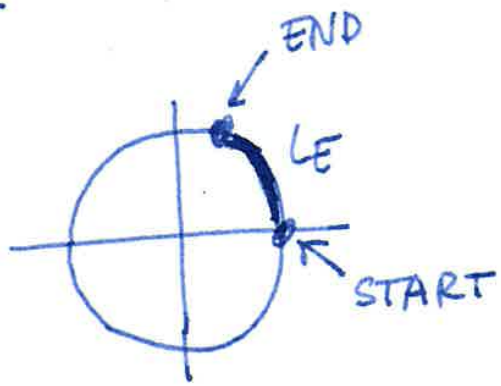
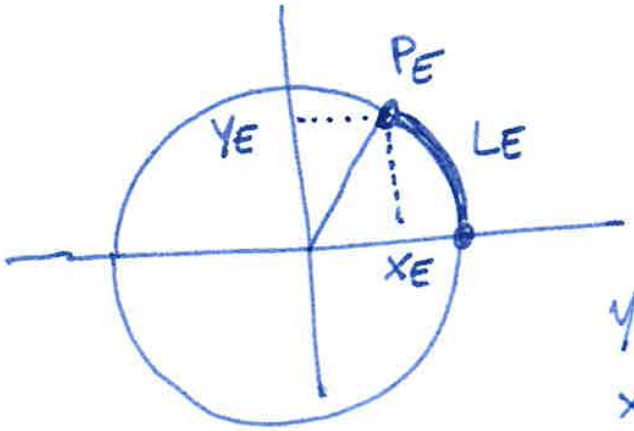
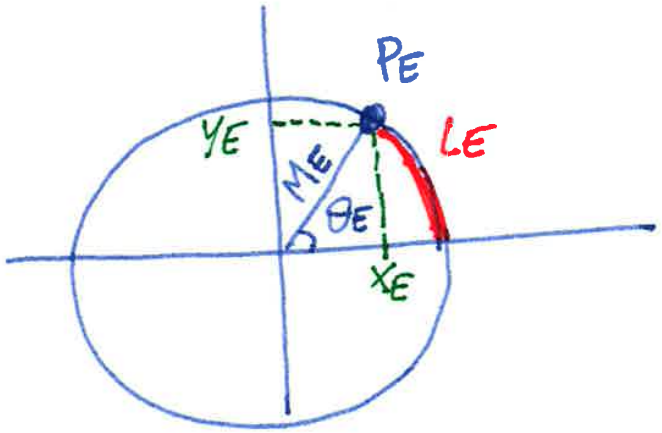
COMPLEX NUMBERS





$$y_E = \sin L_E$$

$$x_E = \cos L_E$$



$y_E = \sin L_E$

$x_E = \cos L_E$

CARTESIAN COORD. ≡ RECTANGULAR COORD.

↖ different names for same thing!

$2\pi = 360^\circ$

$L_E \leftrightarrow \theta_E$

$$\frac{L_E}{2\pi} = \frac{\theta_E}{360^\circ} \longleftrightarrow \boxed{\frac{L_E}{\pi} = \frac{\theta_E}{180^\circ}}$$

$$P_E = (x_E, y_E) = x_E + j\,y_E$$

$$P_E = M_E\, e^{j\theta_E}$$

$$M_E = \sqrt{x_E^2 + y_E^2}$$

$$\theta_E = \operatorname{atan} \frac{y_E}{x_E}$$

$y_E = \sin \theta_E$

$x_E = \cos \theta_E$

↓

$$\tan \theta_E = \frac{\sin \theta_E}{\cos \theta_E} = \frac{y_E}{x_E}$$

$$\theta_E = \operatorname{atan} \frac{y_E}{x_E}$$

ADDITION

$U = a + jb$
$V = c + jd$ $\longrightarrow$ $U + V = (a+c) + j(b+d)$

SUBTRACTION

$U - V = (a-c) + j(b-d)$

MULTIPLICATION

$U = a + jb = M_U \cdot e^{j\theta_U}$
$V = c + jd = M_V \cdot e^{j\theta_V}$ $\longrightarrow$ $U \cdot V = M_U \cdot M_V \, e^{j(\theta_U + \theta_V)}$

$U \cdot V = (a+jb)(c+jd) = ac + jbc + jad - bd = (ac - bd) + j(bc+ad)$

DIVISION

$U = a + jb = M_U \cdot e^{j\theta_U}$
$V = c + jd = M_V \cdot e^{j\theta_V}$ $\longrightarrow$ $\frac{U}{V} = \frac{M_U}{M_V} \cdot e^{j(\theta_U - \theta_V)}$

$$\frac{U}{V} = \frac{a+jb}{c+jd} = \frac{(a+jb)(c-jd)}{c^2+d^2} = \frac{ac + jbc - jad + bd}{c^2+d^2} =$$

$$= \frac{ac + bd + j(bc - ad)}{c^2+d^2}$$

complex numbers : Examples

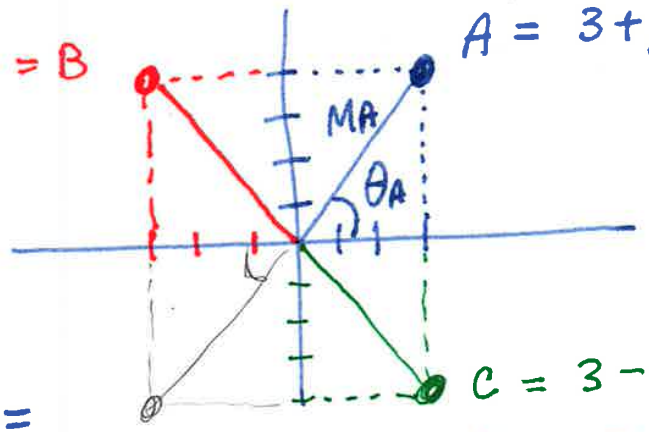
$-3+j4 = B$

$M_B = \sqrt{3^2+4^2} = 5$

$\theta_B = \operatorname{atan} \frac{4}{-3}$

$\theta_B = +180 - 53 =$

$= 127°$



$A = 3+j4$

$M_A = \sqrt{3^2+4^2} = 5$

$\theta_A = \operatorname{atan} \frac{4}{3} \approx 53°$

$C = 3-j4$

$M_C = 5$

$\theta_C = \operatorname{atan} \frac{-4}{3} =$

$\theta_C \cong -53°$

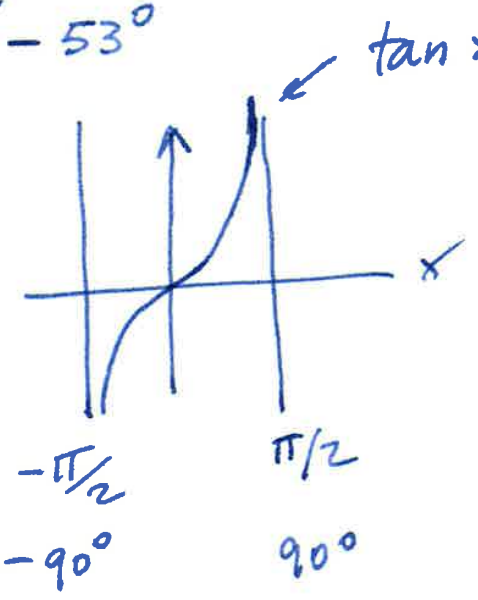
$D = -3-j4$

$M_D = 5$

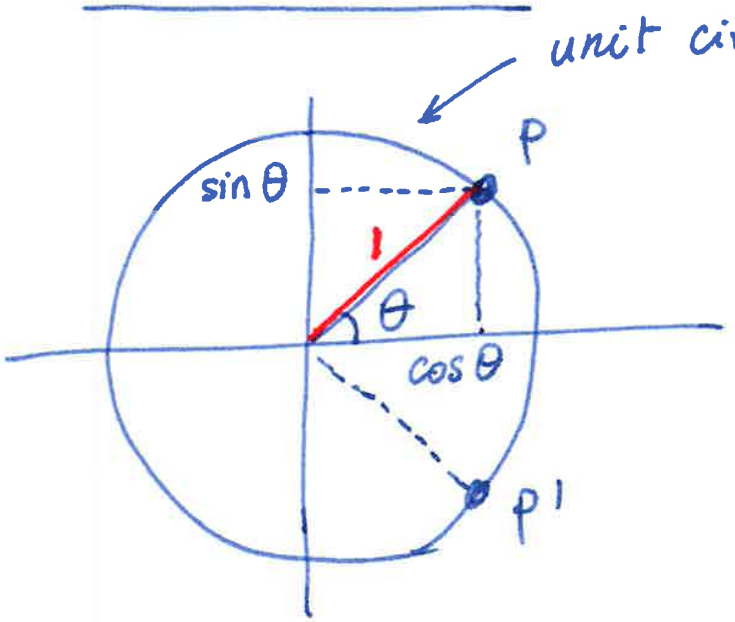
$\theta_D = -180+53 =$

$= -127°$

$\operatorname{atan} \frac{|Y_E|}{|X_E|}$



Euler's Formula



$$P: \quad \cos\theta + j\sin\theta = 1 \cdot e^{j\theta}$$

$$P': \quad \cos\theta - j\sin\theta = 1 \cdot e^{-j\theta}$$

$$2\cos\theta = e^{j\theta} + e^{-j\theta} \rightarrow$$

$$+2j\sin\theta = e^{j\theta} - e^{-j\theta} \rightarrow$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos\theta + j\sin\theta = e^{j\theta} \rightarrow$$

$$Me^{j\theta} = \underbrace{M\cos\theta} + j\underbrace{M\sin\theta} = A$$

$$RE\{A\} = M\cos\theta$$

$$IM\{A\} = M\sin\theta$$

angle in deg.	sin	cos	tan	angle in rad
0°	0	1	0	0 rad
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	$\pi/6$ rad.
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\pi/4$ rad.
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\pi/3$ rad.
90°	1	0	∞	$\pi/2$ rad.