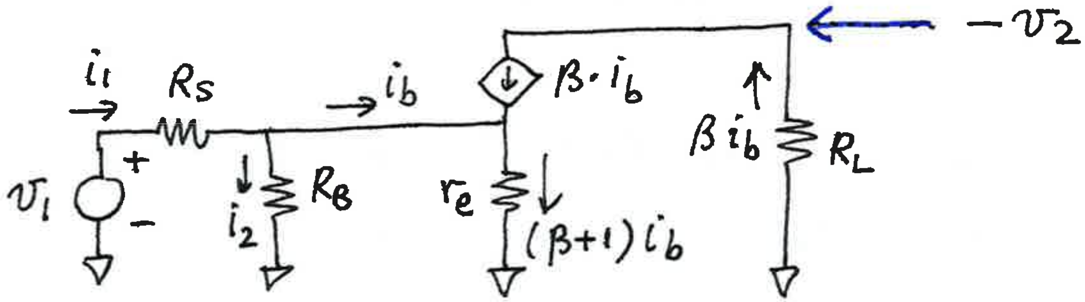
Low Entropy Expressions (the key to DOA)

- <u>Example #1</u> (conventional approach).
  Find voltage gain $A_V = v_2/v_1$



$$\begin{cases} v_1 = R_S i_1 + R_B i_2 \\ R_B \cdot i_2 = r_e (\beta+1) i_b \\ i_1 = i_2 + i_b \end{cases} \longrightarrow \begin{cases} R_S i_1 + R_B \cdot i_2 + \quad = v_1 \\ 0 + R_B i_2 - r_e (\beta+1) i_b = 0 \\ i_1 - i_2 - i_b = 0 \end{cases}$$

and $v_2 = R_L \cdot \beta \cdot i_b$ and $v_2 = R_L \beta i_b$

which leads to:

$$A_V = \frac{v_2}{v_1} = \frac{\beta R_B R_L}{(1+\beta) r_e R_S + (1+\beta) r_e R_B + R_S R_B}$$

HIGH ENTROPY EXPRESSION
(≡ insight: zero !!)

Slide set source: D. Middlebrook

```
% Claudio Talarico:
% common emitter amplifier (small signal T-model)
% bjtampl.m

clc; clear all; close all;
syms v1 v2 i1 i2 iB;
syms RS RB rE beta RL;

eqns = [v1 == RS*i1 + RB*i2; ...
    RB*i2 == rE*(beta+1)*iB; ...
    i1 == i2 + iB];
sol = solve(eqns, i1, i2, iB);
iB = sol.iB;
v2 = RL*beta*iB;
Av = v2/v1;
fprintf('Av = \n');
pretty(Av)
fprintf('\n');
```

```
Av =

                    RB RL beta
    ---------------------------------------------------
    RB RS + RB rE + RS rE + RB beta rE + RS beta rE
```

1bis.

Disadvantages of conventional approach

- No physical interpretation of the result
- Obscures how the element values affect the result
- Difficult to use for design: given the gain A_V (specification), how do you chose element values
- Purely algebraic derivation increases likelihood of mistakes

Lowering the entropy "post-mortem"

$$A_V = \frac{\beta R_B R_L}{(1+\beta) r_e R_S + (1+\beta) r_e R_B + R_S R_B} =$$

$$= \frac{\beta R_B R_L}{(1+\beta) r_e (R_S + R_B) + R_S R_B} =$$

factor $(1+\beta) r_e$

$$= \frac{R_B}{R_S + R_B} \cdot \frac{\beta \cdot R_L}{(1+\beta) r_e + \frac{R_S R_B}{R_S + R_B}} = \frac{R_B}{R_S + R_B} \cdot \frac{\beta \cdot R_L}{(1+\beta) r_e + R_S \| R_B} =$$

$$= \frac{R_B}{R_S + R_B} \cdot \frac{\alpha \cdot R_L}{r_e + \frac{R_B \| R_S}{1+\beta}} \approx \frac{R_B}{R_S + R_B} \cdot \frac{R_L}{r_e}$$

$\alpha \triangleq \frac{\beta}{\beta + 1} \approx 1$ for large β

(a) r_e

(b) $\frac{R_B \| R_S}{1+\beta}$

$\alpha \approx 1$

$\frac{R_B \| R_S}{1+\beta} << r_e$

Additional Information from the low entropy result

- The low entropy result provides the following additional information, not apparent from the high entropy version
  1. The $R_B/(R_S+R_B)$ factor is identified as a voltage divider
  2. Resistances appear in series/parallel combinations, so it is clear which ones are dominant
  3. The relative values of the two terms labeled (a) and (b) determine the sensitivity of the gain A_V to variations of β

- The additional information makes possible a better informed choice of the element values

Advantages of the low entropy form of the result

- Direct physical interpretation of the result
- Clarifies how the element values affect the result
- Easy to use for design: given the gain A_V (specification), how do you chose element values ?
- What about the algebra ???
  - If we try to lower entropy post-mortem is even harder. But …

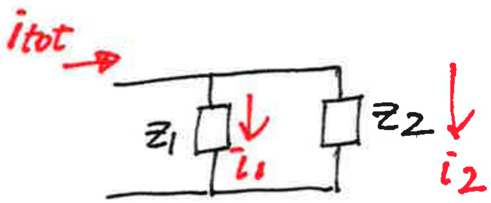
How can we get low entropy expressions ?

- It is easier to keep the entropy low from the start of the analysis than it is to lower the entropy once it has increased

- **The solution is: doing the algebra on the circuit**
  - Reflection of impedances
  - Current and voltage dividers
  - Loop and Node removal
    - Every time we do a Thevenin's reduction, one loop is removed from the circuit
    - Every time we do a Norton's reduction, one node is removed from the circuit

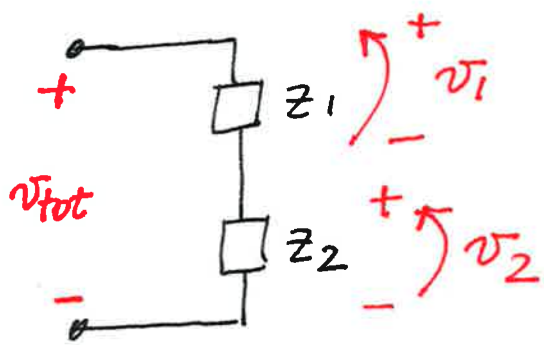
Current and voltage dividers

current divider



$$\frac{i_2}{i_{tot}} = \frac{z_1}{z_1 + z_2}$$

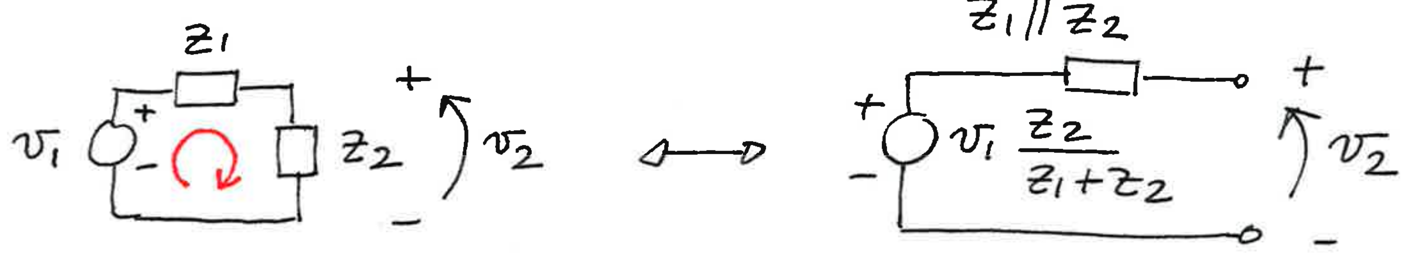
voltage divider



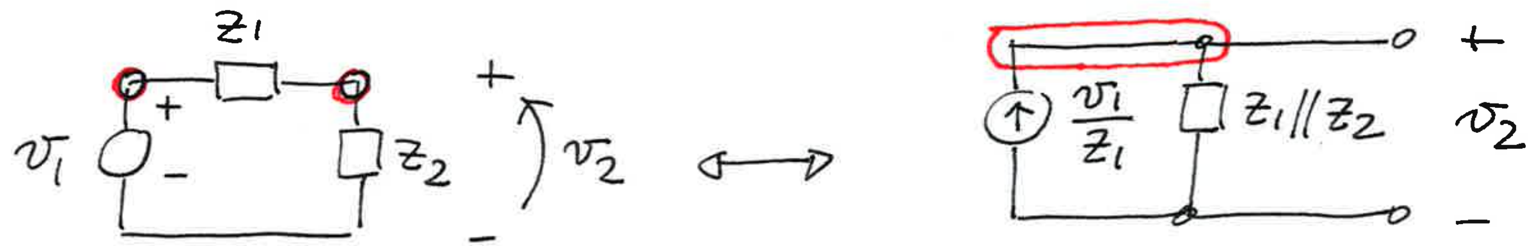
$$\frac{v_2}{v_{tot}} = \frac{z_2}{z_1 + z_2}$$

Loop and node removal

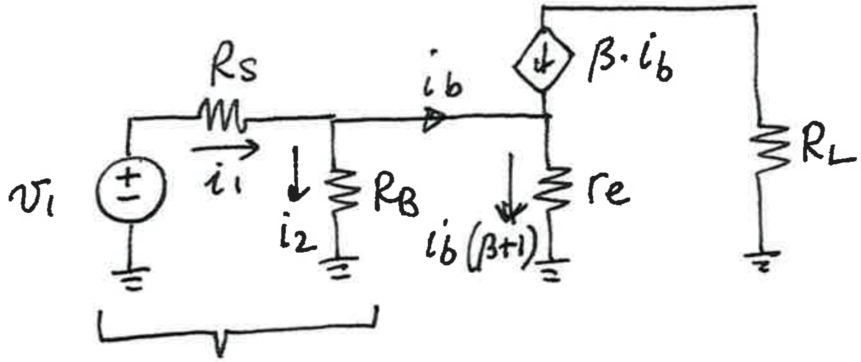
- Every time a Thevenin's reduction is used, one loop is removed from the circuit



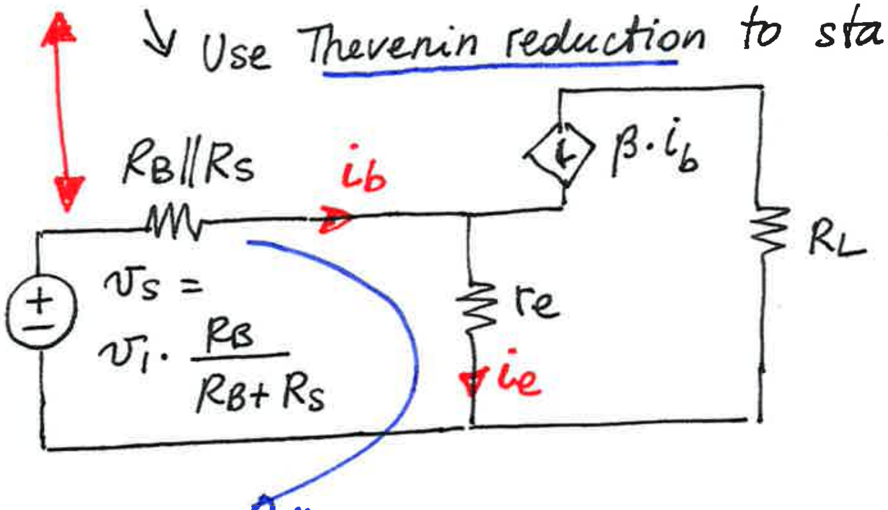
- Every time a Norton's reduction is used, one node is removed from the circuit

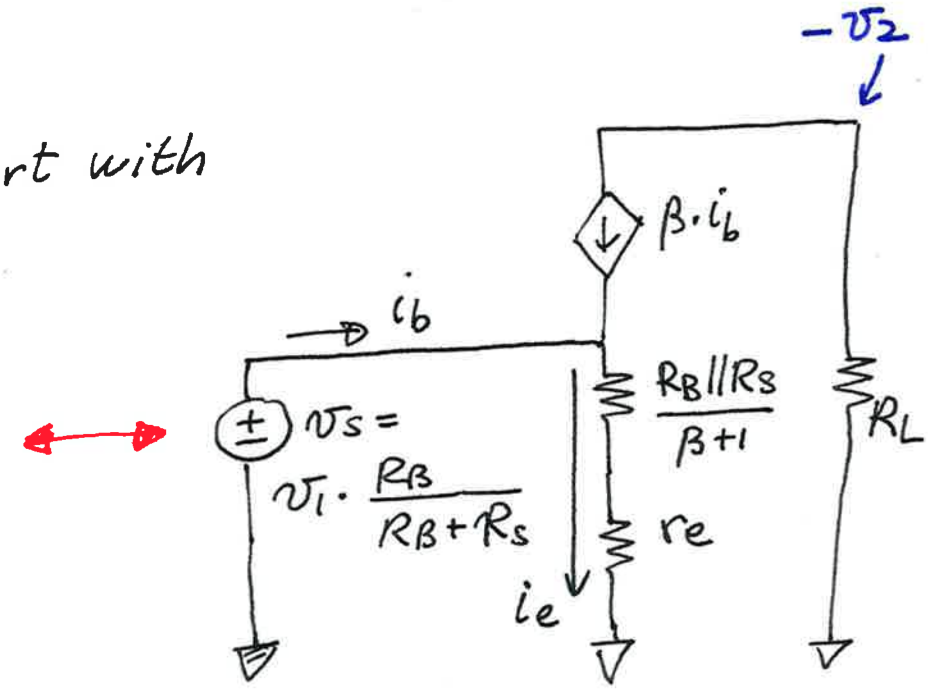


Example #1 done right !



Use Thevenin reduction to start with





"Reflect" $R_B||R_S$ to the R_E side

$i_E = i_b(\beta+1)$

↕

$i_b = \frac{i_e}{\beta+1}$

$$i_e = v_S \cdot \frac{1}{r_e + \frac{(R_S||R_B)}{1+\beta}}$$

$$v_2 = R_L \cdot \beta \cdot i_b = R_L \frac{\beta}{\beta+1} \cdot i_e$$

Example #1 done right !

$$\frac{v_2}{v_S} = R_L \cdot \underbrace{\frac{\beta}{\beta+1}}_{=\alpha} \cdot i_e \cdot \frac{1}{i_e} \cdot \frac{1}{r_e + \frac{R_S \| R_B}{\beta+1}} =$$

$$= \alpha \cdot R_L \cdot \frac{1}{r_e + \frac{R_B \| R_S}{\beta+1}}$$

$$\frac{v_2}{v_1} = A_V = \frac{v_2}{v_S} \cdot \underbrace{\frac{v_S}{v_1}}_{= \frac{R_B}{R_B+R_S}} = \frac{R_B}{R_B+R_S} \cdot \frac{\alpha \cdot R_L}{r_e + \frac{R_B \| R_L}{\beta+1}} =$$

$$\approx \frac{R_B}{R_B+R_S} \cdot \frac{R_L}{r_e}$$

← LOW ENTROPY RESULT !!

↑ β large, $r_e \gg \frac{R_B \| R_L}{\beta+1}$

Advantages of doing the algebra on the circuit

- Simultaneous solution of multiple equations is replaced by **sequential simple steps**

- The element values in the successive reduced models **automatically** appear in useful grouped combinations (facilitate tradeoffs)

- Less likelihood of making algebraic mistakes

- Because the **physical origin** of all terms in the analytic result remain **explicit**, the results are in optimum form for design: element values can be chosen so that the results **meet specifications**

Slide set source: D. Middlebrook

Summary

- Avoid solving simultaneous equations
- Follow the signal path from input to output by Thevenin/Norton reduction, voltage/current dividers, and reflection of impedances
- This automatically generates **low entropy expressions**

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