1. Differentiate the function $f(x) = (\sin x)(\cos x)$.

Solution. $f'(x) = \cos^2 x - \sin^2 x$

2. Differentiate the function $f(x) = \dfrac{\sin x}{x}$

Solution. $f'(x) = \dfrac{x\cos x - \sin x}{x^2}$

3. Differentiate the function $f(x) = \cos\left(1 + x^2\right)$.

Solution. $f'(x) = -2x\sin(1 + x^2)$

4. Differentiate the function $f(x) = \sqrt{1 + \sqrt{x}}$.

Solution. $f'(x) = \dfrac{1}{4}x^{-\frac{1}{2}}\left(1 + x^{\frac{1}{2}}\right)^{-\frac{1}{2}}$

5. Calculate the second derivative of the function $f(x) = \dfrac{3x+1}{2x-1}$.

Solution. $f''(x) = \dfrac{20}{(2x-1)^3}$

6. Calculate the second derivative of the function $f(x) = \sin(x^2)$.

Solution. $f''(x) = 2\cos(x^2) - 4x^2\sin(x^2)$

7. Find the second derivative of $f(x) = \frac{1}{1+x^2}$.

Solution. $f''(x) = -2(1+x^2)^{-2}\left[1 + 4x^2(1+x^2)^{-1}\right]$

8. Use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x^2 + 2xy - y^2 + x = 2$.

Solution. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x + 2y + 1}{2y - 2x}$

9. Find an equation for the tangent line to the curve $y = \cos^2 x$ at the point $\left(\frac{\pi}{4}, \frac{1}{2}\right)$

Solution. $y - \dfrac{1}{2} = -\left(x - \dfrac{\pi}{4}\right)$

10. Find an equation for the tangent line to the curve $x^2 + y^4 = 5$ at the point $(2, 1)$.

Solution. $y - 1 = -(x - 2)$

11. If $y = x^2 - 2x + 2$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$, find $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = 3$.

Solution. $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 8$

12. Two people leave a point at the same time. The first person jogs North at 4 m/s and the second person jogs West at 3 m/s. How fast is the distance between the people increasing 2 seconds after they leave?

Solution. $5\dfrac{\mathrm{m}}{\mathrm{s}}$

13. Use a linear approximation of $f(x) = x^{\frac{3}{2}}$ at 4 to estimate the value of $(4.2)^{\frac{3}{2}}$. (You may make use of the fact that $f(4) = 8$).

Solution. $(4.2)^{\frac{3}{2}} \approx 8.6$

14. Determine if the function

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x & \text{if } x \geq 1 \end{cases}$$

is differentiable at $x = 1$.

Solution. The function is **not** differentiable at $x = 1$ (it has a cusp).

15. Use the definition of the derivative to find $f'(1)$ for $f(x) = x^2 - 5x$.

Solution.

$$\begin{aligned} f'(1) &= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} \\ &= \lim_{h\to 0} \frac{(1+h)^2 - 5(1+h) - (-4)}{h} \\ &= \lim_{h\to 0} \frac{1 + 2h + h^2 - 5 - 5h + 4}{h} \\ &= \lim_{h\to 0} \frac{h(-3+h)}{h} \\ &= -3 \end{aligned}$$