PN Junction and MOS structure

Fig. 1.1 A cross section of a pn diode.

Fig. 1.6 A cross section of a typical n-channel transistor.
Basic electrostatic equations

• We will use simple one-dimensional electrostatic equations to develop insight and basic understanding of how semiconductor devices operate

  – Gauss's Law
  
  – Potential Equation
  
  – Poisson's Equation

It puts together Gauss's Law and the potential equation
Gauss’s Law

\[
\frac{dE}{dx} = \frac{\rho}{\epsilon}
\]

- \(E\) = electric field [V/m]
- \(\rho\) = charge density [C/m³]
- \(\epsilon\) = permittivity [F/m]
- \(\epsilon_x\) = permittivity in the region from \(x_a\) to \(x\)
- \(\epsilon_a\) = permittivity at the material interface

The possibility of a change in permittivity due to a material interface has been accounted for by keeping the permittivity together with the field.

\[
\int_{x_a}^{x} d \left[ \epsilon_x E(x) \right] = \epsilon_x E(x) - \epsilon_a E(x_a) = \int_{x_a}^{x} \rho(x) dx = Q_s(x)
\]
Potential Equation

\[ \phi(x) - \phi(x_R) = \int_{x_R}^{x} -E(x) \, dx \]

\[ E(x) = -\frac{d \phi(x)}{dx} \]
Poisson’s Equation

- It directly links the potential with the charge distribution (there is no need to go through the field)

\[
\begin{align*}
\frac{dE}{dx} &= \frac{\rho}{\epsilon} \\
E(x) &= -\frac{d\phi}{dx}
\end{align*}
\]

\[
\frac{d^2\phi(x)}{dx^2} = -\frac{dE(x)}{dx} = -\frac{\rho}{\epsilon}
\]
Boundary Conditions

- Electronic devices are made of layers of different materials
- We need conditions for $\Phi$ and $E$ at the boundary between two materials
Potential at a boundary

• An abrupt jump of $\Phi$ ("along x") would lead to an infinite electric field at the boundary

• Infinite electric fields are not possible (they would tear the material apart)

• Therefore $\Phi(x)$ must be continuous:

$$E(x) = -\frac{d\phi}{dx}$$

Where the boundary is located at $x = 0$
Electric Field at a boundary

- The electric field usually jumps at a boundary

\[ \int_{-\Delta}^{+\Delta} d [\epsilon E(x)] = \epsilon_2 E(x=+\Delta) - \epsilon_1 E(x=-\Delta) = \int_{-\Delta}^{+\Delta} \rho(x) \, dx \]

- By letting \( \Delta \rightarrow 0 \):

\[ \int_{-\Delta}^{+\Delta} \rho(x) \, dx = 0 \]

\[ \epsilon_2 E(x=0^+) - \epsilon_1 E(x=0^-) = 0 \]

\[ \epsilon_2 E(x=0^+) - \epsilon_1 E(x=0^-) = Q_S \]

There can be a sheet of charge at the boundary

A sheet of charge is an Infinite amount of charge all distributed on the boundary surface (that is a Dirac function)
Boundary between materials

Figure 3.4 (a) Boundary between materials 1 and 2 with permittivities $\varepsilon_1 > \varepsilon_2$ and (b) resulting jump in the electric field. (c) Boundary, with a surface charge $Q$ located at the interface and (d) resulting jump in the electric field.
Boundary conditions

\[ \phi(x = 0^+) = \phi(x = 0^-) \]

continuity of potential at a boundary

\[ +\Delta \int_{-\Delta} \rho(x) \, dx = 0 \]
\[ -\Delta \int_{-\Delta} \rho(x) \, dx = Q_s \]

electric field jump for charge free boundary

\[ E(x = 0^+) = \frac{\epsilon_1}{\epsilon_2} E(x = 0^-) \]

\[ E(x = 0^+) = \frac{\epsilon_1}{\epsilon_2} E(x = 0^-) + \frac{Q_s}{\epsilon_2} \]

electric field jump for charged boundary
Oxide-Silicon interface

• Example of very common interface in microelectronic devices

\[ E_{si}(0^+) = \frac{\varepsilon_{ox}}{\varepsilon_{si}} E_{ox}(0^-) \approx \frac{E_{ox}(0^-)}{3.0} \]

Permittivity of vacuum

\[ \varepsilon_{si} = 11.7 \cdot \varepsilon_0 \]
\[ \varepsilon_{ox} = 3.9 \cdot \varepsilon_0 \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ [F/m]} \]
Metal – Metal Capacitor

- In many IC processes there are two or more levels of metal separated by silicon oxide.
Metal – Metal Capacitor

![Diagram of Metal-Metal Capacitor]

**Figure Ex3.1B** Close-up of metal-metal capacitor with applied voltage, showing surface charges on top and bottom metal plates and electric field lines.
Metal – Metal Capacitor

• Since there is no charge present in the oxide — the electric field in the oxide is constant.

• Since the electric field in the oxide is constant and the voltage drop across the gap is $V$ — it follows that:

$$E_{ox} = \frac{V}{t_d}$$

$$E_{ox} = \frac{\varepsilon_{metal}}{\varepsilon_{ox}} E_{metal} + \frac{Q_S}{\varepsilon_{ox}}$$

$$\frac{V}{t_d} = \frac{Q_S}{\varepsilon_{ox}}$$

No electric field inside metals

$$C_S = \frac{dQ_S}{dV} = \frac{\varepsilon_{ox}}{t_d}$$

Capacitance per unit area $[F/m^2]$
Example

- Find potential, electric field and charge distribution for a metal-metal capacitor with \( t_d = 1 \) \( \mu \text{m} \) and an applied voltage of 1V
M-O-S Capacitor

Figure Ex3.3A  Metal-oxide-silicon capacitor: (a) layout and (b) cross section.
M-O-S charge distribution

\[ Q_B = \rho_0 \cdot X_d = -Q_G \]

\[ Q_G = C_{ox} \cdot V = \frac{\varepsilon_{ox}}{t_{ox}} V \]

\[ X_d = -\frac{\varepsilon_{ox} V}{t_{ox} \rho_0} \approx \frac{\varepsilon_{ox} V}{t_{ox} q N_A} \]
M-O-S charge density profile
The electric field is confined in the region $-t_{ox} < x < X_d$.

- In the metal the electric field is 0.
- In the oxide ($-t_{ox} < x < 0$) the charge density is zero ($\rho(x)=0$), therefore the electric field is constant $E(x)=E_{ox}$.

NOTE: the total excess charge in the region $-t_{ox} < x < X_d$ is zero.

For $-t_{ox} < x < 0$: \[ \frac{dE(x)}{dx} = \frac{\rho(x)}{\varepsilon_{ox}} = 0 \]
Outside the charged region of the silicon ($x > X_d$) the electric field is 0.

In the charged region of the silicon ($0 < x < X_d$) the charge density is constant ($\rho_0$) therefore the electric field is a linear function of $x$:

$$dE(x) = \frac{\rho_0}{\varepsilon_s} \, dx \quad \rightarrow \quad \varepsilon_s E(x_d) - \varepsilon_s E(0^+) = \rho_0 \int_{0^+}^{X_d} dx$$

$$E(0^+) = -\frac{\rho_0 X_d}{\varepsilon_s}$$

**NOTE:**
The electric field jumps only at the interface between two different materials.
• The boundary condition at the oxide/silicon interface is:

\[ \varepsilon_{ox} E_{ox} = \varepsilon_s E(0^+) \rightarrow E_{ox} = \frac{\varepsilon_s}{\varepsilon_{ox}} E(0^+) \approx 3 \]

\[ E_{ox} = -\frac{\rho_0 X_d}{\varepsilon_{ox}} \]
Potential plot through M-O-S

Potential is continuous at boundaries:
\( x = -t_{ox} \) metal/oxide boundary
\( x = 0 \) oxide/silicon boundary

\[ \frac{d^2 \phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon} \]

Poisson's Equation:

Potential at the metal gate:
\( \phi_m \equiv \phi(-t_{ox}) \)

Surface potential:
\( \phi_s \equiv \phi(0) \)

Substrate potential:
\( \phi_{sub} \equiv \phi(X_d) \)

Drop across the oxide:
\( V = \phi_m - \phi_{sub} = V_{ox} + V_B \)

Drop across the charged region of the silicon.
Surface potential

for \(-t_{ox} < x < 0\):

\[
\phi_m - \phi_s = \int_{-t_{ox}}^{0} (-E_{ox}) \, dx = E_{ox} \cdot t_{ox} \equiv V_o.
\]

\[
\phi_s = \phi_m - E_{ox} \cdot t_{ox} = \phi_m - \left( -\rho_0 \right) \frac{X_d}{\varepsilon_{ox}} t_{ox}
\]

\[
= \frac{Q_G}{C_{ox}} = -\frac{Q_B}{C_{ox}}
\]

\[
\varepsilon_{ox}
\]

\[
1/C_{ox}
\]

The drop across the oxide is proportional to the charge stored on each side of the oxide

\[
\phi_m - \phi_s = \int_{-t_{ox}}^{0} (-E_{ox}) \, dx = E_{ox} \cdot t_{ox} \equiv V_o.
\]

\[
= \frac{Q_G}{C_{ox}} = -\frac{Q_B}{C_{ox}}
\]

x is negative in the region considered

Linear

\[
\phi_m - \phi_s = \int_{-t_{ox}}^{0} (-E_{ox}) \, dx = E_{ox} \cdot t_{ox} \equiv V_o.
\]
Potential drop across the substrate

for $0 < x < X_d$:

$$\frac{d^2 \phi(x)}{dx^2} = -\frac{\rho_0}{\epsilon_s} \quad (> 0)$$

$$V_B \equiv \phi(0) - \phi(X_d) = \frac{1}{2} \left( -\frac{\rho_0}{\epsilon_s} \right) X_d^2 = \left( -\frac{\rho_0 X_d}{2\epsilon_s} \right) X_d = -\frac{Q_B}{2\epsilon_s} X_d$$

charged region of silicon

The potential is “concave up” in the charged region

Quadratic
PN Junction in Thermal Equilibrium

• If no external stimulus is applied (zero applied voltage, no external light source, etc) the device will eventually reach a steady state status of thermal equilibrium

• In this state (“open circuit” and steady state condition) the current density must be zero:

\[ J_{\text{tot},0} = J_{p,0} + J_{n,0} = 0 \]

• Eventually, the populations of electrons and holes are each in equilibrium and therefore must have zero current densities

\[
\begin{align*}
J_{p,0} &= 0 \\
J_{n,0} &= 0
\end{align*}
\]
Diffusion Mechanism

The charge on the two sides of the junction must be equal (charge neutrality).

Under “open circuit” and steady state conditions the built in electric field opposes the diffusion of free carriers until there is no net charge movement.

Fig. 1.2 A simplified model of a diode. Note that a depletion region exists at the junction due to diffusion and extends farther into the more lightly doped side.

We assumed the n-side is the more lightly doped.
Diffusion Current

• It’s a manifestation of thermal random motion of particles (statistical phenomenon)

• In a material where the concentration of particles is uniform the random motion balances out and no net movement result (drunk sail-man walk $\rightarrow$ Brownian walks)

• If there is a difference (gradient) in concentration between two parts of a material, statistically there will be more particles crossing from the side of higher concentration to the side of lower concentration than in the reverse direction.

• Then we expect a net flux of particles
**Diffusion Equations**

Charge in the cross section:

\[ I_n \propto A q_e \frac{dn}{dx} \]

The more non-uniform the concentration the larger the current.

\[ I_n = -D_n A q_e \frac{dn}{dx} = D_n A q \frac{dn}{dx} \]

Assuming the charge concentration decreases with increasing x, it means that \( \frac{dn}{dx} \) and \( \frac{dp}{dx} \) are negative, so to conform with conventions we must put a – sign in front of the equations.

Proportionality Constants

\[ J_n = D_n q \frac{dn}{dx} \]

\[ J_p = -D_p q \frac{dp}{dx} \]
Drift Current

\[ \vec{v}_n = -\mu_n \vec{E} \]
\[ \vec{v}_p = +\mu_p \vec{E} \]

Eventually \( v \) saturates
- too many collisions
- effective electrons' mass increases

\[ I_n = v_n W h n q_e \]
\[ I_p = v_p W h p q_h \]

\[ J_n = -\mu_n E n q_e = \mu_n E n q \]
\[ J_p = \mu_p E p q_h = \mu_p E p q \]

- cross section area
- charge per unit of volume
- volume travelled per unit of time
Drift and Diffusion currents

**Drift Current**

\[ J_{n,\text{drift}} = -\mu_n E n q_e = \mu_n E n q \]
\[ J_{p,\text{drift}} = \mu_p E p q_h = \mu_p E p q \]

\[ q = q_h = -q_e = 1.6 \times 10^{-19} \text{ C} \]

**Diffusion Current**

\[ J_{n,\text{diff}} = -D_n q_e \frac{dn}{dx} = D_n q \frac{dn}{dx} \]
\[ J_{p,\text{diff}} = -D_p q_h \frac{dp}{dx} = -D_p q \frac{dp}{dx} \]
Built in Voltage

- At equilibrium:
  (drift and diffusion balance out)

\[
J_n = J_{n,\text{drift}} + J_{n,\text{diff}} = \mu_n E n q + D_n q \frac{dn}{dx} = 0
\]

\[
J_p = J_{p,\text{drift}} + J_{p,\text{diff}} = \mu_p E p q - D_p q \frac{dp}{dx} = 0
\]

- Let's consider the second equation:

\[
\int_{V(x_n)}^{V(x_p)} dV = D_p \int_{p(x_n)}^{p(x_p)} \frac{dp}{p} \rightarrow V(x_p) - V(x_n) = -\mu_p \int_{p(x_n)}^{p(x_p)} \frac{dp}{p} = \frac{D_p}{\mu_p} \ln \left( \frac{p(x_p)}{p(x_n)} \right) \rightarrow
\]
Built in Voltage

\[ \phi_0 \equiv V(x_n) - V(x_p) = \frac{D_p}{\mu_p} \ln \left( \frac{p(x_p)}{p(x_n)} \right) \]

\( n_n : \text{Concentration of electrons on n side} \approx N_D \ (e.g. 10^{17}) \)

\( p_n : \text{Concentration of holes on n side} \approx n_i^2 / N_D \)

\( p_p : \text{Concentration of holes on p side} \approx N_A \ (e.g. 10^{18}) \)

\( n_p : \text{Concentration of electrons on p side} \approx n_i^2 / N_A \)

Since both \( \mu \) and \( D \) are manifestations of thermal random motion (i.e. statistical thermodynamics phenomena) they are not independent

**Einstein's Relation:**

\[ \frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{K}{q} = V_T \]

**Mass – Action Law:**

\[ n \cdot p \equiv n_i^2 \]

If \( n \uparrow \) then \( p \downarrow \)
A larger number of electrons causes the recombination rate of electrons with holes to increase
Applying KVL to the PN junction in equilibrium

We cannot have current. Something is wrong!

\[ I = \frac{\Phi_0}{R} \]
If a free electron in the P region or a hole in the N region somehow reach the edge of the depletion region get swept by the electric field (drift)
Neutrality of charge

- There are 4 charged particles in silicon, two mobiles (holes and electrons) and two fixed (ionized donors and ionized acceptors).
- The total positive charge density and the total negative charge must be equal.

\[ N_D + p = N_A + n \]
Depletion region in equilibrium

- The doping concentrations $N_A$ on the p side and $N_D$ on the n side are assumed constants.

\[ \rho(x) = q \left( p - n + N_D - N_A \right) \]

The depletion region is free of electrons and holes:

\[ N_D - N_A \gg n - p \]

\[ \rho(x) \approx q(N_D - N_A) \]

* On the P side $N_D = 0$:

\[ \rho(x) = -qN_A \]

* On the N side $N_A = 0$:

\[ \rho(x) = qN_D \]

Positive and negative excess charge in the depletion region must balance out (neutrality of charge):

\[ qN_D x_n = qN_A x_p \]

**Gauss' Law:**

\[ \frac{dE(x)}{dx} = \frac{\rho(x)}{\varepsilon_s} \]

The depletion region is free of electrons and holes on the P side $N_D = 0$:

\[ qNA x_p \]

on the N side $N_A = 0$:

\[ qND x_n \]
Depletion region in equilibrium

Potential Equation:
\[ d\phi = -E(x)\,dx \]

Equation:
\[ E_{\text{max}} = -qN_A x_p / \varepsilon_s = -qN_D x_n / \varepsilon_s \]

\[ \frac{1}{2} |E_{\text{max}}| x_n = \frac{1}{2} \frac{qN_D x_n^2}{\varepsilon_s} \iff \text{Area Triangle 0} \overline{E_{\text{max}}} x_n \]

\[ \frac{1}{2} |E_{\text{max}}| x_p = \frac{1}{2} \frac{qN_A x_p^2}{\varepsilon_s} \iff \text{Area Triangle 0} \overline{E_{\text{max}}} x_p \]

\[ \phi_0 = \frac{1}{2} \frac{qN_D}{\varepsilon_s} x_n^2 + \frac{1}{2} \frac{qN_A}{\varepsilon_s} x_p^2 \]

\[ N_D x_n = N_A x_p \]

\[ x_n = \frac{N_A}{N_D} x_p \]

\[ x_p = \frac{N_D}{N_A} x_n \]
Depletion region in equilibrium

\[
\begin{aligned}
\begin{align*}
x_n &= \frac{N_A}{N_D} x_p \\
x_p &= \frac{N_D}{N_A} x_n
\end{align*}
\end{aligned}
\Rightarrow X_{dep} = x_n + x_p = \left(1 + \frac{N_A}{N_D}\right) x_p = \left(1 + \frac{N_D}{N_A}\right) x_n
\]

\[
x_p = X_{dep} \left(\frac{N_D}{N_A + N_D}\right) \quad \text{and} \quad x_n = X_{dep} \left(\frac{N_A}{N_A + N_D}\right)
\]

\[
\phi_0 = \frac{1}{2} \frac{q N_D}{\epsilon_s} x_n^2 + \frac{1}{2} \frac{q N_A}{\epsilon_s} x_p^2 = \frac{1}{2} \frac{q N_D}{\epsilon_s} X_{dep}^2 \left(\frac{N_A}{N_A + N_D}\right)^2 + \frac{1}{2} \frac{q N_A}{\epsilon_s} X_{dep}^2 \left(\frac{N_D}{N_A + N_D}\right)^2 = \frac{1}{2} \frac{q X_{dep}^2}{2 \epsilon_s} \frac{N_A N_D}{N_A + N_D}
\]
Width and max field of the depletion region in equilibrium

\[ X_{dep} = \sqrt{\frac{2 \epsilon_s}{q} \frac{N_A + N_D}{N_A \bar{N}_D}} \phi_0 \]

with: \( \phi_0 = \frac{KT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \)

\[ |E_{max}| = \frac{q N_A}{\epsilon_s} x_p = \frac{q N_A}{\epsilon_s} X_{dep} \frac{N_D}{N_A + N_D} = \frac{q}{\epsilon_s} \frac{N_A N_D}{N_A + N_D} X_{dep} \]

\[ |E_{max}| = \sqrt{\frac{2 \epsilon_s}{q} \frac{N_A N_D}{N_A + N_D}} \phi_0 \]
Biased PN Junction

\[ v + \phi_{pm} + \phi_{mn} + v_j = 0 \]

\[ v_j = \phi_0 - v \]

\[ E(x) \]

\[ x \]

\[ v > 0 \]

\[ v = 0 \]

\[ v < 0 \]
Biased PN Junction

- if we keep decreasing the voltage eventually we'll break the material. For silicon the breakdown point is reached for an electric field of approx. 10^7 V/m. **NOTE: the depletion region can't get bigger than the length of the bar!**

- if we keep increasing the voltage the depletion region will disappear (v=Φ₀). As v becomes comparable with Φ₀ the PN junction behave like a sort of resistor (the current is determined by the ohmic contacts and the resistance of the semiconductor)
Reverse Biased PN Junction

- Under reverse bias the depletion region becomes wider
- Then, it gets harder for the majority carriers to cross (diffuse through) the junction and easier for the minority carrier to be swept (drifted) across the junction
- Since there are only a FEW minority carriers, the current carried under reverse bias is negligible
Reverse Biased PN Junction

NOTE:

As soon as a minority carrier, let's say an electron on the P side, is swept across the junction, on the N side it becomes a majority carrier. Every time a minority carrier on the P side is swept toward the N side it leaves one less minority carrier on the edge of the depletion region in the P region. The same is true for holes swept from the N side to the P side.

Under reverse bias the minority carrier concentration at the edges of the depletion regions is depleted below their equilibrium value. Since the number of minority carriers is small anyway this won't be a major difference.
Forward Biased PN Junction

- Under forward bias the depletion region shrinks
- Then, it gets easier for the majority carriers to cross (diffuse through) the junction and harder for the minority carrier to be swept (drifted) across the junction.
- Since there are a LOT of majority carriers we expect the current to be considerable
Forward Biased PN Junction

NOTE:
As soon as a majority carrier (let's say a hole on the P side) crosses the junction it becomes a minority carrier. Thus at the edge of the N side of the depletion region we have an excess of minority carriers compared with the concentration of minority carriers on the rest of the N region far from the junction. This gradient causes a considerable diffusion current. The same is true for electrons crossing from the N side to the P side.

As VF is increased the excess minority concentration is increased. If VF = 0 (equilibrium) there is no excess minority concentration.
I/V characteristic of PN Junction

\[ I_{\text{diod}} = \left( q \frac{A D_n}{L_n} \frac{n_i^2}{N_A} + q \frac{A D_p}{L_p} \frac{n_i^2}{N_D} \right) \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) = I_s \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]

with:

- \( A \) = cross section area of the diode
- \( \frac{n_i^2}{N_D} \) = holes' concentration in the N region (minority carriers)
- \( \frac{n_i^2}{N_A} \) = electrons' concentration in the P region (minority carriers)
- \( D_p \) = diffusion constant for the holes in the N region
- \( L_p \) = diffusion length for the holes in the N region = \( \sqrt{D_p \tau_p} \)
- \( \tau_p \) = average time it takes for a hole into the N region to recombine with a majority electron

...
I/V characteristic of PN Junction

Since the only region where we have “net charge” is between $-x_p$ and $x_n$ such region (a.k.a. space charge region) is the only one where there is electric field.

The regions from A to $-x_p$ and from $x_n$ to K are quasi-neutral (it is like they were perfect conductors and in perfect conductors there is no electric field inside)
The situation is similar to the one at equilibrium but now the “built in voltage” is $\Phi_0 - V_{\text{diode}}$ instead of $\Phi_0$. 
I/V characteristic of PN Junction

- Under forward bias close to the depletion edges we have:
  - a greater hole concentration than normal on the N side (minority carriers)
  - a greater electrons concentration than normal on the P side (minority carriers)

\[ n_{p0} = \frac{n_i^2}{N_A} \]

In the P region we have a lot of holes that will diffuse toward the N region

In the N region we have a lot of electrons that will diffuse toward the P region
I/V characteristic of PN Junction

Extending the result derived at equilibrium we can write the voltage across the space charge region (between $-x_p$ and $x_n$) as:

$$V_j \equiv V(x_n) - V(-x_p) = \phi_0 - V_{\text{diode}} = V_T \ln \left( \frac{p(-x_p)}{p(x_n)} \right) = V_T \ln \left( \frac{n(x_n)}{n(-x_p)} \right)$$

\[
\begin{align*}
\frac{\phi_0 - V_{\text{diode}}}{V_T} &= \ln \frac{p(-x_p)}{p(x_n)} \\
\frac{\phi_0 - V_{\text{diode}}}{V_T} &= \ln \frac{n(x_n)}{n(-x_p)}
\end{align*}
\]

\[
\begin{align*}
p(x_n) &= \frac{p(-x_p)}{\frac{\phi_0 - V_{\text{diode}}}{V_T}} = \frac{p(-x_p)}{\frac{\phi_0}{V_T}} e^{\frac{V_{\text{diode}}}{V_T}} \\
n(-x_p) &= \frac{n(x_n)}{\frac{\phi_0 - V_{\text{diode}}}{V_T}} = \frac{n(x_n)}{\frac{\phi_0}{V_T}} e^{\frac{V_{\text{diode}}}{V_T}}
\end{align*}
\]
I/V characteristic of PN Junction

- And noting that:
  
  - at the boundary of the quasi neutral P region at -Xp the hole density (majority carriers) is approximately equal at equilibrium as well as under bias,
  
  - and the same is true for the electron density (majority carriers) at the boundary of the quasi neutral N region (at Xn)

\[
\frac{p(-x_p)}{e^{\frac{\phi_0}{V_T}}} \approx \frac{p_{p0}}{e^{\frac{\phi_0}{V_T}}} = n_{p0} = \frac{n_i^2}{N_D}
\]

\[
\frac{n(x_n)}{e^{\frac{\phi_0}{V_T}}} \approx \frac{n_{n0}}{e^{\frac{\phi_0}{V_T}}} = n_{p0} = \frac{n_i^2}{N_A}
\]

the concentration of the majority carriers in the quasi neutral regions is approximately the same as the concentration at equilibrium
I/V characteristic of PN Junction

Thus:

\[
\begin{align*}
p(x_n) &= \frac{p(-x_p)}{\phi_0 - e V_{\text{diode}}} = \frac{p(-x_p)}{V_T} \frac{V_{\text{diode}}}{e} \\
n(-x_p) &= \frac{n(x_n)}{\phi_0 - e V_{\text{diode}}} = \frac{n(x_n)}{V_T} \frac{V_{\text{diode}}}{e}
\end{align*}
\]

\[
\begin{align*}
p(x_n) &\approx \frac{n_i^2}{N_D} e^{\frac{V_{\text{diode}}}{V_T}} \\
n(-x_p) &\approx \frac{n_i^2}{N_A} e^{\frac{V_{\text{diode}}}{V_T}}
\end{align*}
\]
I/V characteristic of PN Junction

\[ I_{\text{diode}} = I_n + I_p = I_{n,\text{drift}} + I_{n,\text{diff}} + I_{p,\text{drift}} + I_{p,\text{diff}} \]

• If we consider the quasi neutral regions, since in the quasi neutral regions there is no field there will be no drift

\[ I_n \approx I_{n,\text{diff}} \quad (\text{neutral region } W_N) \]
\[ I_p \approx I_{p,\text{diff}} \quad (\text{neutral region } W_P) \]

• Then, the most suitable traverse sections for the evaluation of the total current \( I_{\text{diode}} \) are those at the boundary of the depletion layer \((x=-x_p \text{ or } x=x_n)\)

\[ I_{\text{diode}} = I_n(-x_p) + I_p(-x_p) \approx I_{n,\text{diff}}(-x_p) + I_{p,\text{diff}}(-x_p) \]
I/V characteristic of PN Junction

- If we make the simplifying assumption that the flow of the carriers in the depletion region is approximately constant (in other words we assume the recombination in the depletion region is negligible)

\[ I_{p,\text{diff}}(-x_p) \approx I_{p,\text{diff}}(x_n) \]

\[ \downarrow \]

\[ I_{\text{diode}} \approx I_{n,\text{diff}}(-x_p) + I_{p,\text{diff}}(-x_p) \approx I_{n,\text{diff}}(-x_p) + I_{p,\text{diff}}(x_n) \]

- The currents due to diffusing carriers moving away from the junction are given by the well know diffusion equations:

\[ I_{n,\text{diff}}(x) = qA D_n \frac{dn_p(x)}{dx} \]

\[ I_{p,\text{diff}}(x) = -qA D_p \frac{dp_n(x)}{dx} \]
I/V characteristic of PN Junction

- If we assume that the carriers distribution is linear (SHORT DIODE)

\[
\frac{dn_p(x)}{dx} \bigg|_{-x_p} = \frac{n_p(-x_p) - n_p(A)}{W_p} = - \frac{n_{p0} e^{V_{\text{diode}}/V_T} - n_{p0}}{W_p} = \frac{n_{p0} e^{V_{\text{diode}}/V_T} - 1}{W_p}
\]

\[
\frac{dp_n(x)}{dx} \bigg|_{x_n} = - \frac{p_n(x_n) - p_n(K)}{W_n} = - \frac{p_{n0} e^{V_{\text{diode}}/V_T} - p_{n0}}{W_n} = - \frac{p_{n0} e^{V_{\text{diode}}/V_T} - 1}{W_n}
\]
PN Junction: I/V characteristic

- Recalling that:

\[ n_{p0} = \frac{n_i^2}{N_A} \]  
\[ p_{n0} = \frac{n_i^2}{N_D} \]

(electrons are minority carriers in P)

(holes are minority carriers in N)

\[ \frac{dn_p(-x_p)}{dx} = \frac{n_{p0}(e^{\frac{V_{\text{diode}}}{V_T}} - 1)}{W_p} = \frac{n_i^2}{N_A}(e^{\frac{V_{\text{diode}}}{V_T}} - 1) \]

\[ \frac{dp_n(x_n)}{dx} = -\frac{p_{n0}(e^{\frac{V_{\text{diode}}}{V_T}} - 1)}{W_n} = -\frac{n_i^2}{N_D}(e^{\frac{V_{\text{diode}}}{V_T}} - 1) \]

\[ I_{n, \text{diff}}(-x_p) = q AD_n \frac{dn_p(-x_p)}{dx} = q A n_i^2 \frac{D_n}{N_A W_p} \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]

\[ I_{p, \text{diff}}(x_n) = -q AD_p \frac{dp_n(x_n)}{dx} = q A n_i^2 \frac{D_p}{N_D W_n} \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]
PN Junction: I/V characteristic

- And finally:

\[ I_{n,\text{diff}}(-x_p) = q \, A \, n_i^2 \, \frac{D_n}{N_A \, W_p} \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]

\[ I_{p,\text{diff}}(x_n) = q \, A \, n_i^2 \, \frac{D_p}{N_D \, W_n} \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]

\[ I_{\text{diode}} \approx I_{n,\text{diff}}(-x_p) + I_{p,\text{diff}}(-x_p) = q \, A \, n_i^2 \left( \frac{D_n}{N_A \, W_p} + \frac{D_p}{N_D \, W_n} \right) \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]

- In the case of a **LONG DIODE** the minority carriers will recombine before reaching the diode terminals
PN Junction: I/V characteristic

**SHORT DIODE:**

\[ I_{\text{diode}} \approx I_{n,\text{diff}}(-x_p) + I_{p,\text{diff}}(-x_p) = qA n_i^2 \left( \frac{D_n}{N_A W_p} + \frac{D_p}{N_D W_n} \right) \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]

**LONG DIODE:**

\[ I_{\text{diode}} \approx I_{n,\text{diff}}(-x_p) + I_{p,\text{diff}}(-x_p) = qA n_i^2 \left( \frac{D_n}{N_A L_n} + \frac{D_p}{N_D L_p} \right) \left( e^{\frac{V_{\text{diode}}}{V_T}} - 1 \right) \]

Where \( L_n \) is a constant known as the diffusion length of electrons in the P side and \( L_p \) is a constant known as the diffusion length for holes in the N side. The constants \( L_n \) and \( L_p \) are dependent on the doping concentrations \( N_A \) and \( N_D \) respectively.
Diode Capacitances

- Depletion Capacitance (= Junction Capacitance) \( C_j \)
- Diffusion Capacitance \( C_d \)

- Reverse Biased Diode
  - Depletion Capacitance

- Forward Biased Diode
  - Diffusion Capacitance + Depletion Capacitance
Depletion Charge

- The depletion region stores an immobile charge of equal amount on each side of the junction (it forms a capacitance !!)

\[ q_J = q_P = -q N_A x_p A = -q N_N x_n A \]

N side has “+” ions

P side has “−” ions

\[ x_p = X_{dep} \left( \frac{N_D}{N_A + N_D} \right) \quad \text{and} \quad x_n = X_{dep} \left( \frac{N_A}{N_A + N_D} \right) \]

\[ q_J = -q \frac{N_A N_D}{N_A + N_D} A X_{dep} \]

\[ X_{dep} = \sqrt{\frac{2 \varepsilon_s}{q} \frac{N_A + N_D}{N_A \cdot N_D} (\phi_0 - v_D)} \]

The charge of the depletion region is a function of the voltage \( v_D \) applied to the diode.

The decision to take \( q_J \) negative is totally arbitrary. (But it turns out to be a good one if we prefer to work with positive capacitances)
Depletion Capacitance

• Since the depletion charge does not change linearly with the applied voltage the resulting capacitor is non linear !!

\[ q_J(v_D) = -A \sqrt{2q \varepsilon_s \frac{N_A \cdot N_D}{N_A + N_D}} (\phi_0 - v_D) \]

• An important physical consideration: we are dealing with a capacitor that no matter where I put the + of the applied voltage it always accumulate the positive charge on the N side of the junction, and the negative charge on the P side of the junction.
Small Signal Depletion Capacitance

- For small changes of the applied voltage about a specified DC voltage $V_D$ we can derive an equivalent linear capacitor approximation.

$$q_J(v_D) - q_J(V_D) = \left( \frac{dq_J}{dv_D} \right)_{V_D} \times (v_D - V_D)$$

since $q_J$ vs. $v_D$ relationship is non linear

$$q_J \propto \sqrt{(-v_D)}$$

$$q_J = \frac{dq_J}{dv_D} \bigg|_{V_D} \times v_d$$

Common conventions:
- total “quantities” (applied)
- DC “quantities”
Small Signal Depletion Capacitance

\[ C_j \equiv C_j(V_D) = \frac{d q_J}{d v_D} \bigg|_{V_D} = \frac{d}{d v_D} \left[ -A \left( 2 q \epsilon_s \frac{N_A \cdot N_D}{N_A + N_D} \left( \phi_0 - v_D \right)^{1/2} \right) \right]_{V_D} = \]

\[ = -A \left( 2 q \epsilon_s \frac{N_A \cdot N_D}{N_A + N_D} \right)^{1/2} \left( \frac{d}{dv_D} \left( \phi_0 - v_D \right)^{1/2} \right)_{V_D} = \]

\[ = A \left( \frac{q \epsilon_s}{2} \frac{N_A \cdot N_D}{N_A + N_D} \right)^{1/2} \left( \phi_0 - V_D \right)^{-1/2} = A \left( \frac{2}{q \epsilon_s} \frac{N_A + N_D}{N_A \cdot N_D} \left( \phi_0 - V_D \right) \right)^{-1/2} = \]

\[ = \frac{A}{\sqrt{\frac{2}{q \epsilon_s} \frac{N_A + N_D}{N_A \cdot N_D} \phi_0 \left( 1 - \frac{V_D}{\phi_0} \right)}} = C_{j0} \]
Small Signal Depletion Capacitance

Zero Bias Capacitance = junction capacitance in thermal equilibrium ($V_D = 0$)

$$C_{j0} = C_j(V_D = 0) \equiv \frac{A}{\sqrt{\frac{2}{q \varepsilon_s} \frac{N_A + N_D}{N_A \cdot N_D}}} \phi_0 = \frac{A \varepsilon_s}{X_{dep,0}}$$

$$X_{dep,0} = \sqrt{\frac{2 \varepsilon_s}{q \varepsilon_s} \frac{N_A + N_D}{N_A \cdot N_D}} \phi_0 = \varepsilon_s \sqrt{\frac{2}{q \varepsilon_s} \frac{N_A + N_D}{N_A \cdot N_D}} \phi_0$$

$$C_j \equiv C_j(V_D) = \frac{q_j}{V_d} = \frac{dq_j}{dV_D} \bigg|_{V_D} = \frac{C_{j0}}{\sqrt{(1 - \frac{V_D}{\phi_0})}} = \frac{A \varepsilon_s}{X_{dep}}$$
Small Signal Depletion Capacitance: Physical Interpretation

$$C_j = \frac{q_j}{v_d} = \frac{A \varepsilon_s}{X_{\text{dep}}}$$

Capacitance of a parallel plate capacitor with its plates separated by the depletion width $X_{\text{dep}}(V_D)$ at the particular DC voltage $V_D$.

The charges separated by $X_{\text{dep}}$ are the small signal charge layers $\pm q_j$.

For $v_d \rightarrow 0$, the small signal charges become sheets that are separated by a gap width of exactly $X_{\text{dep}}$. 
Physical Interpretation

Figure 3.19 (a) Charge density $\rho(x)$ in depletion region for a reverse bias of $V_D < 0$, (b) charge density $\rho'(x)$ in depletion region for a perturbed reverse bias $V_D + v_d > V_D$, and (c) difference $\Delta \rho(x)$ between (b) and (a) showing incremental depletion charge $\pm q_j$ separated by approximately the depletion width $X_{dep}$. Note that the magnitude of $v_d$ is exaggerated in order to clarify its effect on the depletion region width.
Small Signal Depletion Capacitance

In the practice the depletion capacitance is usually provided per cross-section area:

\[ C_{j0} = C_j(V_D = 0) \equiv \frac{\varepsilon_s}{X_{dep,0}} = \frac{1}{\sqrt{\frac{2}{q} \varepsilon_s \frac{N_A + N_D}{N_A \cdot N_D} \phi_0}} \]

\[ C_j = C_j(V_D) \equiv \frac{dq_J}{dv_D} \bigg|_{V_D} = \frac{\varepsilon_s}{X_{dep}} = \frac{\varepsilon_s}{X_{dep,0} \sqrt{1 - \frac{V_D}{\phi_0}}} = \sqrt{\frac{C_{j0}}{(1 - \frac{V_D}{\phi_0})}} \]

NOTE: when the diode is forward biased with \( V_D = \phi_0 \) the equation for \( C_j \) “blows up” (i.e., is equal to infinity). As \( V_D \) approaches \( \phi_0 \) the assumption that the depletion region is free of charged carriers is no longer true.
Graded Junctions

• All the equations derived for the depletion capacitance are based on the assumption that the doping concentration change abruptly at the junction. Although this is a good approximation for many integrated circuits is not always true.

• More in general:

\[ C_j = \frac{C_{j0}}{\left(1 - \frac{V_D}{\phi_0}\right)^{M_j}} \]

• M\(j\) is a constant called grading coefficient and its value ranges from \(1/3\) to \(1/2\) depending on the way the concentration changes from the P to the N side of the junction
Large Signal Depletion Capacitance

- The equations for the depletion capacitance given before are valid only for small changes in the applied voltage.
- It is extremely difficult and time consuming to accurately take this non-linear capacitance into account when calculating the time to charge or discharge a junction over a large voltage change.
- A commonly used approximation is to calculate the charge stored in the junction for the two extreme values of applied voltage, and then through the use of $\Delta Q = C \Delta V$, calculate the average capacitance accordingly.

$$C_{j-av} = \frac{Q(V_2) - Q(V_1)}{V_2 - V_1}$$

The approximation is pessimistic.
Large Signal Depletion Capacitance

\[ Q(V) = \sqrt{2 q \varepsilon_s \frac{N_A \cdot N_D}{N_A + N_D} (\phi_0 - V)} = \sqrt{2 q \varepsilon_s \frac{N_A \cdot N_D}{N_A + N_D} \phi_0 \left(1 - \frac{V}{\phi_0}\right)} = 2 \phi_0 C_{j0} \sqrt{1 - \frac{V}{\phi_0}} \]

\[ C_{j0} = \sqrt{\frac{q \varepsilon_s}{2 \phi_0} \frac{N_A \cdot N_D}{N_A + N_D}} \]

\[ C_{j_{-av}} = \frac{|Q(V_2) - Q(V_1)|}{|V_2 - V_1|} = 2 \phi_0 C_{j0} \left| \frac{1 - \frac{V_2}{\phi_0}}{V_2 - V_1} - \frac{1 - \frac{V_1}{\phi_0}}{V_2 - V_1} \right| \]
Example

Find a rough approximation for the junction capacitance to be used to estimate the charging time of a reverse biased junction from 0V to 5V (or vice versa). Assume $\phi_0 \approx 0.9$ V

$$C_{j-av} = 2 \phi_0 C_{j0} \left( \left| \frac{\sqrt{1 - \frac{V_2}{\phi_0}} - \sqrt{1 - \frac{V_1}{\phi_0}}}{V_2 - V_1} \right| \right) \approx 2 \cdot 0.9 C_{j0} \left( \left| \frac{\sqrt{1 - \frac{-5}{0.9}} - 1}{-5 - 0} \right| \right)$$