1. Sketch the graph of a function that is continuous on the interval $[0, 5]$, has an absolute maximum at $x = 0$, and absolute minimum at $x = 4$, and critical points at $x = 1$ and $x = 3$.

2. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 3x + 1$ over the interval $[0, 3]$

Solution. The function has a maximum value of 19 at $x = 3$ and a minimum value of -1 at $x = 1$.

3. Find the absolute maximum and absolute minimum values of $f(x) = (x^3 - 1)^2$ over the interval $[-2, 2]$.

Solution. The function has a maximum value of 81 at $x = -2$ and a minimum value of 0 at $x = 1$.

4. Find the intervals of increase and the intervals of decrease of the function $f(x) = \dfrac{x^2}{x-4}$.

Solution. The function is increasing on $(-\infty, 0)$ and on $(8, \infty)$ and decreasing everywhere else.

5. Find the intervals on which the function $f(x) = 1 - 3x - 24x^2 + x^4$ is concave up and those on which it is concave down.

Solution. The function is concave down on $(-2, 2)$ and concave up everywhere else.

6. Let f be a continuous function with critical points at $x = -1$ and at $x = 2$ and such that f'' is continuous and $f''(-1) = 4$ and $f''(2) = -1$. Determine if $x = -1$ is a local minimum or maximum and if $x = 2$ is a local minumum or maximum.

Solution. The function has a local minimum at $x = -1$ and a local maximum at $x = 2$.

7. Let $f(x) = \dfrac{x}{x^2 - 1}$. Use the Mean Value Theorem to show that $f'(x) = \frac{1}{3}$ for some x in the interval $[0, 2]$ or explain why the Mean Value Theorem does not apply.

Solution. The MVT does not apply because f is not continuous on the interval.

8. Let $g(x) = x^2 + \sin x$. Use the Mean Value Theorem to show that $g'(x) = \pi$ for some x in the interval $[0, \pi]$ or explain why the Mean Value Theorem does not apply.

Solution. By the MVT there is a number c in $(0, \pi)$ such that $g'(c) = \dfrac{g(\pi) - g(0)}{\pi - 0} = \pi$.

9. Sketch the graph of $f(x) = 8x^2 - x^4$. Clearly indicate the location of all axis intercepts, asymptotes, and local extremes.

10. Sketch the graph of $f(x) = \dfrac{1}{x^2 - 2x}$. Clearly indicate the location of all axis intercepts, asymptotes, and local extremes.

11. Find the positive number x such that $f(x) = 4x^2 + \frac{1}{x}$ is a small as possible.

Solution. $x = \frac{1}{2}$

12. A cylindrical capsule with a total volume of 16π cm^3 has radius r and height h. What radius and height minimize the surface area of the capsule? Hint: The volume of the capsule is $\pi r^2 h$ and its surface area is $2\pi r^2 + 2\pi rh$.

Solution. $r = 2$ cm and $h = 4$ cm.

13. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation of a solution to the equation $x^3 - 2 = 0$.

Solution. $x_2 = \frac{4}{3}$

14. Find the general antiderivative of $f(x) = \dfrac{\sin x}{4} + x^{\frac{3}{4}}$

Solution. $F(x) = -\dfrac{\cos x}{4} + \dfrac{4}{7}x^{\frac{7}{4}} + C$

15. The velocity of a particle at time t is given by the function $v(t) = 3t^2 + 4t$ and after 1 second its position is $p(1) = 1$. Find an equation for the position of the particle at time t.

Solution. $p(t) = t^3 + 2t^2 - 2$