

1. Use the midpoint rule with  $n = 3$  to estimate the value of  $\int_{-2}^4 \frac{x}{1+x^2} dx$ .

**Solution.**  $M_3 = 2 \left( \frac{-1}{2} + \frac{1}{2} + \frac{3}{10} \right)$

2. Evaluate the integral  $\int_{-1}^1 x(4+3x) dx$ .

**Solution.**  $\int_{-1}^1 x(4+3x) dx = \int_{-1}^1 4x + 3x^2 dx = 2$

3. Evaluate the integral  $\int_{-3}^3 x^3 dx$ .

**Solution.**  $f(x) = x^3$  is odd, so the answer is 0.

4. Evaluate the integral  $\int_0^1 \sin(\pi x) dx$ .

**Solution.**  $\frac{2}{\pi}$

5. Which is greater,  $\int_{-1}^1 x^2 dx$  or  $\int_{-1}^1 |x| dx$ ?

**Solution.**  $x^2 \leq |x|$  over the interval  $[-1, 1]$  and so  $\int_{-1}^1 x^2 dx \leq \int_{-1}^1 |x| dx$ .

6. Find the average value of the function  $f(x) = \frac{1}{3}x^{\frac{1}{3}}$  over the interval  $[0, 8]$ .

**Solution.**  $\frac{1}{2}$

7. The velocity of an object is  $v(t) = 2t - 6$ . Find the total distance traveled over  $0 \leq t \leq 4$ .

**Solution.**  $\int_1^4 |2t - 6| dt = 10$ .

8. Evaluate the indefinite integral  $\int \frac{1}{\sqrt{x}} + \sec^2 x dx$ .

**Solution.**  $2\sqrt{x} + \tan x + C$

9. Evaluate the indefinite integral  $\int -2 \sin(\theta) \cos^3(\theta) d\theta$ .

**Solution.**  $\frac{1}{2} \cos^4 \theta + C$

10. Differentiate the function  $g(x) = \int_x^0 \frac{\sin t}{t} dt$ .

**Solution.**  $g'(x) = -\frac{\sin x}{x}$

11. Differentiate the function  $g(x) = \int_0^{x^2} \frac{1}{1+t} dt$ .

**Solution.**  $g'(x) = \frac{2x}{1+x^2}$

12. Find the area enclosed between the curve  $y = 1 - x^4$  and the  $x$ -axis.

**Solution.** The area is  $A = \int_{-1}^1 1 - x^4 dx = \frac{8}{5}$ .

13. Evaluate the integral  $\int_{-2}^2 x \cos x dx$

**Solution.**  $f(x) = x \cos x$  is an odd function and consequently  $\int_{-2}^2 x \cos x dx = 0$ .

14. Which of the following are equal?

a)  $\int_{\frac{1}{4}}^1 \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx$

b)  $\int_{\frac{1}{4}}^1 \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$

c)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(x) e^{\sin x} dx$

d)  $\int_{\frac{1}{4}}^{\frac{1}{2}} e^{2x} dx$

e)  $\int_{\frac{1}{2}}^1 2e^x dx$

**Solution.** Items a and c are equal. Items b and e are equal.

15. Differentiate  $y = (e^{\sqrt{x}})^2$ .

**Solution.**  $y' = \frac{e^{2\sqrt{x}}}{\sqrt{x}}$ .

16. Let  $g(x) = \frac{3x}{x+1}$ . Find a formula for  $g^{-1}(x)$ .

**Solution.**  $g^{-1}(x) = \frac{x}{3-x}$ .

17. Let  $y = \sin(\ln x)$ . Find  $y''$ .

**Solution.**  $y'' = \frac{-\sin(\ln x) - \cos(\ln x)}{x^2}$ .

18. Evaluate the integral  $\int \frac{t^2}{4-t^3} dt$ .

**Solution.**  $-\frac{1}{3} \ln |4-t^3| + C$

19. Evaluate the integral  $\int_0^{\frac{\pi}{2}} (\cos x) e^{\sin x} dx$ .

**Solution.**  $e - 1$

20. Differentiate  $y = \frac{x^3 \sqrt{x^2+1}}{(x+2)^5}$ .

**Solution.**  $y' = \frac{x^3 \sqrt{x^2+1}}{(x+2)^5} \left( \frac{3}{x} + \frac{x}{x^2+1} - \frac{5}{x+2} \right)$