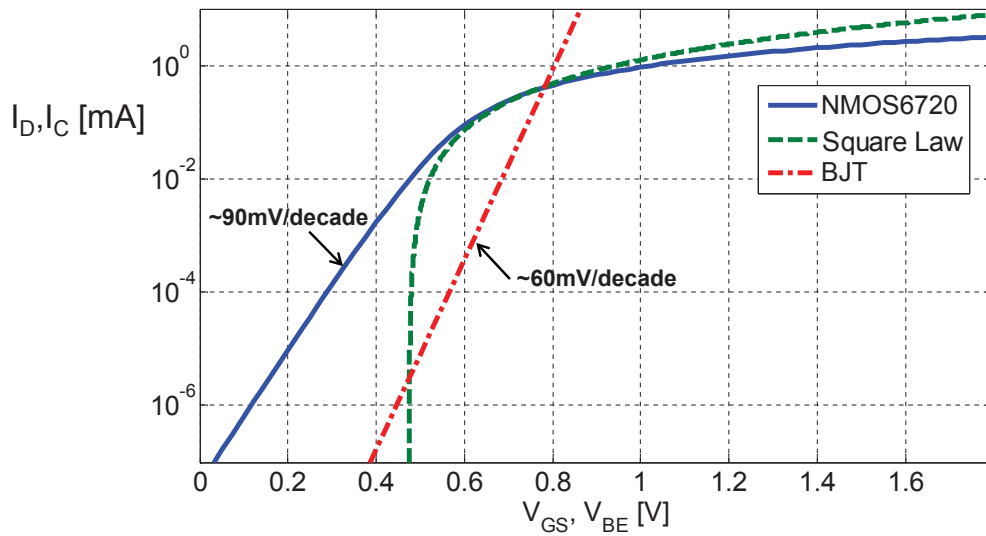
- Two observations
  - The transistor does not abruptly turn off at some  $V_t$
  - The current is not perfectly quadratic with  $(V_{GS}-V_t)$

## $g_m/I_D$ at Low $V_{GS}$



- The square law fails miserably at predicting  $g_m/I_D$  for low  $V_{GS}$

## Currents on a Log Scale

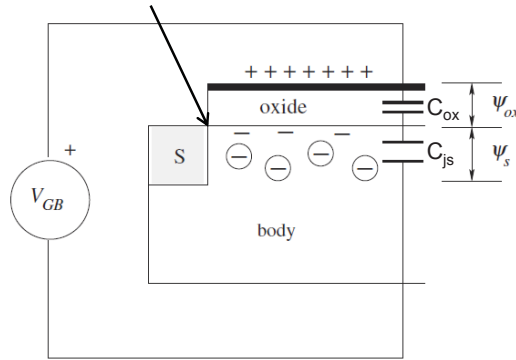


- What is  $V_t$ , anyway? The device does not turn off at all, but really approaches an exponential IV law for low  $V_{GS}$
- What determines the current at low  $V_{GS}$ ?

## Weak Inversion (Subthreshold) Operation

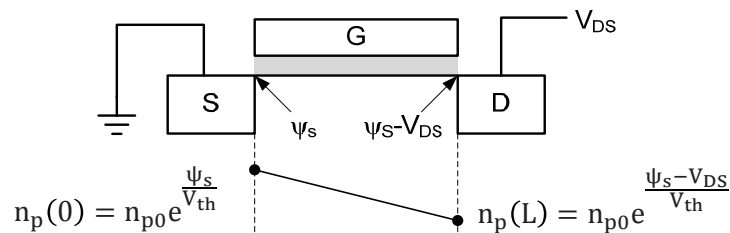
- Before inversion occurs, the electrostatic field from the gate forward-biases the source-side pn junction at the surface
- Physics governed by a “gated diode” model

Potential at this point is higher than body/source potential → forward bias



D.L. Pulfrey, Understanding Modern Transistors and Diodes, Cambridge University Press, 2010.

## Resulting Diffusion Current



$$I_D = qAD_n \frac{n_p(0) - n_p(L)}{L}$$

$$I_D = \frac{1}{L} qAD_n n_{p0} e^{\frac{\psi_s}{V_{th}}} (1 - e^{-\frac{V_{DS}}{V_{th}}})$$

Here  $v_{th} = kT/q$  is the “thermal voltage”

- The current grows exponentially with  $\psi_s$
- The current becomes independent of  $V_{DS}$  for  $V_{DS} > 3V_{th}$  (~78mV)

## Capacitive Divider

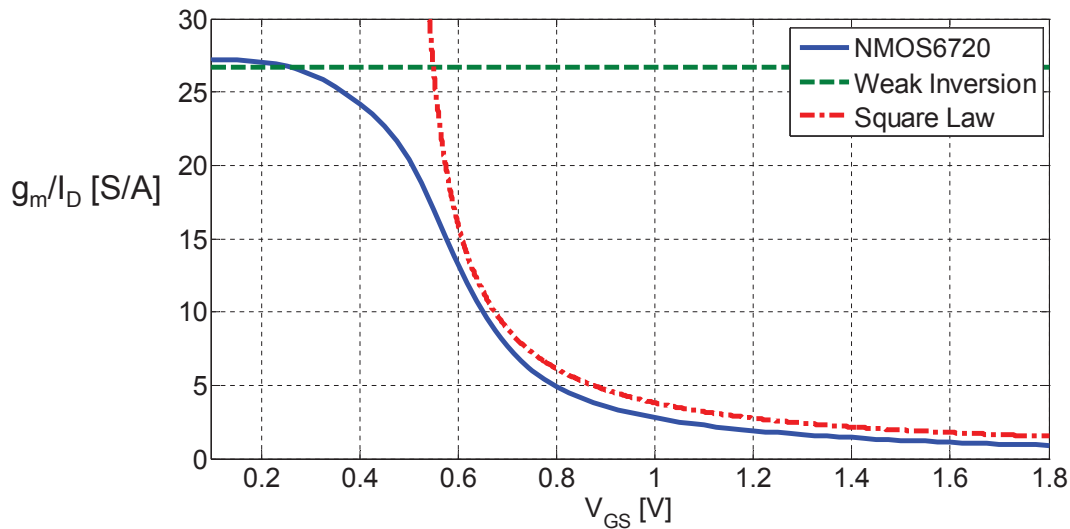
$$\frac{d\psi_s}{dV_{GS}} = \frac{C_{ox}}{C_{js} + C_{ox}} = \frac{1}{n}$$

- $n$  is called “subthreshold factor” or “nonideality factor”
- $n \cong 1.45$  for an NMOS device in the ECE6720 technology
- After including this relationship between  $\psi_s$  and  $V_{GS}$  and after a few additional manipulations, the final expression for the drain current becomes:

$$I_D = \frac{W}{L} I_{D0} e^{\frac{V_{GS}-V_t}{nV_{th}}} \left(1 - e^{-\frac{V_{DS}}{V_{th}}}\right)$$

where  $I_{D0}$  depends on technology ( $I_{D0} \cong 0.43\mu\text{A}$  for an NMOS device in ECE6720 technology)

## $g_m/I_D$



- We now have a better idea about the maximum possible  $g_m/I_D$ , but we still do not know how to model the transition region between the two IV laws

## Moderate Inversion

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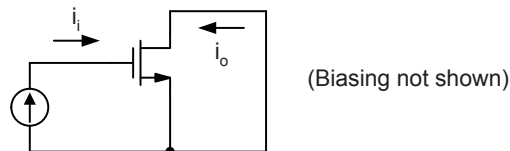
- In the transition region between weak and strong inversion, the drain current consists of both drift and diffusion currents
- One can show that the ratio of drift/diffusion current in moderate inversion and beyond is approximately  $(V_{GS}-V_t)/(kT/q)$
- This means that the square law equation (which assumes 100% drift current) does not work unless the gate overdrive is several  $kT/q$
  
- Is there a simple expression that works for all three regions (weak, moderate and strong inversion)?
  - No, there is no closed-form expression that captures all modes of operation as well as “short channel effects”
  - This is why  $g_m/I_D$ -based design is very useful...

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## FOM #2: Transit Frequency

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- Transistor figure of merit that relates to circuit speed and efficiency
  - The transit frequency of a transistor has "historically" been defined as the frequency where the magnitude of the common source current gain ( $|i_o/i_i|$ ) falls to unity



- Ignoring extrinsic capacitance and  $C_{gb}$ , and using the square law model:

$$\omega_T = \frac{g_m}{C_{gs}} \cong \frac{3 \mu V_{OV}}{2 L^2}$$

- We'll use this improved definition for  $g_m/I_D$ -based design:

$$\omega_T \triangleq \frac{g_m}{C_{gg}} = \frac{g_m}{C_{gs} + C_{gb} + C_{gd}}$$

## Transit Frequency Interpretation

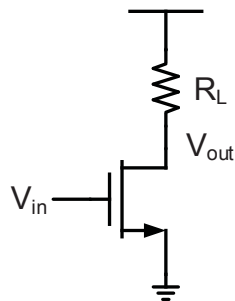
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- The transit frequency is only useful as a figure of merit in that it quantifies  $g_m/C_{gg}$ 
  - Tells you how much  $C_{gg}$  you get for a given  $g_m$
- It does **not** accurately predict up to which frequency you can use the device
  - See slide 46 of Chapter 2
- At a high level,  $C_{gg}$  affects circuit speed
  - At a lower level,  $C_{gs}$ ,  $C_{gb}$ , and  $C_{gd}$  affect a circuit's bandwidth differently
    - e.g. Miller effect in a common source stage,  $(1+|Av|)C_{gd}$  versus  $C_{gs}$
- We'll see that  $C_{gs}/C_{gg}$ ,  $C_{gd}/C_{gg}$ , etc. are fairly stable ratios
  - Having a small  $C_{gg}$  means the individual components are small too

## FOM #3: Intrinsic Gain

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- Transistor figure of merit related to the maximum gain a circuit can achieve



$$\frac{V_{out}}{V_{in}} = -g_m(r_o || R_L)$$

$$\max\left(\left|\frac{V_{out}}{V_{in}}\right|\right) = g_m r_o = \frac{g_m}{g_{ds}}$$

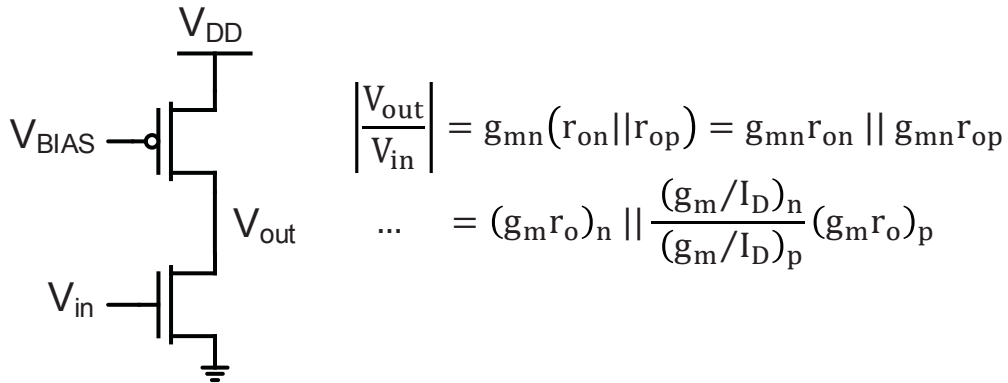
$$\text{Square law: } g_m r_o = \frac{1}{\lambda} \frac{g_m}{I_D} = \frac{1}{\lambda} \frac{2}{V_{OV}} \quad \left. \vphantom{\frac{1}{\lambda} \frac{2}{V_{OV}}} \right\} \text{A very poor model}$$

- Intrinsic gain is typically related to circuit accuracy
  - e.g. in feedback circuits, we use high open-loop gain to make an accurate closed-loop gain



## Example: Intrinsic Gain and Active Loads

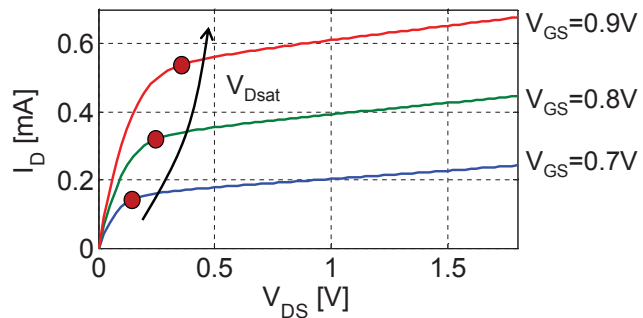
- The gain of amplifiers with active loads is fundamentally linked to intrinsic gain



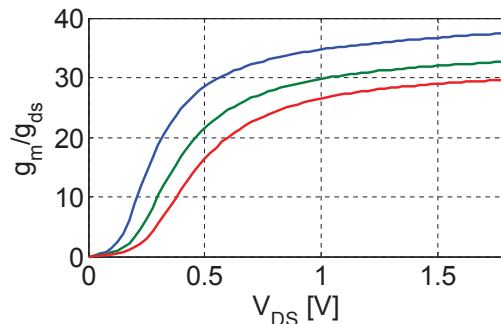
- The ratio of  $g_{mp}/g_{mn}$  is usually designed to be low, for low noise
    - Generally constrained by practical biasing ( $V_{Dsat} \approx V_{OV}$ )
- $\frac{(g_m/I_D)_n}{(g_m/I_D)_p}$  is typically 2-4

## How about $V_{Dsat}$ ?

- $V_{Dsat}$  tells us how much voltage we need across the channel to operate in saturation
  - “High gain region”



- It is important to note that  $V_{Dsat}$  is not crisply defined in modern devices
  - $g_m/g_{ds}$  gradually increases with  $V_{DS}$



## Relationship Between $V_{Dsat}$ and $g_m/I_D$

- It turns out that  $2/(g_m/I_D)$  is a reasonable first-order estimate for  $V_{Dsat}$

### Square Law

$$I_D = K(V_{GS} - V_t)^2$$

$$g_m = 2K(V_{GS} - V_t)$$

$$\frac{2}{(g_m/I_D)} = (V_{GS} - V_t) = V_{OV} = V_{Dsat}$$

Consistent with the classical first-order relationship

### Weak Inversion

$$I_D = I_{D0} e^{\frac{V_{GS}-V_t}{nV_{th}}} \left( 1 - e^{-\frac{V_{DS}}{V_{th}}} \right)$$

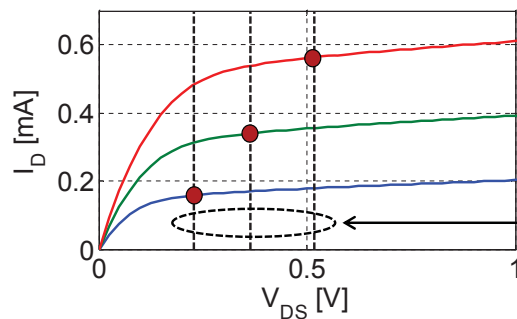
Need about  $3V_{th}$  for saturation

$$g_m = \frac{I_{D0}}{nV_{th}} e^{\frac{V_{GS}-V_t}{nV_{th}}} \left( 1 - e^{-\frac{V_{DS}}{V_{th}}} \right)$$

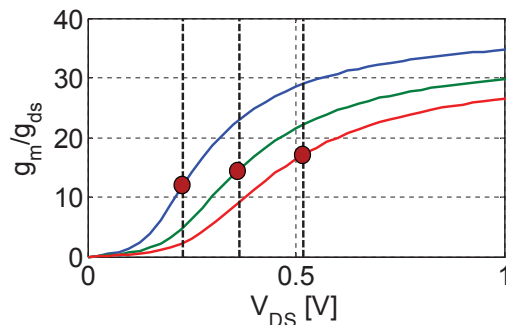
$$\frac{2}{(g_m/I_D)} = 2nV_{th} \cong 3V_{th}$$

Corresponds well with the required minimum  $V_{DS}$

## Reality Check



Computed  $2/(g_m/I_D)$  values

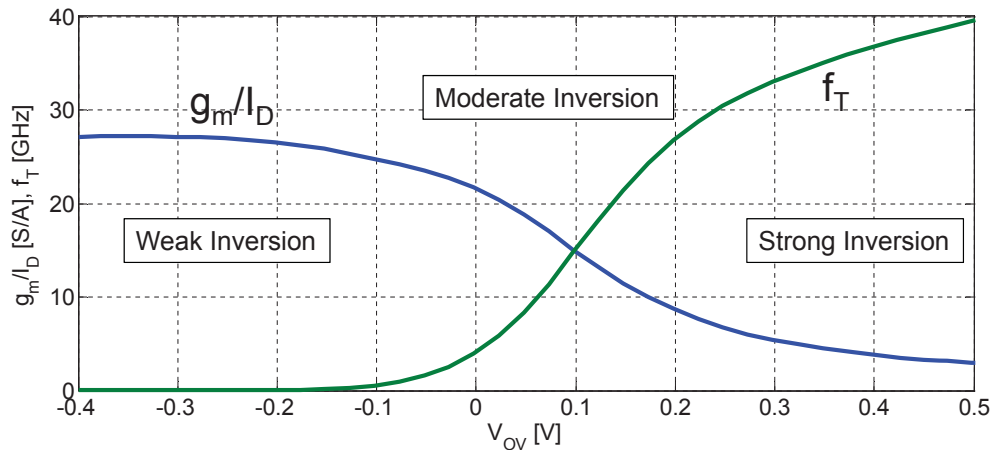


The SPICE model data confirms that  $2/(g_m/I_D)$  is a good estimate for the minimum reasonable  $V_{DS}$

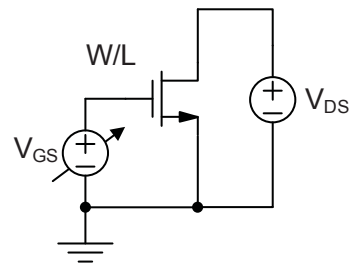
## Summary: Transistor Figures of Merit for Design

		<b>Square Law</b>
<ul style="list-style-type: none"> <li>Transconductance efficiency                             <ul style="list-style-type: none"> <li>– Want large <math>g_m</math>, for as little current as possible</li> </ul> </li> </ul>	$\frac{g_m}{I_D}$	$= \frac{2}{V_{OV}}$
<ul style="list-style-type: none"> <li>Transit frequency                             <ul style="list-style-type: none"> <li>– Want large <math>g_m</math>, without large <math>C_{gg}</math></li> </ul> </li> </ul>	$\frac{g_m}{C_{gg}}$	$\cong \frac{3}{2} \frac{\mu V_{OV}}{L^2}$
<ul style="list-style-type: none"> <li>Intrinsic gain                             <ul style="list-style-type: none"> <li>– Want large <math>g_m</math>, but no <math>g_{ds}</math></li> </ul> </li> </ul>	$\frac{g_m}{g_{ds}}$	$\cong \frac{2}{\lambda V_{OV}}$

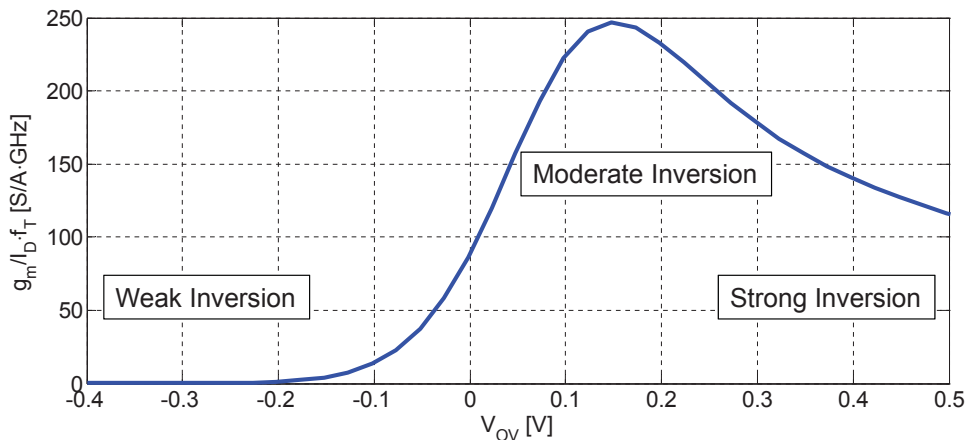
## Design Tradeoff: $g_m/I_D$ and $f_T$



- Weak inversion: Large  $g_m/I_D$  ( $>20$  S/A), but small  $f_T$
- Strong inversion: Small  $g_m/I_D$  ( $<10$  S/A), but large  $f_T$

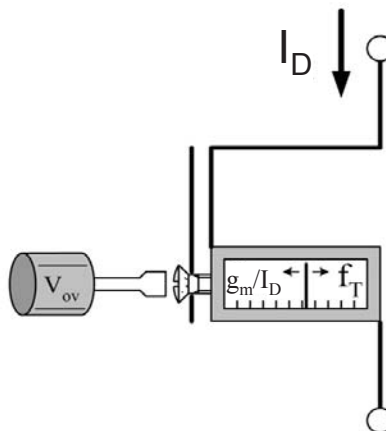


## Product of $g_m/I_D$ and $f_T$



- Interestingly, the product of  $g_m/I_D$  and  $f_T$  peaks in moderate inversion
  - Peaks around  $g_m/I_D \approx 10$  S/A in ECE6720 technology
- Operating the transistor in moderate inversion makes sense when we value speed and power efficiency equally
  - This is just a heuristic, not always the case

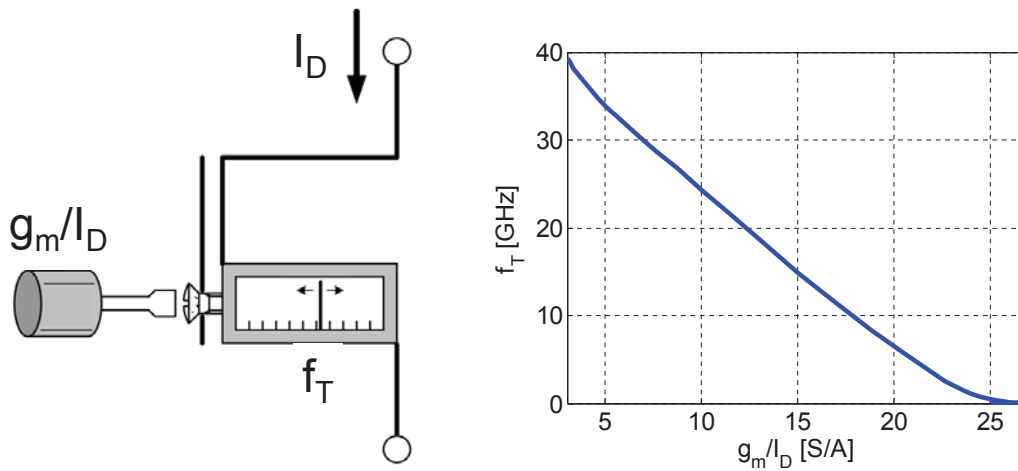
## Design in a Nutshell



- Choose length such that the circuit has 'enough' gain
- Choose the inversion level according to the proper tradeoff between speed ( $f_T$ ) and transconductance efficiency ( $g_m/I_D$ ) for the given circuit
- The inversion level is fully determined by the gate overdrive  $V_{OV}$ 
  - But,  $V_{OV}$  is not a very interesting parameter outside the square law framework; not much can be computed from  $V_{OV}$

## Eliminating $V_{OV}$

- The inversion level is also fully defined once we pick  $g_m/I_D$ , so there is no need to know  $V_{OV}$

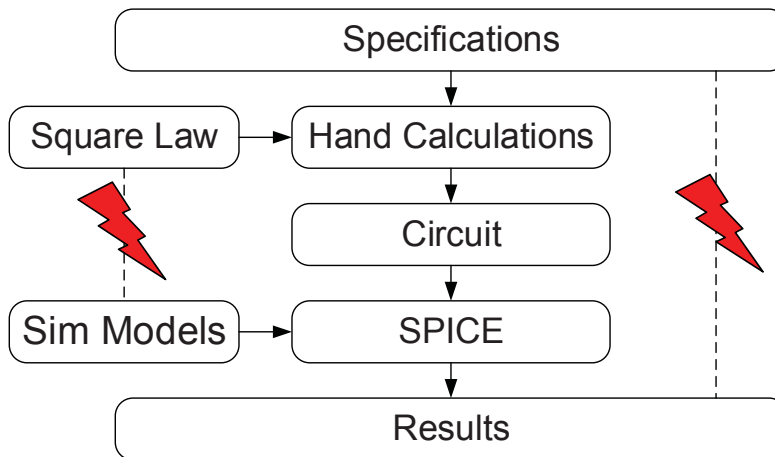


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## The Problem

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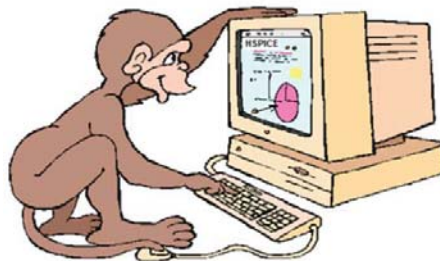


- Since there is a disconnect between actual transistor behavior and the simple square law model, any square-law driven design optimization will be far off from SPICE results

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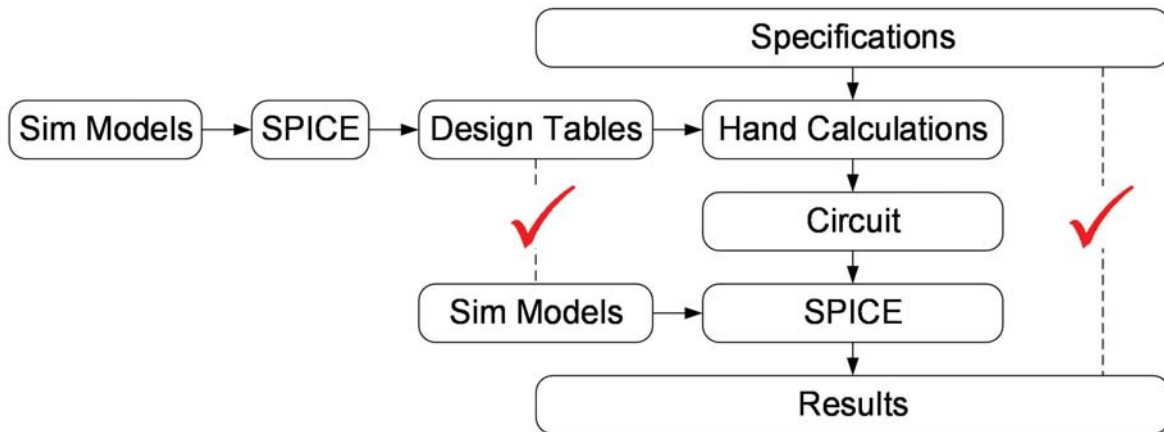
## Unfortunate Consequence

- In the absence of a simple set of equations for hand analysis, many designers tend to converge toward a “SPICE monkey” design methodology
  - No hand calculations, play SPICE like a video game until the circuit “somehow” meets the specifications
  - Typically results in sub-optimal designs, uninformed design decisions, circuit marginalities, etc.
- Our goal
  - Maintain a systematic design methodology in the absence of a set of useful compact MOS equations
- Strategy
  - Design using look-up tables or charts



[Courtesy Isaac Martinez]

## The Solution



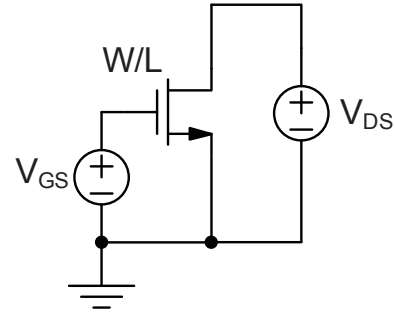
- Use pre-computed SPICE data in hand calculations

## $g_m/I_D$ -centric Technology Characterization

- Tabulate the figures of merit considering  $g_m/I_D$  as an index, over a reasonable range of  $g_m/I_D$  and channel lengths
  - Transit frequency ( $f_T$ )
  - Intrinsic gain ( $g_m/g_{ds}$ )
- Also tabulate relative estimates of capacitances
  - $C_{gd}/C_{gg}$  and  $C_{dd}/C_{gg}$
- In order to compute device widths, we need one more table that links  $g_m/I_D$  and current density  $I_D/W$
- Note that all of these parameters are (to first order) independent of device width

## Starting Point: Technology Characterization via DC Sweep

- Obtain these tables through a DC sweep simulation of the transistor models
  - Measure transistor .op parameters at each point of the sweep
    - $g_m$ ,  $I_D$ ,  $C_{gg}$ ,  $g_{ds}$ , etc.
  - Repeat the sweep for different lengths
    - 180nm, 200nm, ..... 3 $\mu$ m
  - Generate charts, Matlab arrays, etc.
- Simple version: sweep  $V_{GS}$  with  $V_{DS}$  held fixed at  $V_{DS} = V_{DD}/2$ 
  - The figures of merit and  $I_D/W$  don't vary too much with  $V_{DS}$
- Advanced version: sweep  $V_{DS}$  and  $V_{BS}$  also



## HSPICE Example

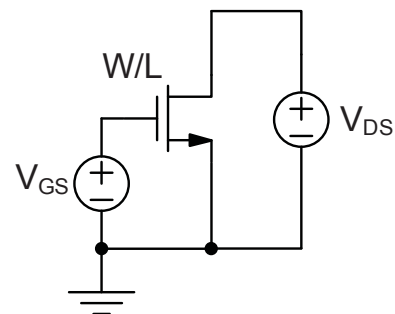
```
* Example HSPICE netlist/simulation, techchar.sp
* Model file
.inc './models/ece6720.mod'
* Define 'width' and 'length' in a parameter file,
* along with 'gsmax' and 'gsstep'
.inc 'techchar_params.sp'

.param ds = 0.9
.param gs = 0.9

vdsn vdn 0 dc 'ds'
vgsn vgn 0 dc 'gs'
mn vdn vgn 0 0 nmos6720 L='length' W='width'

.options dccap post brief accurate nomod
.dc gs 0 'gsmax' 'gsstep'

.probe n_id = par('i(mn)')
.probe n_gm = par('gmo(mn)')
.probe n_gds = par('gds0(mn)')
.probe n_cgg = par('cggbo(mn)')
```





## Example Matlab Wrapper

```
% Example Matlab wrapper

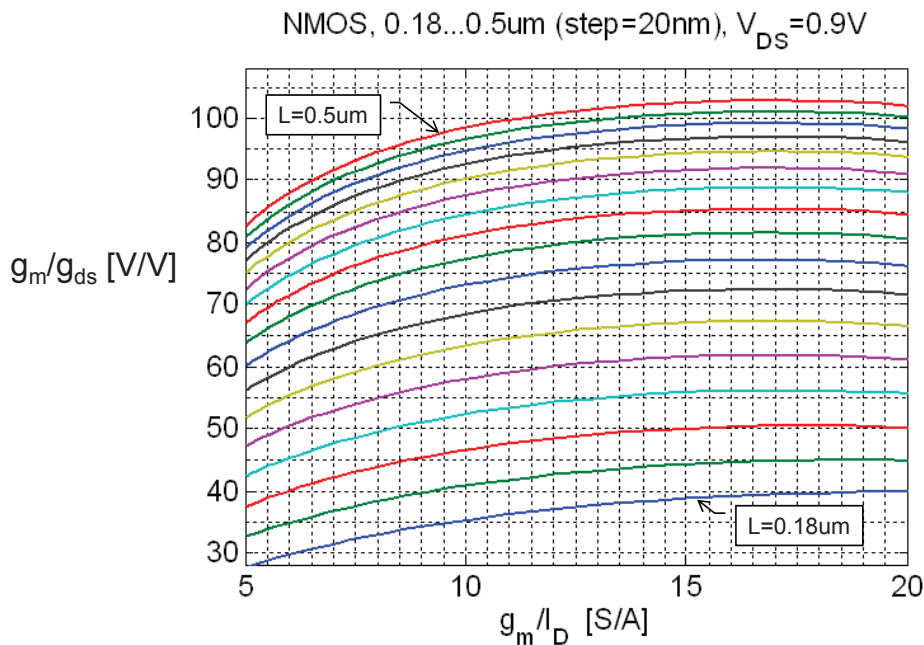
% HSPICE toolbox
addpath('./HSPICEToolBox')

% Parameters for HSPICE runs
VGS_step = 25e-3; VGS_max = 1.8;
VGS = 0:VGS_step:VGS_max;
W = 5e-6; L = [(0.18:0.02:0.5) (0.6:0.1:1.0) (1.2:0.2:3)]*1e-6;

% HSPICE simulation loop
for i = 1:length(L)
    % write out circuit parameters and run HSPICE
    fid = fopen('techchar_params.sp', 'w');
    fprintf(fid, '*** simulation parameters ***\n', datestr(now));
    fprintf(fid, '.param width = %d\n', W);
    fprintf(fid, '.param length = %d\n', L(i));
    fprintf(fid, '.param gsstep = %d\n', VGS_step);
    fprintf(fid, '.param gsmax = %d\n', VGS_max);
    fclose(fid);
    system('/uusoc/facility/cad_common/Synopsys/hspice_G-2012.06-SP1/hspice/bin/hspice...
           techchar.sp >! techchar.out');

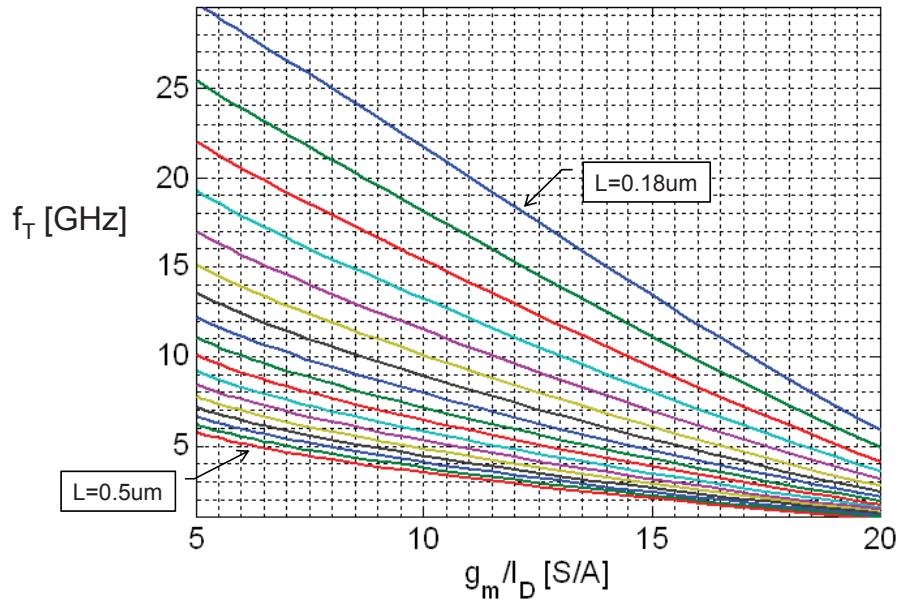
    %Read and store results
    h = loadsig('techchar.sw0');
    nch.GM(i,:) = evalsig(h, 'n_gm');
    nch.ID(i,:) = evalsig(h, 'n_id');
    nch.CGG(i,:) = evalsig(h, 'n_cgg');
    nch.GDS(i,:) = evalsig(h, 'n_gds');
end
```

## Intrinsic Gain Chart



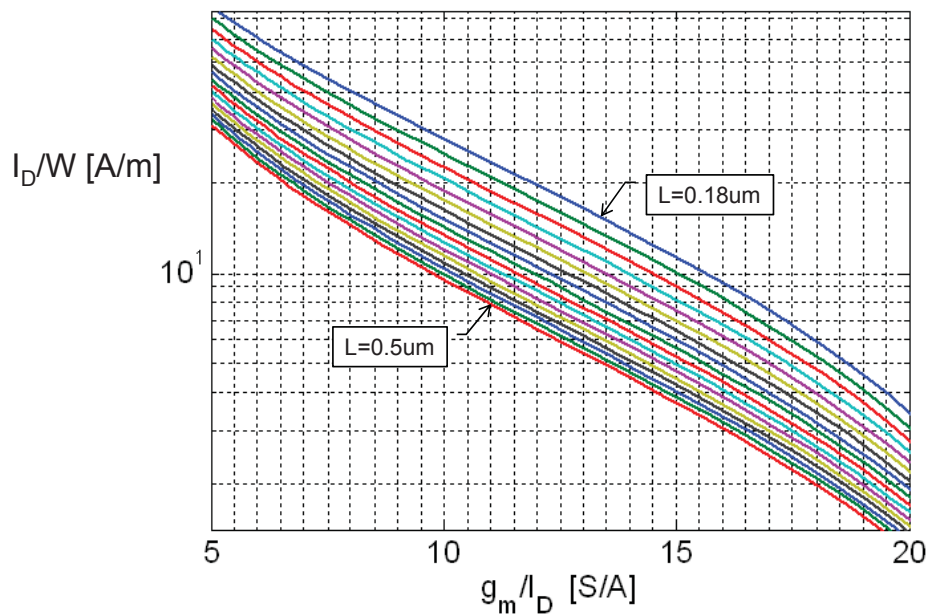
## Transit Frequency Chart

NMOS, 0.18...0.5um (step=20nm),  $V_{DS}=0.9V$

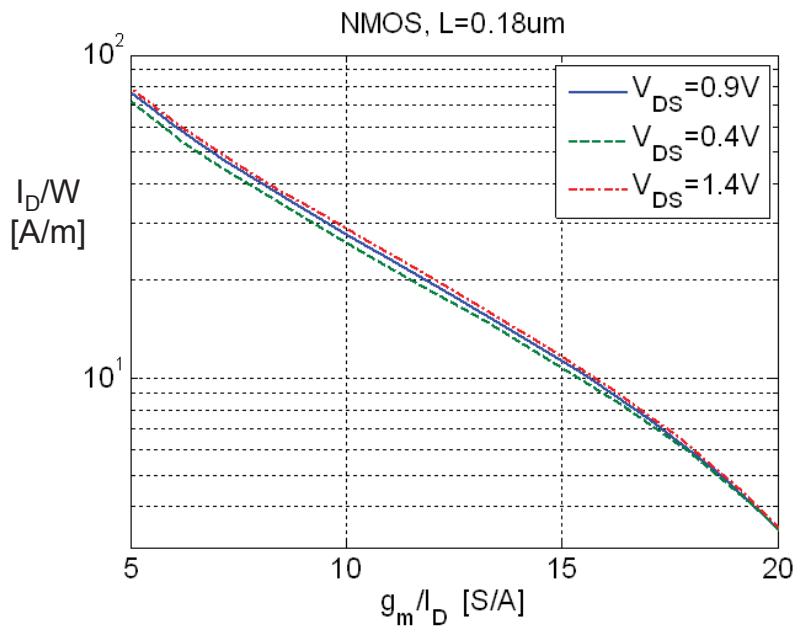


## Current Density Chart

NMOS, 0.18...0.5um (step=20nm),  $V_{DS}=0.9V$

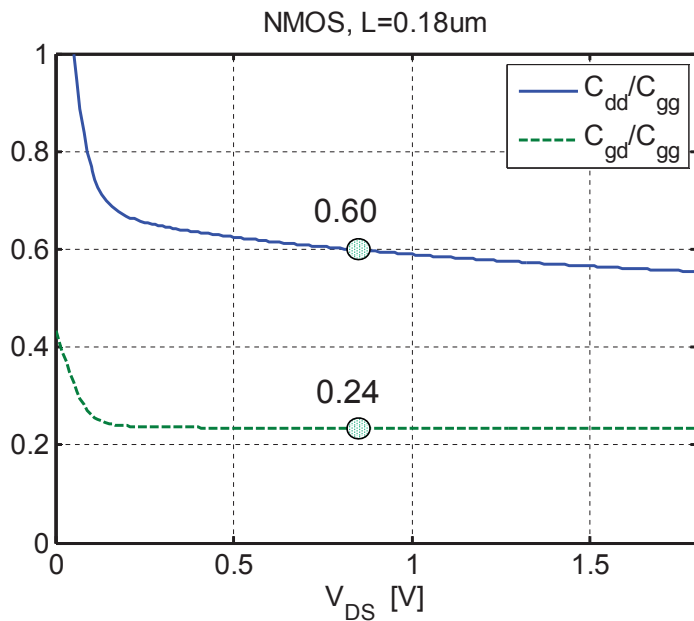


## V<sub>DS</sub> Dependence



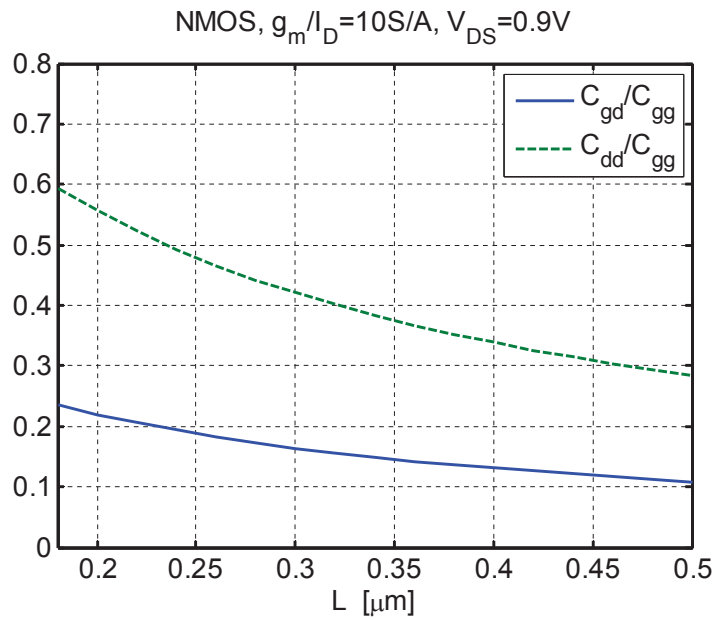
- V<sub>DS</sub> dependence is relatively weak
- Typically OK to work with data generated for V<sub>DD</sub>/2

## Handling Extrinsic Capacitances

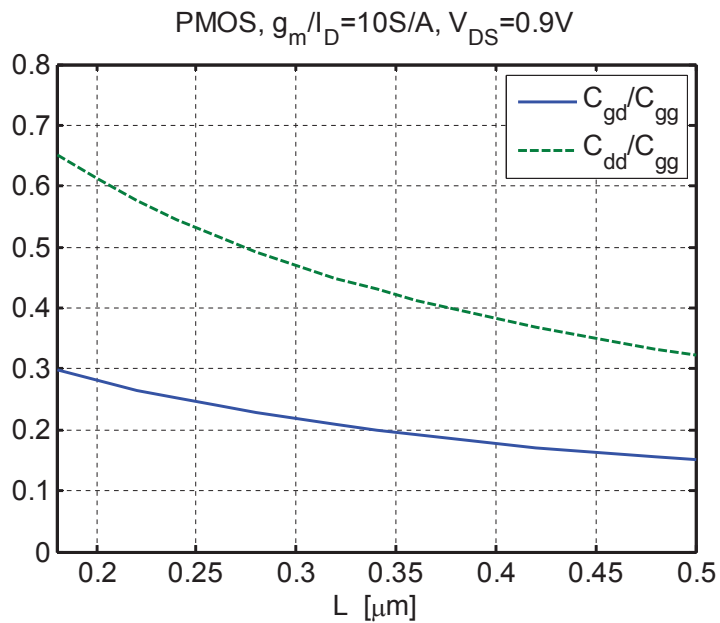


- Again, it's usually OK to work with estimates taken at V<sub>DS</sub>=V<sub>DD</sub>/2

## Extrinsic Capacitances – Length Dependence



## Extrinsic Capacitances (PMOS) – Length Dependence



## Key Question

---

- How can we use all this data for systematic design?
  
- Many options exist
  - And you can invent your own, if you like
  
- Method taught in ECE6720
  - Look at the transistor in terms of width-independent figures of merit that are intimately linked to design specifications and performance
    - Rather than physical modeling parameters that do not directly relate to circuit specs
  - Think about the design tradeoffs in terms of the MOSFET's inversion level (bias point), using  $g_m/I_D$  as a proxy

---

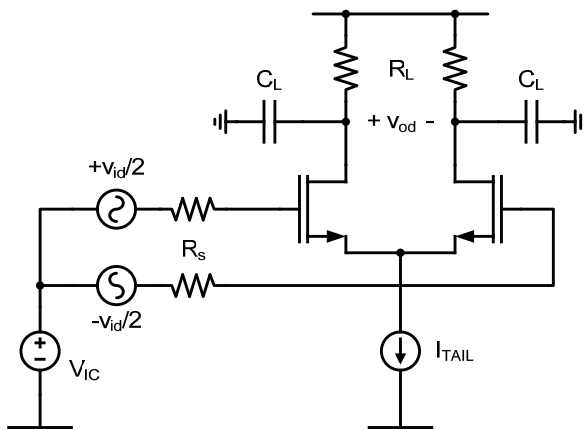
## A Generic Design Flow

---

- 1) Determine  $g_m$  (from design objectives)
- 2) Pick  $L$ 
  - Short channel  $\rightarrow$  high  $f_T$  (high speed)
  - Long channel  $\rightarrow$  high intrinsic gain
- 3) Pick  $g_m/I_D$ 
  - Large  $g_m/I_D \rightarrow$  low power, large signal swing (low  $V_{DSsat}$ )
  - Small  $g_m/I_D \rightarrow$  high  $f_T$  (high speed)
- 4) Determine  $I_D$  (from  $g_m$  and  $g_m/I_D$ )
- 5) Determine  $W$  (from  $I_D/W$ )

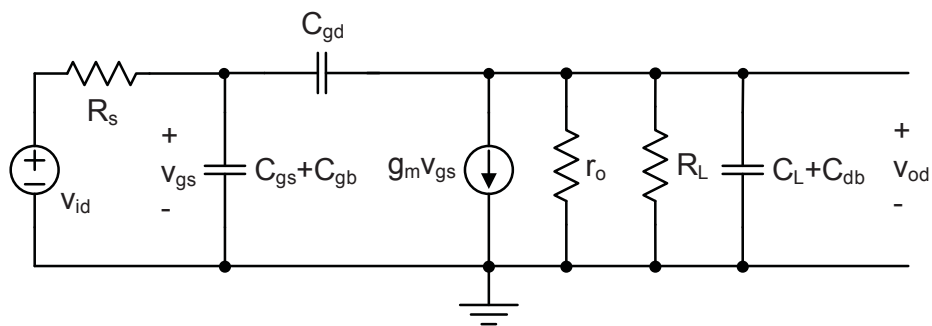
Many other possibilities exist (depends on circuit specifics, design constraints and objectives)

## Basic Design Example



- Given specifications and objectives
  - 0.18 $\mu\text{m}$  technology
  - Low frequency gain = -4
  - $R_L = 1\text{k}\Omega$ ,  $C_L = 50\text{fF}$ ,  $R_s = 10\text{k}\Omega$
  - Maximize bandwidth while keeping  $I_{\text{TAIL}} \leq 600\mu\text{A}$ 
    - Implies  $L = L_{\text{min}} = 0.18\mu\text{m}$
  - Determine device width
  - Estimate dominant and non-dominant pole

## Small-Signal Half-Circuit Model



- Calculate  $g_m$  and  $g_m/I_D$

$$|A_{v0}| \cong g_m R_L = 4 \quad \Rightarrow \quad g_m = \frac{4}{1\text{k}\Omega} = 4\text{mS}$$

$$\frac{g_m}{I_D} = \frac{4\text{mS}}{300\mu\text{A}} = 13.3 \frac{\text{S}}{\text{A}}$$

## Aside: Why can we Neglect $r_o$ ?

$$|A_{v0}| = g_m (R_L \parallel r_o)$$

$$\therefore 4 = g_m R_L \parallel g_m r_o$$

$$\frac{1}{4} = \frac{1}{g_m R_L} + \frac{1}{g_m r_o}$$

- Even at  $L=L_{\min} = 0.18\mu\text{m}$ , we have  $g_m r_o > 30$

$$\frac{1}{4} \gg \frac{1}{30}$$

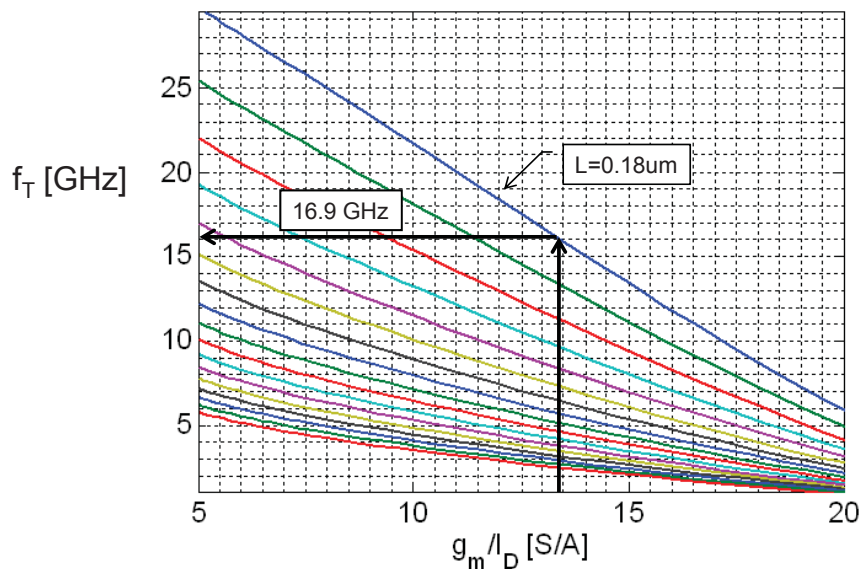
$$\therefore \frac{1}{4} \cong \frac{1}{4} - \frac{1}{g_m r_o} = \frac{1}{g_m R_L}$$

$$4 \cong g_m R_L$$

- $r_o$  is negligible in this design problem

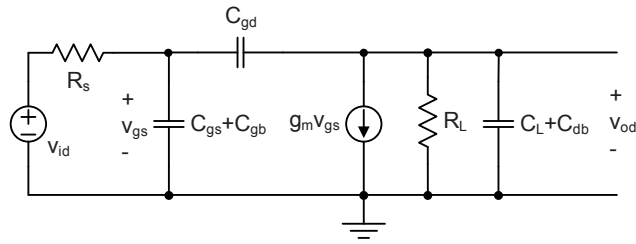
## Determine $C_{gg}$ via $f_T$ Look-up

NMOS, 0.18...0.5 $\mu\text{m}$  (step=20nm),  $V_{DS} = 0.9\text{V}$



$$\frac{g_m}{I_D} = 13.3 \frac{\text{S}}{\text{A}}$$

## Calculate Capacitance Values



$$C_{gg} = \frac{1}{2\pi} \frac{4\text{mS}}{16.9\text{GHz}} = 37.7\text{fF}$$

$$C_{gd} = \frac{C_{gd}}{C_{gg}} C_{gg} = 0.24 \cdot 37.7\text{fF} = 9.0\text{fF}$$

$$C_{dd} = \frac{C_{dd}}{C_{gg}} C_{gg} = 0.60 \cdot 37.7\text{fF} = 22.6\text{fF}$$

$$C_{db} = C_{dd} - C_{gd} = 13.6\text{fF}$$

$$C_{gs} + C_{gb} = C_{gg} - C_{gd} = 28.7\text{fF}$$

## Zero and Pole Expressions

KCL analysis shows you: 
$$H(s) = \frac{-g_m R_L (1 - sC_{gd}/g_m)}{1 + b_1 s + b_2 s^2}$$

High frequency zero: 
$$\omega_z = \frac{g_m}{C_{gd}} \gg \omega_T \rightarrow \text{negligible}$$

Denominator coefficients: 
$$b_1 = R_s [C_{gs} + C_{gd} (1 + |A_{v0}|)] + R_L (C_L + C_{gd})$$
  

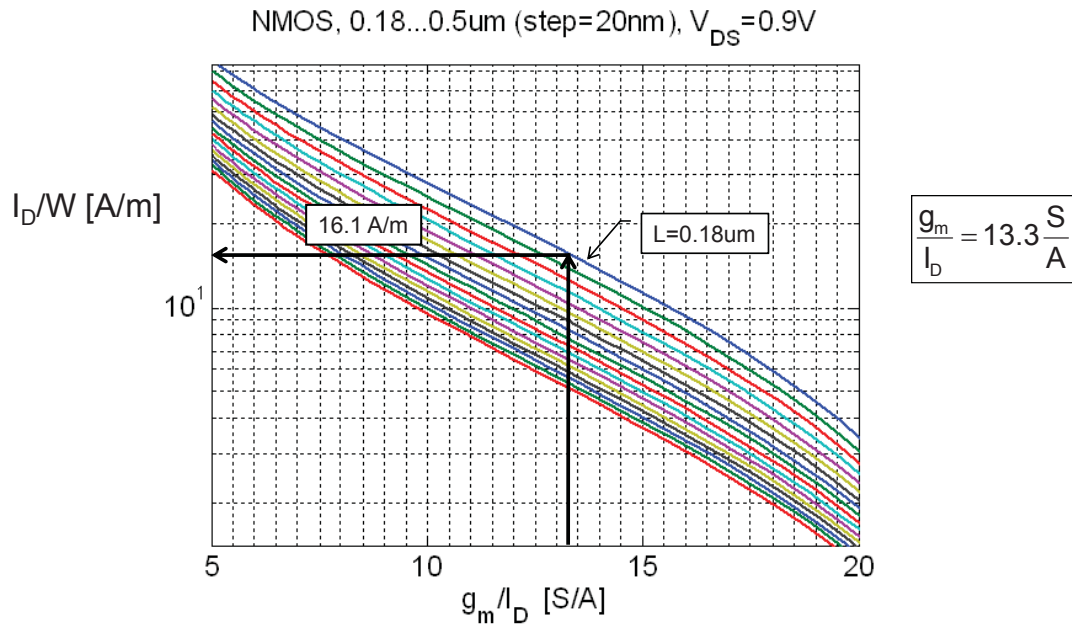
$$b_2 = R_s R_L (C_{gs} C_L + C_{gs} C_{gd} + C_L C_{gd})$$
  
 ( $C_{db}$  can be added to  $C_L$  if significant)

Dominant pole approximation: 
$$\omega_{p1} \cong \frac{1}{b_1} \quad \boxed{f_{p1} \cong 200 \text{ MHz}}$$

Non-dominant pole approximation: 
$$\omega_{p2} \cong \frac{b_1}{b_2} \quad \boxed{f_{p2} \cong 5.8 \text{ GHz}}$$



## Device Sizing



## A Note on Current Density

- Designing with current density charts in a normalized, width-independent space works because
  - Current density and  $g_m/I_D$  are independent of  $W$ 
    - $I_D/W \sim W/W$
    - $g_m/I_D \sim W/W$
  - There is a one-to-one mapping from  $g_m/I_D$  to current density

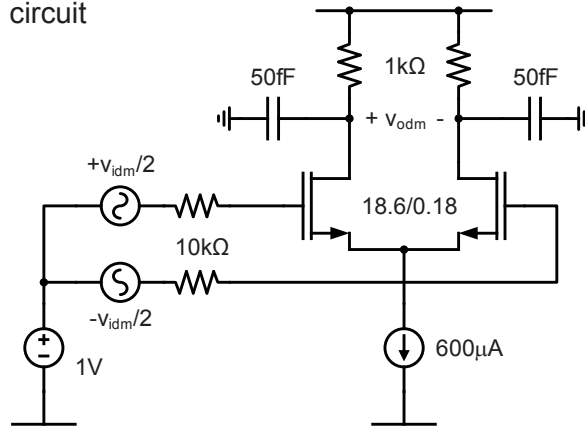
Square law:  $\frac{g_m}{I_D} = \frac{2}{V_{OV}}$        $\frac{I_D}{W} = \frac{1}{2} \mu C_{ox} \frac{1}{L} V_{OV}^2 = \mu C_{ox} \frac{1}{L} \left( \frac{1}{2} \frac{g_m}{I_D} \right)^{-2}$

General case:  $\frac{g_m}{I_D} = f(V_{OV})$        $\frac{I_D}{W} = g(V_{OV}) = g\left(f^{-1}\left(\frac{g_m}{I_D}\right)\right)$

# Circuit For SPICE Verification

- Device width  $W = \frac{I_D}{\frac{I_D}{W}} = \frac{300\mu A}{16.1A/m} = 18.6\mu m$

- Simulation circuit



## Simulated DC Operating Point (Cadence/Spectre)

Good agreement with simulation!

Hand analysis:	Simulation:
$g_m = 4 \text{ mS}$	$g_m = 4.05 \text{ mS}$
$C_{dd} = 22.6 \text{ fF}$	$C_{dd} = 24.63 \text{ fF}$
$C_{gg} = 37.8 \text{ fF}$	$C_{gg} = 37.49 \text{ fF}$
$C_{gd} = 9.0 \text{ fF}$	$C_{gd} = 8.97 \text{ fF}$

Signals Search

```

betaeff(A/V^2)=0.036684862  cdb(F)=-1.5498007e-14  cgd(F)=-8.9670839e-15  cgsovl(F)=9.12769e-15  cssbi(F)=1.8601233e-14
cbb(F)=3.9480359e-14  cdd(F)=2.4825335e-14  cgdbi(F)=1.6060608e-16  cjd(F)=1.5496277e-14  gbd(S)=0
cbd(F)=-1.553178e-14  cddbi(F)=1.3683095e-18  cgdovl(F)=9.12789e-15  cjs(F)=1.8683964e-14  gbs(S)=0
cbdbi(F)=-3.5502974e-17  cdg(F)=-9.1349816e-15  cgg(F)=3.7488874e-14  csb(F)=-2.2254659e-14  gds(S)=0.00010035656
cbg(F)=-4.3221562e-15  cds(F)=7.6335644e-18  cggbi(F)=1.9233347e-14  csd(F)=-1.2847141e-16  gmi(S)=0.0040505737
cbs(F)=-1.9626423e-14  cgb(F)=-1.7276924e-15  cgs(F)=-2.6794098e-14  csg(F)=-2.4031757e-14  gmbs(S)=0.00086131562
cbsbi(F)=-3.4245885e-16  cgbovl(F)=1.4765368e-19  cgsbi(F)=-1.7666408e-14  css(F)=4.6412687e-14  gmoverid(1/V)=13.501677
    
```

Shell

13(31) plot new graph subwindow

## Spectre dcOpInfo Capacitance Output Parameters

### Spectre dcOpInfo output parameter

cdd	24.6253f
cgg	37.4887f
css	46.4128f
cbb	39.4803f
cgs	-26.7941f
cgd	-8.9671f

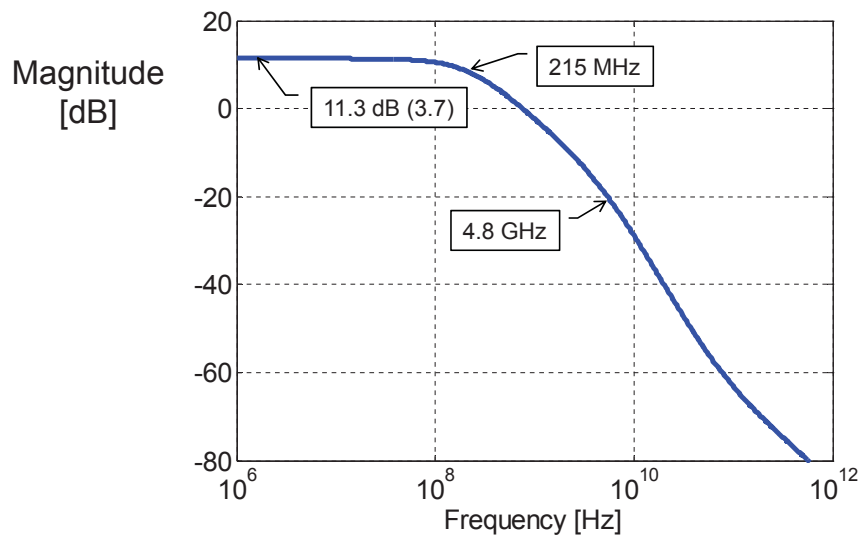
### Corresponding Small Signal Model Elements

$$\begin{aligned} \text{cdd} &\equiv C_{gd} + C_{db} \\ \text{cgg} &\equiv C_{gs} + C_{gd} + C_{gb} \\ \text{css} &\equiv C_{gs} + C_{sb} \\ \text{cbb} &\equiv C_{gb} + C_{sb} + C_{db} \\ \text{cgs} &\equiv C_{gs} \\ \text{cgd} &\equiv C_{gd} \end{aligned}$$

Note: you can ignore the negative signs, they are artifacts from the way Spectre computes small signal capacitances, e.g.  $C_{gs} = dQ_g/dV_s$

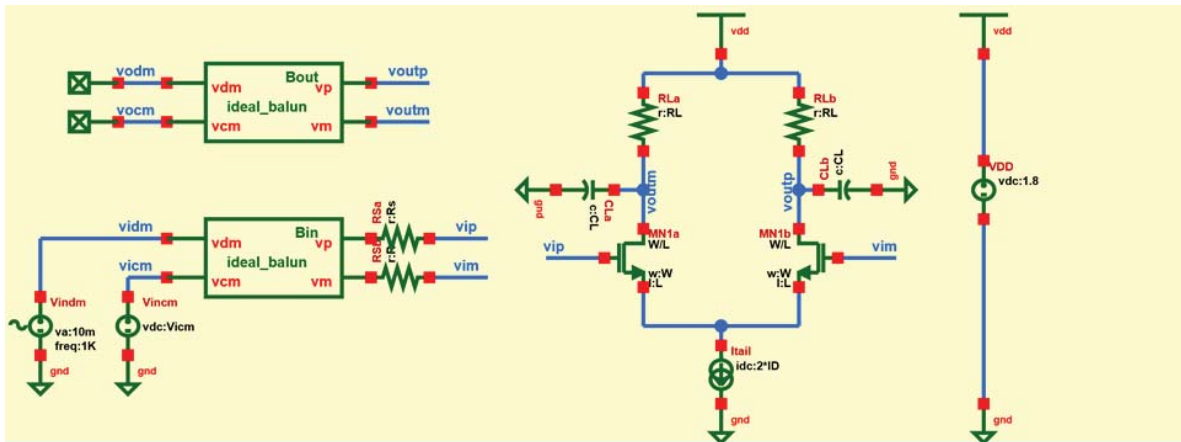
See Cadence help/manuals for more parameters and details: 'cdnshep' from the command prompt, then search for bsim3v3 and see the section "Operating-Point Parameters"

## Simulated AC Response



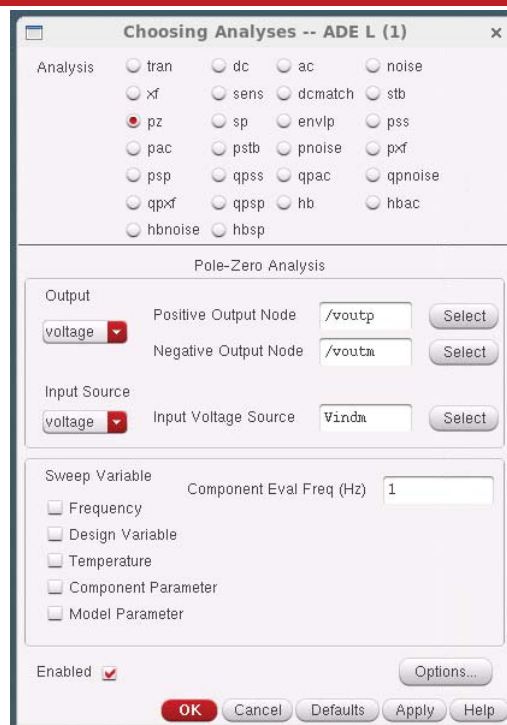
- Calculated values:  $|A_{v0}| = 12 \text{ dB (4.0)}$ ,  $f_{p1} = 200 \text{ MHz}$ ,  $f_{p2} = 5.8 \text{ GHz}$

# Cadence Schematic

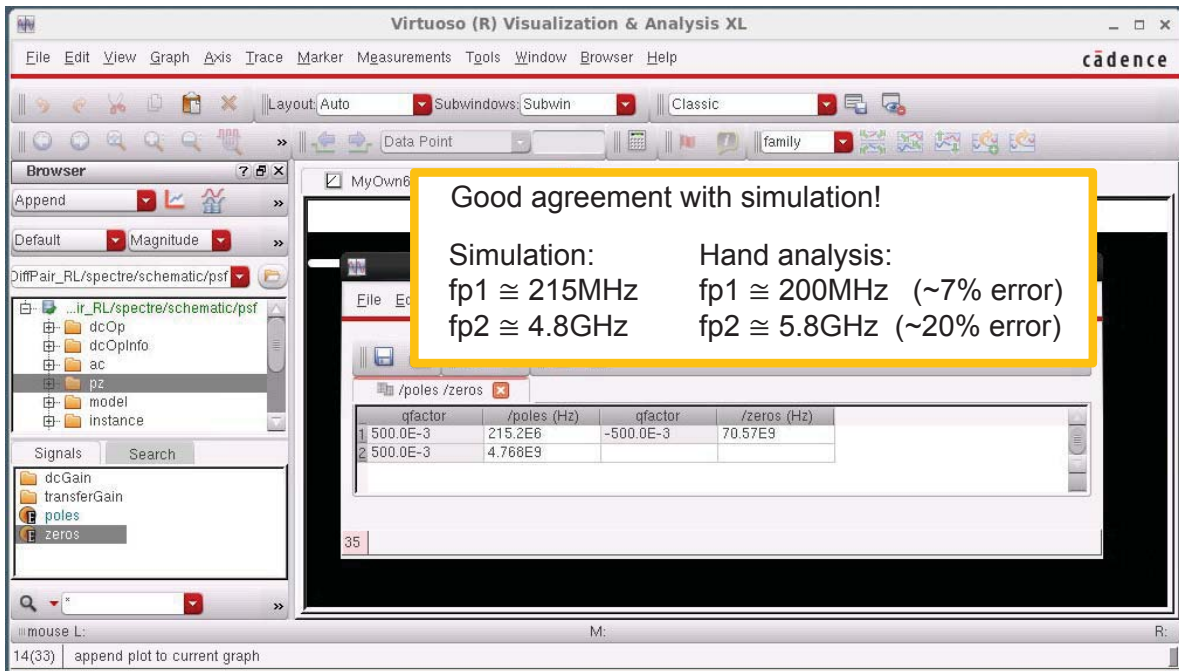


- W, L, Rs, RL, CL, ID are setup as design variables in ADE L
- Ideal baluns (from the ECE6720 library) are used to handle differential and common mode conversion

# Using Pole/Zero Analysis (pz)



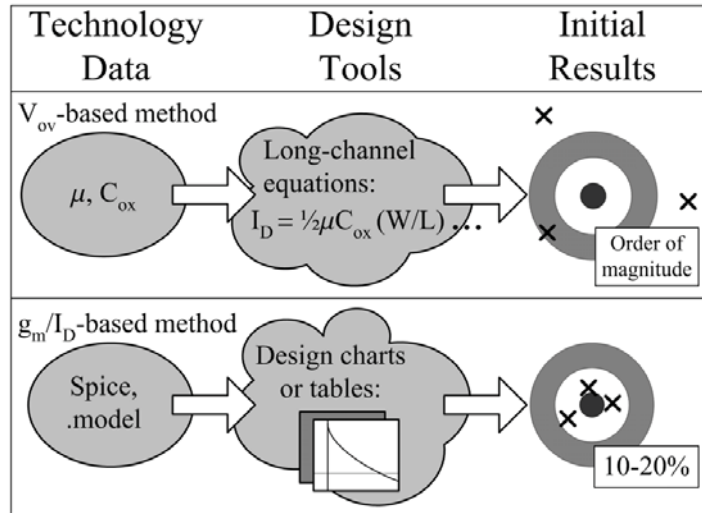
## pz Analysis Results



## Observations

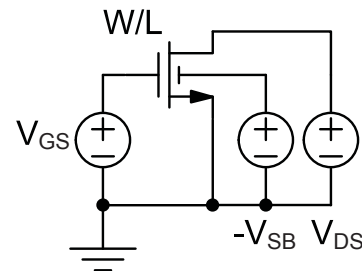
- The design is essentially right on target!
  - Typical discrepancies are no more than 10-20%, due to  $V_{DS}$  dependencies, finite output resistance, etc.
- We accomplished this by using pre-computed SPICE data in the design process
- Even if discrepancies are more significant, there's always the possibility to track down the root causes
  - Hand calculations are based on parameters that also exist in SPICE, e.g.  $g_m/I_D$ ,  $f_T$ , etc.
  - Different from square law calculations using  $\mu C_{ox}$ ,  $V_{OV}$ , etc.
    - Based on artificial parameters that do not exist or have no significance in the SPICE model

# Comparison



# Advanced Characterization

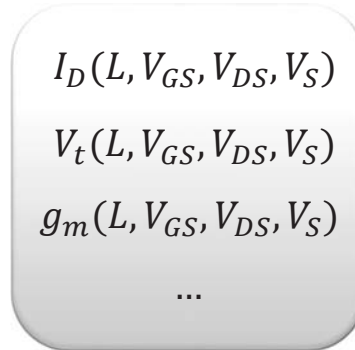
- While variations due to  $V_{DS}$  are fairly small for most parameters, we can get better accuracy by sweeping  $V_{DS}$  as well
  - Adds another dimension to the tables
    - Harder to visualize
    - Easy to deal with using Matlab lookup functions instead of charts
- In many circuits  $V_S \neq V_B$ 
  - e.g. a differential pair
  - Back-gate effect!
    - Sweeping  $V_{SB}$  generates characterization data that captures this effect as well



## Simulation Data in Matlab

```
% data stored in /home/ece6720/matlab
>> load 180n.mat
>> nch
nch =
      ID: [4-D double]
      VT: [4-D double]
      GM: [4-D double]
      GMB: [4-D double]
      GDS: [4-D double]
      CGG: [4-D double]
      CGS: [4-D double]
      CGD: [4-D double]
      CGB: [4-D double]
      CDD: [4-D double]
      CSS: [4-D double]
      INFO: 'Univ of Utah ECE6720 models,
180nm CMOS, BSIM3'
      VGS: [73x1 double]
      VDS: [73x1 double]
      VSB: [11x1 double]
      L: [32x1 double]
      W: 5.0000e-06
      NFING: 1
>> size(nch.ID)
ans =
     32     73     73     11
```

Four-dimensional arrays



## Lookup Function in /home/ece6720/matlab

```
>> lookup(nch, 'ID', 'VGS', 0.5, 'VDS', 0.5)
ans =
     8.4181e-06
>> help lookup
Rev. 20140731, Boris Murmann
The function "lookup" extracts a desired subset from the 4-dimensional simulation data
The function interpolates when the requested points lie off the simulation grid
There are three basic usage modes:
(1) Simple lookup of parameters at given (L, VGS, VDS, VSB)
(2) Lookup of arbitrary ratios of parameters, e.g. GM_ID, GM_CGG at given (L, VGS,
VDS, VSB)
(3) Cross-lookup of one ratio against another, e.g. GM_CGG for some GM_ID
In usage modes (1) and (2) the input parameters (L, VGS, VDS, VSB) can be
listed in any order and default to the following values when not specified:
L = min(data.L); (minimum length used in simulation)
VGS = data.VGS; (VGS vector used during simulation)
VDS = max(data.VDS)/2; (VDD/2)
VSB = 0;
```

## Matlab Design Script for the Diff Pair Example

```
clear all; close all;
load 180n.mat;

% Specs
Av0 = 4; RL = 1e3; CL = 50e-15; Rs = 10e3; ITAIL = 600e-6;

% Component calculations
gm = Av0/RL;
gm_id = gm/(ITAIL/2);
wT = lookup(nch, 'GM_CGG', 'GM_ID', gm_id);
cgd_cgg = lookup(nch, 'CGD_CGG', 'GM_ID', gm_id);
cdd_cgg = lookup(nch, 'CDD_CGG', 'GM_ID', gm_id);
cgg = gm/wT;
cgd = cgd_cgg*cgg;
cdd = cdd_cgg*cgg;
cdb = cdd - cgd;
cgs = cgg - cgd;

% Pole calculations
b1 = Rs*(cgs + cgd*(1+Av0))+RL*(CL+cgd);
b2 = Rs*RL*(cgs*CL + cgs*cgd + CL*cgd);
fp1 = 1/2/pi/b1
fp2 = 1/2/pi*b1/b2

% Device sizing
id_w = lookup(nch, 'ID_W', 'GM_ID', gm_id)
w = ITAIL/2 / id_w
```

## lookup\_examples.m

```
% Basic usage examples for function "lookup"
% Boris Murmann
% Stanford University
% Rev. 20140731
% Modified by Ross Walker, Univ of Utah, 8/1/2014
clear all; close all;

%Plot settings
font_size = 20; font_name = 'Arial'; linewidth = 1;

% Load data table
load 180n.mat;

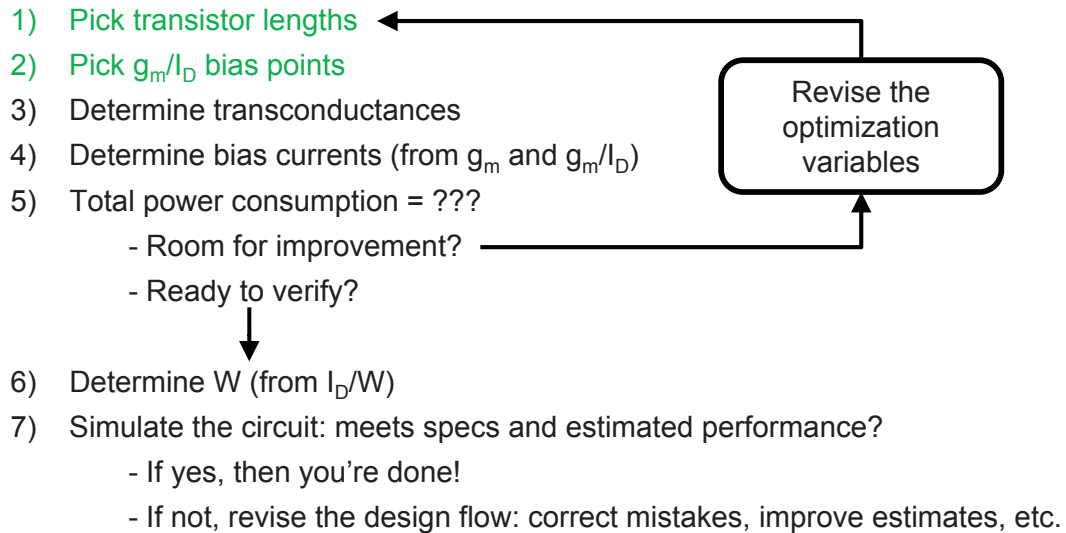
% Plot drain characteristics for different VGS at minimum L (default)
vgs = 0:0.2:max(nch.VGS);
id = lookup(nch, 'ID', 'VGS', vgs, 'VDS', nch.VDS);
figure(1)
set(gca, 'FontSize', font_size, 'FontName', font_name);
plot(nch.VDS, id, 'LineWidth', linewidth)
xlabel('V_D_S [V]')
ylabel('I_D [A]')

% Plot fT against gm_ID for different L
gm_id = 5:0.1:20;
wt = lookup(nch, 'GM_CGG', 'GM_ID', gm_id, 'L', nch.L);
figure(2)
```



## An Example Optimization Flow

- Complicated circuits have \*many\* degrees of freedom and objectives
  - Usually we must make some **heuristic** choices up front
  - Charts and lookup tables help you iterate through possible designs rapidly



## A Note on Tools

- Modern IC design depends on \*many\* tools
  - Schematic editors, simulators, layout editors, verification tools, etc...
  - 'Hand' analysis tools
    - MATLAB, Octave, MathCAD, Maple, Excel, etc...
    - Technology characterization charts and lookup tables
  - GUIs & graphics versus scripting
- The designer **uses the tools** to design the circuit
  - Tools don't do the design for you
  - Tools sometimes break, or are incompatible with each other
    - Search/read the manuals, learn to use tools correctly
      - Tips: ctrl + f, google, cdnshelp, CAD tutorial
- Pick the right tool for the job
  - Balance complexity, overhead, connection to the design objectives
    - This can be a very personal choice
  - Sometimes biting the bullet is better than fancy solutions

## Chapter 5 Summary

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- Revisited the three figures of merit for transistors, in the context of **design**
  - $g_m/I_D$ ,  $\omega_T$ ,  $g_m/g_{ds}$
  - Device characteristics that are directly linked to circuit performance
    - Used as tradeoff 'knobs' or indices to balance power efficiency, speed, gain
    - Powerful parameters to characterize a process technology
- Saw how the square law breaks down in practice
  - Weak/moderate/strong inversion, short channel effects
- Discussed  $g_m/I_D$ -based design using charts and lookup tables
  - Provides a systematic design/optimization methodology
    - Accurately estimate capacitances, gain, power dissipation
    - Minimize tweaking of W and L
- Worked through a differential pair design example
  - Good agreement between hand analysis and simulation
  - This was a basic design exercise: nothing to optimize

---

## Chapter 5 Learning Objectives

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- Learning objectives
  - Understand tradeoffs between speed, power efficiency, and gain
    - In transistors, and in circuits
  - Be able to use technology characterization tables/charts to size transistors
    - Get practice in the design project
  - Develop a systematic design methodology in the absence of simple analytical models like the square law equation

## References

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- F. Silveira et al. "A  $g_m/I_D$  based methodology for the design of CMOS analog circuits and its application to the synthesis of a silicon-on-insulator micropower OTA," IEEE J. Solid-State Circuits, Sep. 1996, pp. 1314-1319.
- D. Foty, M. Bucher, D. Binkley, "Re-interpreting the MOS transistor via the inversion coefficient and the continuum of  $g_{ms}/I_d$ ," Proc. Int. Conf. on Electronics, Circuits and Systems, pp. 1179-1182, Sep. 2002.
- B. E. Boser, "Analog Circuit Design with Submicron Transistors," IEEE SSCS Meeting, Santa Clara Valley, May 19, 2005, <http://www.ewh.ieee.org/r6/scv/ssc/May1905.htm>
- P. Jespers, The  $g_m/I_D$  Methodology, a sizing tool for low-voltage analog CMOS Circuits, Springer, 2010.
- T. Konishi, K. Inazu, J.G. Lee, M. Natsu, S. Masui, and B. Murmann, "Optimization of High-Speed and Low-Power Operational Transconductance Amplifier Using  $g_m/I_D$  Lookup Table Methodology," IEICE Trans. Electronics, Vol. E94-C, No.3, Mar. 2011.